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$\Delta I = 1/2$ RULE FOR MESON DECAYS AND QUARK STATISTICS[1]

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A proof of the $\Delta I = 1/2$ rule for hadronic weak decays has been given for a pure $V-A$ current interaction and pointlike para-quarks [1]. The argument is based on the Fierz transformation, which applied to the outgoing pointlike para—quarks implies an antisymmetric isospin wave function, i. e. $I = 0$. Since there is only one incoming strange quark, one has $I = 1/2$ and the exact rule follows. On the other hand, it is well known that in the quark model the ratio G_A/G_V determines the coefficient of the axial vector current ϱ as 0.74 [2]. A similar result has been obtained recently from chiral $SU(2)$ X $SU(2)$ mixing and the transformation between constituent and current quarks [3]. However, to the case of $\varrho \neq 1$ the Fierz symmetry does not apply.

One can nevertheless investigate the connection between quark statistics and the $\Delta I = 1/2$ rule for an arbitrary renormalization of the axial vector current and any quark form factor $F(q^2)$ if one can compute all the overlap integrals. This is possible in a relativistic spin 1/2 quark model for mesons based on the Bethe-Salpeter equation, which has proved to be quite succesful for electromagnetic decays [4][2]. The 0^- meson Bethe—Salpeter amplitudes are given by [5]

$$\chi(q,p) = \frac{4\pi}{\sqrt{3\beta}}\left(1 + \frac{p\gamma}{M}\right)\gamma_5 \exp\left(-\frac{q_E^2}{2\sqrt{\beta}}\right)|q\bar{q}\rangle \tag{1}$$

where $|q\bar{q}\rangle$ are the corresponding quark wave functions of the mesons, $(2\sqrt{\beta})^{-1}$ the universal Regge slope $\sim$ 1 GeV^{-2} and M the mass of the quarks. One can thus explicitly compute the amplitude for $K^+ \to \pi^+\pi^0$, which is pure $I = 3/2$ and show that it vanishes identically for any ϱ and $F(q^2) = 1$ if para statistics are used. For the usual Cabibbo current—current interaction, the graphs relevant for the process are shown in Fig. 1.

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[2] Flamm D., Kielanowski P., Sánchez J., Electromagnetic meson decays and mixing angles in a relativistic quark model. Ac. Phys. Austr. (in print).

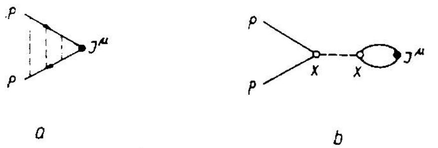

Fig. 1. Diagrams for the $K^+ \to \pi^+\pi^0$ decay. The circles are B-S amplitudes and the dots weak currents. p, n and λ denote three quarks.

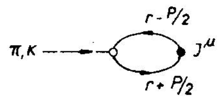

Fig. 2. Diagram for $K_s^0 \to 2\pi$ decay with the ε meson dominance.

The contribution of the graphs c, d, e, f, vanishes because of the Clebsh—Gordan coefficients. Graph (a) involves two loops, while in graph (b) there is one loop only. For para—quarks (a) and (b) give following contribution

$$g_a = -g_b = -\frac{2\sqrt{\beta}\varrho}{\pi\sqrt{3}M}(m_K^2 - m_\pi^2)\, G \sin\Theta\cos\Theta. \tag{2}$$

Therefore, if quarks obey para statistics, the two graphs cancel each other and the decay $K^+ \to \pi^+\pi^0$ is strictly forbidden as required by the $\Delta I = 1/2$ rule, independently of the axial vector current renormalization.

In the case of Fermi statistics, however, the graph (b) has an additional minus sign and (a) and (b) would add to give an $I = 3/2$ amplitude.

As a final remark let us mention, that if one computed along these lines the decay $K_s \to 2\pi$, the result would be too small. One sees the necessity of a dynamic octet enhancement, which can be easily achieved in this model via O^+, $I = 0$ dominance. The corresponding diagram is shown in Fig. 2. The $B-S$ amplitude for O^+ mesons is

$$\chi_{O^+} = \frac{4\pi\sqrt{2}}{3}\beta^{-3/4}\left\{q\gamma - \frac{P\cdot q}{m_\varepsilon^2}P\gamma + O\left(\frac{1}{M}\right)\right\}\exp\left(\frac{-q^2}{2\sqrt{\beta}}\right)|q\bar{q}\rangle. \tag{3}$$

Using the effective weak current discussed in [5] one obtains the following expression for the $K_s \to 2\pi$ amplitude

$$\mathbf{M}(K_s^0 \to \pi^+\pi^-) = \frac{8CF_kG\sin\Theta\cos\Theta}{3\sqrt{3\pi}}\left(1 - \frac{1}{4}\frac{m_K^2}{m_\varepsilon^2}\right)\times$$
$$\times\left\{\frac{m_\varepsilon^2\Gamma_\varepsilon(\varepsilon - 2\pi)\beta^{3/2}}{(m_\varepsilon^2 - 4m_\pi^2)^{1/2}[(m_K^2 - m_\varepsilon^2 + \Gamma_\varepsilon^2/4)^2 + m_\varepsilon^2\Gamma_\varepsilon^2]}\right\}^{1/2} \tag{4}$$

where $C = -0.9$ is the $SU(3)$ breaking parameter which is determined from the $K\mu3$ decay[3]. With $m_\varepsilon = 660 \pm 100$ MeV and $\Gamma_\varepsilon/2 = 320 \pm 70$ MeV [6] one obtains the decay rate $\Gamma(K_s^0 \to \pi^+\pi^-) \simeq C^2 \cdot 10^{-12}$ MeV, which is of the right order of magnitude.

The contribution of Fig. 2 to the $K^+ \to \pi^+\pi^0$ decay vanishes. However, inserting the

[3] Flamm D., Kielanowski P., Sánchez J., Weak meson decays in a relativistic quark model. Ac. phys. slov. *24* (1974).

$SU(3)$ breaking term in the diagrams of Fig. 1 gives the correct order of magnitude for this decay $\Gamma(K^+ \to \pi^+\pi^0) \sim 10^{-14}$ MeV in the case of para—statistics. The deviation from the $\Delta I = 1/2$ rule, responsible for the $K^+ \to \pi^+\pi^0$ decay may thus also be explained by $SU(3)$ breaking.

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