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PROBABILISTIC MODELS FOR THE PROCESSES OF MULTIPARTICLE PRODUCTION[1]

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In this paper we deal with models for multiparticle production which use mainly statistical-probabilistic arguments. We describe especially the stationary and non-stationary Markov models, within the framework of which one can account for several experimental facts by the multiparticle production.

I. INTRODUCTION

Recently a lot of work has been done in constructing theoretical models to explain the data obtained in high-energy inclusive experiments. We can divide these theoretical models into three groups: (*i*) *dynamical models*, the starting point of which are certain assumptions about the dynamical mechanisms of multiparticle reactions, e.g. multiperipheral, multi-Regge, [1] etc. (*ii*) *probabilistic models*, which use mainly statistical-probabilistic arguments to explain the multiparticle production. The following models belong to this class: the Gaussian [2], the information-theoretical [3] and the Markov [4] models. (*iii*) *other models*, in which use is made of various arguments to construct these models, such as, for example, the thermodynamical [5], the string [6] and many other models.

Let us deal more in detail with the probabilistic models. The first of them represents the Wang model [7], which among other things leads to the Poisson multiplicity distribution given by the well-known formula:

$$P(j) = \frac{\lambda^j e^{-\lambda}}{j!} \tag{1}$$

The second and more soplistaceted model is the information-theoretical one [3] in which the use has been made of Jaynes' principle of the maximum missing information [8] in inclusive multiparticle reactions. In the description of the inclusive experiments there are quantities that are sums of distribution functions integrated over many variables. It is, therefore, clear that a considerable amount of information gets lost and in a limit of very high-energy we can apply the Jaynes' principle. All the known phenomena in inclusive reactions can be calculated from the Jaynes' principle with appropriate constraints [3].

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II. THE MARKOV MODEL

The last model here is the Markov model [4, 9] for multiparticle production. In this model one assumes that the particle production is realized via the excited states of the hadrons (resonances) in two basic stages: (*i*) the interaction stage in which the hadrons are excited into higher energy levels by means of the incoming particles; (*ii*) the decay stage in which the excited hadron decays producing secondary particles.

We assume that the transfers between the hadronic excited states represent a Markov process describing by means of the Kolmogorov equations. Let $S_0, S_1, \ldots, S_n$ be the hadronic states and $\mathbf{L}$ the transfer matrix; the Kolmogorov equation of such a Markov system has the form

$$\frac{d\mathbf{p}}{dt} = \mathbf{L}\mathbf{p}, \tag{2}$$

where p represents the vector of the probability distribution of the hadronic states $[p_0, p_1, \ldots, p_n]$ and

$$\mathbf{L} = \begin{pmatrix} \lambda_{00} & \lambda_{01} & \ldots & \lambda_{0n} \\ \lambda_{10} & \lambda_{11} & \ldots & \lambda_{1n} \\ \vdots & & & \\ \lambda_{n0} & \lambda_{n1} & \ldots & \lambda_{nn} \end{pmatrix}.$$

Here $\lambda_{00}, \lambda_{01}, \ldots$ represent the transfer probabilities between corresponding states which are given by the (generally unknown) internal dynamics. Due to selection rules some of the transfer probabilities might be equal to zero. Since by means of the set of transfer probabilities the final multiplicity distribution is given, we can reach from the final multiplicity distribution some important conclusions about the internal hadron dynamics.

Let us suppose that due to a selection rule between the hadronic states, the hadron can change its state only to the next one with the transfer probability $\lambda_{i,i+1}$. This assumption represents a considerable restriction on the internal dynamics of the hadron. The transfer matrix has then the form

$$\mathbf{L} = \begin{pmatrix} -\lambda_0 & 0 & 0 & 0 \dots \\ \lambda_0 & -\lambda_1 & 0 & 0 \\ 0 & \lambda_1 & -\lambda_2 & 0 \\ 0 & 0 & \lambda_2 & -\lambda_3 \\ \vdots & & & \end{pmatrix}$$

and our Markov system [10] is described by the differential equations

$$\begin{aligned} \dot{p}_0(t) &= -\lambda_0 p_0(t) \\ &\vdots \\ \dot{p}_k(t) &= \lambda_{k-1} p_{k-1}(t) - \lambda_k p_k(t) \\ &\vdots \\ \dot{p}_{n-1}(t) &= \lambda_{n-2} p_{n-2}(t) - \lambda_{n-1} p_{n-1}(t) \qquad k = 1, 2, \dots \end{aligned} \tag{3}$$

