WEAK MESON—DECAYS IN A RELATIVISTIC QUARK—MODEL[1]

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Böhm, Joos and Krammer [1] have recently solved the Bethe-Salpeter (*B-S*) equation n the limit of the small meson $q\bar{q}$-system bound by a 4-dimensional harmonic—oscilator potential of the form

$$K(R) = \alpha + \beta R^2,\ R = (x_4^2 + \vec{x}^2)^{1/2}. \tag{1}$$

Choosing for the spinor structure of the interaction the combination pseudoscalar, vector, scalar $(P+V+S)$ they have been able to reproduce the experimental mass spectrum of mesons. For the groudn state pseudoscalar-and-vector-mesons the *B-S* amplitudes in this model aer as follows

$$\chi_P(r, p) = \frac{4\pi}{\sqrt{3\beta_P}}\left(1 + \frac{p\gamma}{M}\right)\gamma_5 \exp\left(-r^2/2\sqrt{\beta_P}\right) \mid q\bar{q}\rangle \tag{2}$$

$$\chi_V(r,p) = \frac{4\pi}{\sqrt{3\beta_V}}\left\{\left(1 + \frac{p\gamma}{M}\right)\varepsilon_V - \frac{i\varepsilon_V r}{M}\right\}.$$

$$. \exp\left(-r^2/2\sqrt{\beta_V}\right) \mid q\bar{q}\rangle_V \tag{3}$$

where r is the relative momentum of the quarks,

$$(2\sqrt{\beta_P})^{-1} \simeq (2\sqrt{\beta_V})^{-1} = \alpha' \simeq 1\ \mathrm{GeV}^{-2} \tag{4}$$

is the universal Regge slope and $q\bar{q}$ is the $SU(3)$ part of the amplitude and ε_V the polarization vector of the vector mesons. Using these amplitudes and an effective electromagnetic current for the constituent quarks we have obtained good results[2] [2] for the radiative meson decays. It is thus interesing to study the weak decays of mesons by the same method.

Since hadrons are bound states of constituent quarks, we need the matrix element of the

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[1] Talk given at the Triangle Meeting on Weak Interactions at SMOLENICE, June 4—6, 1973 by D. Flamm.

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[2] Flamm D., Kielanowski P., Sánchez J., Preprint HEP IV/73. Institute of High Energy Physics, Vienna and Ac. Phys. Austr. (in print).

weak current beetwen the constituent quarks. As has been stressed by Gell—Mann[3], there is an important difference between constituent and current quarks. We assume the following form for the effective matrix element of the weak current

$$\langle q \text{ constituent } | J^{\text{weak}}(0) | \, q \text{ constituent} \rangle = N\{u_p[\gamma_\mu F_1(q^2) + \varrho\gamma_\mu\gamma_5 F_2(q^2)]u_n \cos\vartheta_c + u_p[\gamma_\mu F_3(q^2) + \frac{c}{2M} q^\mu F_4(q^2) + \varrho\gamma^\mu\gamma_5 F_5(q^2)u_\lambda \sin\vartheta_c\} \quad (5)$$

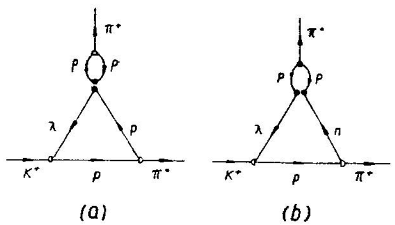
In eq. (5) N denotes the usual normalization factor. $F_i(q^2)$ $i = 1, \ldots, 5$ are form factors which take into account the quark-quark interactions at the vertex. The interaction which is responsible for the $q\bar{q}$ bound states could be accounted for by the meson spectrum itself, suggesting pole dominance for the form-factors as indicated by Fig. 1. ϱ is the axial vector renormalization. $c/2M$ is the coefficient of the $SU(3)$ breakig term in $\Delta S = 1$ part of the vector current. The inclusion of the $SU(3)$ breaking is necessary if one intends to account for the $K_{\mu 3}$ form faktor $f_-(q^2)$ and for the $K^0_s \to 2\pi$ decay rate within a current-current picture. Indeed such a term can be derived using the mechanism shown in Fig. 1b.

The first decays to test this model are the leptonic decays $\pi \to \mu\nu(e\nu)$ and $K \to \mu\nu(e\nu)$ for which the relevant diagram is shown in Fig. 2. These loop diagrams are finite, due to the cut—off provided by the B-S amplitude in contrast to the corresponding ordinary Feynman diagrams. One finds

$$F_\pi = \frac{4}{\pi\sqrt{3}} \frac{\sqrt{\beta}}{M} \varrho F_2(m_\pi^2) \quad (6)$$

and

$$F_K = \frac{4}{\pi\sqrt{3}} \frac{\sqrt{\beta}}{M} \varrho F_5(m_K^2). \quad (7)$$



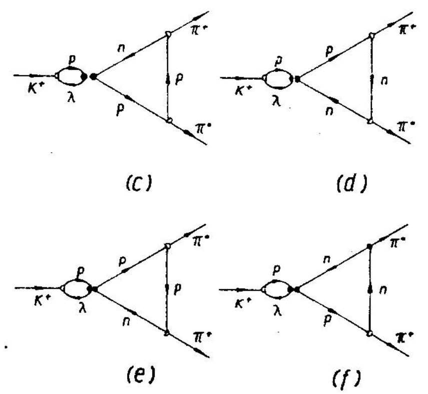
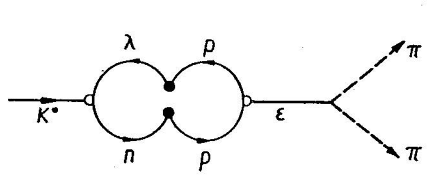


Fig. 1. The quark—quark interactions (a) are approximated by the meson spectrum (b). The circles are the B—S amplitudes and the dots weak currents.