It can be shown that the system (1*a*) has for the boundary conditions $p_0(t = 0) = 1$ $p_i(t = 0) = 0$, $i = 1, 2, \dots$ the following solution [4]

$$\begin{aligned} p_0 &= \exp(-\lambda_0 t) \\ p_1 &= \exp(-\lambda_1 t) \frac{\lambda_0}{\lambda_1 - \lambda_0} [\exp\{(\lambda_1 - \lambda_0)t\} - 1] \\ &\vdots \\ p_k &= \exp(-\lambda_k t) \Big(\prod_{i=0}^{k-1} \lambda_i\Big) \int_0^t \int_0^{\xi_{k-1}} \dots \int_0^{\xi_3} \int_0^{\xi_2} \exp\Big\{\sum_{i=1}^{k} (\lambda_i - \lambda_{i-1})\xi_i\Big\} \, d\xi_1 \dots d\xi_{k-1}. \end{aligned} \tag{4}$$

Putting $\lambda_0 = \lambda_1 = \lambda_2 = \dots = \lambda$, which represent another restriction on the internal dynamics, we get

$$p_k(t) = \frac{(\lambda t)^k}{k!} \exp\{-\lambda t\} \qquad k = 0, 1, \dots, n,$$

which represents the well-known Poisson distribution (see formula (1)). If the interaction stage has the duration T, then at the end of the interaction we have the probability distribution of the excited hadronic state given as

$$p_k(T) = \frac{(\lambda T)^k}{k!} \exp(-\lambda T) \qquad k = 0, 1, \dots$$

In the decay stage the excited hadron goes over to its ground state whereby the k-th state decays into k secondaries so that the probability p_k represents the probability of finding k secondaries in the final state as well.

As it is known the mean multiplicity by the probability distribution (1) is given as

$$\langle n \rangle = \lambda T .$$

We get so a connection between the mean multiplicity and interaction time. If we take for the mean multiplicity the equation $\langle n \rangle = kE_L^{1/2}$, as it results within some statistical models for multiparticle production [13], then we get

$$T = \frac{kE_L^{1/2}}{\lambda}. \tag{4a}$$

The plot of the experimental mean multiplicities [11] and the square root of laboratory energy is shown in Fig. 1. We see that the function $\langle n \rangle = kE_L^{1/2}$ fits the experimental values well so that the relation (4*a*) holds within a large range of the laboratory energies. Eq. (4*a*) shows that the energy of primary particles and the interaction time are functionally connected.

If the Markov process is large enough, we can consider this process to be stationary. In order to obtain a nontrivial solution, we change slightly the Markov system taking the transfer matrix as follows

$$\mathbf{L}' = \begin{pmatrix} -\lambda & \mu & 0 & 0 & 0 & \dots \\ 0 & -\lambda & \mu & 0 & 0 & \\ 0 & 0 & 0 & -\lambda & \mu & 0 \\ \vdots & & & & & \end{pmatrix}$$

The equation (3) has then the form

$$\begin{aligned} \dot{p} &= -\lambda p_0 + \mu p_1 \\ &\vdots \\ \dot{p}_i &= -\lambda p_i + \mu p_{i+1} \qquad i = 1, 2, \dots, n . \end{aligned} \tag{5}$$

We get the stationary solution of equations (5), if we put $\dot{p}_0 = \dot{p}_1 = \dots = \dot{p}_n = 0$. Doing this one obtains ($\varkappa = \lambda/\mu$) $p_1 = \varkappa p_0, \dots, p_{i+1} = \varkappa p_i$, or $p_n = (\varkappa)^n p_0$.

Since $\Sigma p_n = 1$, $p_0 = (1 - \varkappa)$ and

$$p_n = \varkappa^n (1 - \varkappa) . \tag{6}$$

By means of the formula (6), the elements of probability distribution of multiplicity are given and we obtain a power probability distribution. The mean multiplicity is given as

$$\langle n \rangle = \sum_{n=0}^{\infty} (n + 1) p_n = 1/(1 - \varkappa) ,$$

the square of dispersion

$$D^2 = \sum_n (n + 1 - \langle n \rangle)^2 p_n = \langle n \rangle + \varkappa^2 \langle n \rangle^2$$