[3] Gell—Mann M., Lectures at the Schladming Winter School 1972. Act. Phys. Austr., Suppl. IX (1972), 733.

Fig. 2. Diagrams for the decays $\pi \to \mu\nu(e\nu)$ and $K \to \mu\nu(e\nu)$. r is the relative and p the total momentum of the $q\bar{q}$ system.



The effective quark mass M is a parameter of the model which should be the same for similar processes and thus can be eliminated by taking appropriate ratios. For F_π/F_K we obtain, for instance, using A_1-meson dominance for $F_2(q^2)$ and K_A-meson dominance for $F_5(q^2)$ in Eqs. (6) and (7)

$$\frac{F_K}{F_\pi} = \frac{1 - \frac{m_\pi^2}{m_{A_1}^2}}{1 - \frac{m_K^2}{m_{K_A}^2}} = 1.17 \quad (8)$$

in excelent agreement with the experimental value[4] $F_K/F_\pi = 1.18 \pm 0.003$. As one can see from Eq. (8) there is a small $SU(3)$-symmetry breaking due to the weak form factors but the Weisskopf—Van Royen paradox [3] does not arise.

Next consider the semileptonic decays $\pi^+ \to \pi^0 e^+ \nu$ and $K \to \pi\mu\nu(e\nu)$. Here the relevant graphs are given in Fig. 3. For these processes the matrix element does not depend on the quark mass up to terms of the order $(m/M)^2$. For the pion β-decay we find using the notation of [4]

$$ {}^{\pi}_{+}(q^2) = \sqrt{2} \frac{1}{1 - \frac{q^2}{m_\varrho^2}} \quad (9)$$

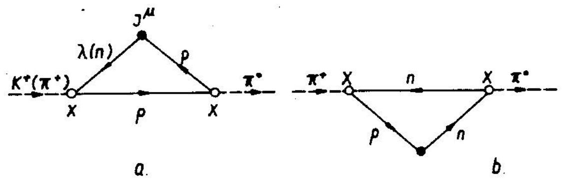


Fig. 3. Diagrams for the decays $K \to \pi e\nu(\mu\nu)$ and $\pi^+ \to \pi^0 e\nu$. p, n and λ denote the three quarks. To the K-decay only (a) contributes.

assuming ϱ dominance for $F_1(q^2)$. For the $K_{\mu 3}$ decay we find

$$f_+^k(q^2) = F_3(q^2) \exp(q^2/16\sqrt{\beta})$$

$$f_-^k(q^2) = \frac{2C}{3} F_4(q^2) \exp(q^2/16\sqrt{\beta}). \quad (10)$$

[4] Pascual P., CERN Yellow report 68-23, p. 125 and "K decays", GIFT Report 1972 Zaragoza University.

Our result in eq. (10) includes al [illegible] integrals. The $SU(3)$ symetric result

$$[illegible](0) = 1 \quad (11)$$

is thus obtained in a non trivial [illegible] to corrections of the order $m/6M^2$. Futher we obtained

$$[illegible] = \frac{2}{3}C. \quad (12)$$

There are contradictory experim[illegible] but all recent results based on $K_{\mu 3}/Ke3$ spectra analized without putting $\lambda_+ = 0$ [illegible] between -0.35 ± 0.22 and -0.8 ± 0.5 [5]. From the experiment with λ_+ closest [illegible] prediction [6], which gives $\xi = -0.62 \pm 0.28$, one has

$$[illegible] = -0.93 \pm 0.42. \quad (13)$$

This value of C also gives the [illegible] results for $K \rightarrow 2\pi$ decay rates[5]. Both $f_-(0)$ and the $K \rightarrow 2\pi$ amplitudes are in fa[illegible] proportional to C. They are therefore explained by the same symmetry breaking pa[illegible] and vanish in the limit of exact $SU(3)$ $(C=0)$. Expanding $f_+^k(q^2)$ in powers of q^2 [illegible]

$$[illegible] = 0.027 \quad (14)$$

which is in good agreement with [illegible] experimental value $\lambda_+ = 0.028 \pm 0.005$ [5]. For the coefficient of the q^4 term we ob[illegible] value 0.0007. Results on hadronic weak meson decays are reported in note-.

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Note added in proof: In a la[illegible] ..Weak meson decays in a quark current $\otimes$ current picture with symmetry br[illegible] HEP VI/1973 and Nuovo Cimento (in print) we were able to estimate theoreti[illegible] symmetry breaking coefficient C and found

$$[illegible] = -\frac{[illegible]}{[illegible]} \frac{m_k^2 - m_\pi^2}{m_{k^*}^2},$$

which is in better agreement with [illegible] measurements of $f_-(0)$.

[illegible]RENCES

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[5] Flamm D., Kielanowski P., [illegible] *24* (1974) and Nuovo Cimento (in print)