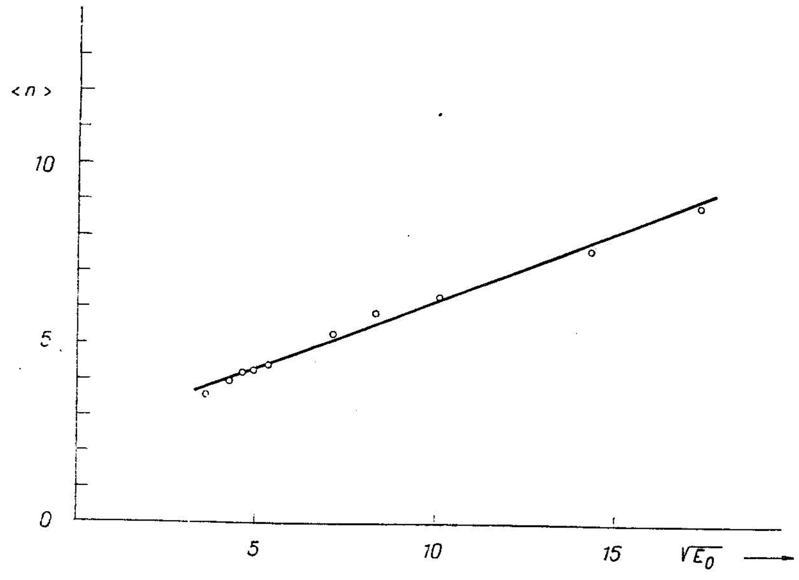

Fig. 1. Mean multiplicity versus square root of laboratory energy E_0.

and the ratio

$$\frac{\langle n\rangle}{D^2} = \frac{\langle n\rangle}{\langle n\rangle + \varkappa^2\langle n\rangle^2}\,.$$

The Wroblewski relation $D = A\langle n\rangle + B$ [12], in our case $D = \varkappa\langle n\rangle + (1/2)\varkappa^{-1}$, is satisfied for the stationary Markov process as far as $\langle n\rangle \gg 1$. In this model we can easily introduce the concept of temperature. Putting $\ln \varkappa = -E_0/kT$ and assuming that the energy of the n-th hadronic state $E_n = nE_0$, we get

$$p(E_n) = \frac{\exp\left\{-\dfrac{nE_0}{kT}\right\}}{\left[1 - \exp\left\{-\dfrac{E_0}{kT}\right\}\right]^{-1}}$$

which represents Gibb's energy distribution of the hadronic states. This energy distribution is supposed within some models for multiparticle production, e.g. in the thermodynamic one [5]. How near to the stationary state

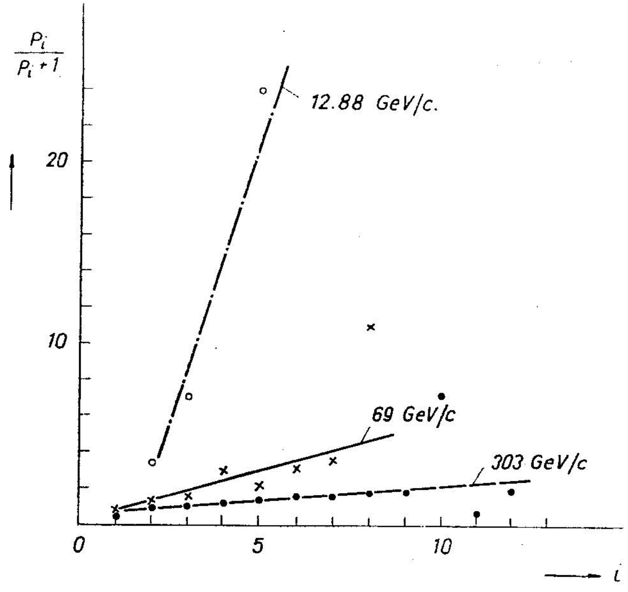

Fig. 2. The ratio p_i/p_{i+1} plotted against the multiplicity i. Parameter is the laboratory energy.

the Markov process is can be shown by means of the dependence of the ratio $\gamma_i = p_i/p_{i+1}$ on the multiplicity. This dependence is plotted in Fig. 2. We see that we get a strong dependence for the ratio γ on the multiplicity for a low laboratory energy $E = 12$ GeV, where for the highest energy this ratio is practically constant. This suggests that the higher the energy of the incoming particle is, the nearer to the stationary is our Markov process.

III. CONCLUSIONS

From what has been said so far it follows that (*i*) In a non-stationary stage of the Markov model of multiparticle reactions we get the Poisson multiplicity distribution. (*ii*) The stationary Markov model can be applied at the highest accelerator energy.

The described models represent only the simplest of the class of the general Markov processes, it may be therefore expected that it can be extended and then one could obtain a better agreement with the data.

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