acta phys. slov. 24 (1974), No. 3

Letters to the Editor

CP VIOLATION AND UNITARITY PROBLEM IN THE K^0 DECAY[1]

MILOŠ LOKAJÍČEK*, VOJTĚCH KUNDRÁT*, VÁCLAV ALDA**, Praha

Positivity of energy is inconsistent with other assumptions normally used in the description of decay processes. Some consequences of this requirement for the K^0 decay problem are considered.

The time evolution of any quantum-mechanical physical system is usually described by the pair $\{\mathscr{H}, U(t)\}$, where $\mathscr{H}$ is a Hilbert space defining all possible states of this system and $U(t)$ is a corresponding evolution operator. If the given pair is to describe a decay of an unstable particle it is required as a rule that $\mathscr{H}$ and $U(t)$ exhibit the following properties:

$$\mathscr{H} = \mathscr{H}_A \oplus \mathscr{H}_D \tag{1}$$

$$U(0) = 1, \; U(t + t') = U(t)\, U(t'), \; \forall t, \; t' \geqslant 0 \tag{2}$$

$$A(t + t') = A(t)\, A(t'), \forall t, \; t' \geqslant 0 \tag{3}$$

where

$$A(t) = P_A U(t) P_A$$

and P_A is the projection operator into the subspace $\mathscr{H}_A$;

$$U^+(t)\, U(t) = U(t)\, U^+(t) = 1, \; \forall t \geqslant 0; \tag{4}$$

$$A^+(t)\, A(t) \neq P_A, \; t > 0; \tag{5}$$

$$\langle \psi, H\psi \rangle \geqslant 0, \; \forall \psi \in \mathscr{H}, \tag{6}$$

where H is defined by

$$U(t) = e^{-iHt} \tag{6a}$$

According to Eq. (1) the space $\mathscr{H}$ is divided into two mutually orthogonal subspaces: $\mathscr{H}_A$ consists of all internal states of the unstable particle and $\mathscr{H}_D$ of all possible decay-

---

[1] Talk given at the Triangle Meeting on Weak Interactions at SMOLENICE, June 4—6, 1973.

* Fysikální ústav ČSAV, Na Slovance 2, 180 40 PRAHA 8, Czechoslovakia.
** Matematický ústav ČSAV, Národní 18, 110 00 PRAHA 1, Czechoslovakia.

-product states of it. Eq. (2) corresponds to the requirement of time homogeneity and the condition (3) guarantees an exponential decay law of the given unstable particle. The unitarity condition (4) is a consequence of the probability conservation and superposition principle. The condition (5) ensures that the particle described by $\mathscr{H}_A$ actually decays and the last condition expresses the fact that the total energy of any physical state should be positive.

We find the first five conditions also in the present description of the K^0-meson decay[2]. In this case the subspace $\mathscr{H}_A$ is assumed to be two-dimensional and the condition (3) has the consequence that two states exist in $\mathscr{H}_A$ which exhibit a strict exponential decay. The consequences of the condition (6) have not yet been examined in connection with the K^0 problem although the given physical system should exhibit such a property as well. And this is just the problem we will follow now.

It was shown in some papers [1, 2] that the conditions (1–6) are mutually inconsistent. This inconsistency does not disappear even if the condition (3) is limited to a finite interval of positive t, t'[3]. And we must ask which of the given assumption should be abandoned or at least weakened. By a detailed examinatoin we can come to the conclusion that there are practically only two possibilities: the condition (3) or the superposition principle which represents a part of the condition (4).

Let us suppose first that the condition (3) is abandoned. In such a case no state of $\mathscr{H}_A$ need exhibit an exponential decay and, e.g. in the K^0 problem no necessity exists to relate the delayed two-pion decay to a violation of the CP symmetry. And we must conclude that abandoning Eq. (3) does not give any possibility of taking any standpoint as regards the problem of the CP violation.

Let us turn now to the other possibility. Recently, two papers [3][4] have appeared which have attempted to build up a quantum theory without the superposition principle which would, however, conserve probability. The classes of non-unitary theories proposed in these papers do not fulfil the condition (2). We will now mention another class of such theories which will satisfy all assumptions introduced at the beginning with the exception of (4), which will be replaced by a weaker one.

First, it is necessary to introduce some consequence following from the first three conditions (1–3) only. The space $\mathscr{H}_D$ can be divided into two orthogonal subspaces (see Theorem 1 of[3]

$$\mathscr{H}_D = \mathscr{D}_- \oplus \mathscr{D}_+,$$

with the following properties

$$\langle \mathscr{D}_- \mid U(t)\mathscr{H}_A\rangle = \langle \mathscr{D}_- \mid U(t)\mathscr{D}_+\rangle = \langle \mathscr{H}_A \mid U(t)\mathscr{D}_+\rangle = 0.$$

It means that the pair $\{\mathscr{H}, U(t)\}$ describes a physical system with an irreversible character.

Now, instead of (4) we can introduce the weaker condition

$$\|U(t)b_\pm\| = \|b_\pm\|, \ \forall t \geqslant 0, \ \forall b_\pm \in \mathscr{B}_\pm, \qquad (4')$$

[2] See e.g. Caneshi L., Van Hove L., CERN 67–27, Geneva 1967, Steinberger J., CERN 70–1, Geneva 1970.

[3] See Alda et al., *Exponential decay law and irreversibility of decay and collision processes*, to appear in Apl. mat. *19* (1974).

[4] Eberhard P., CERN 72–1, Geneva 1972.

where $\mathscr{B}_\pm$ is a special subset (containing a basis) of $\mathscr{D}_\pm$. Condition (4′) means that all states of $\mathscr{B}_\pm$ can be physically realizable but it is not necessary that also their superpositions be physically realizable states. It follows from (1–4′–5) that the pair $\{\mathscr{H}, U(t)\}$ fulfils all conditions the scattering theory of Lax and Phillips [4] is based on.

The mathematical details are given in[5] and we cannot repeat them here. Instead we would like to mention a model which fulfils the requirements of this class of nonunitary theories and to apply it to the K^0 meson. It seems to be quite natural to suppose that an unstable particle can exist in different internal states, which can be regarded to some extent as bound states of other particles and that at least some of these internal states correspond in their structures to the individual decay modes of the given unstable particle. In other words, we shall suppose that the corresponding decay particles are first formed inside the unstable particle and only then a decay can happen. It is, of course, possible that the given internal state will pass to another one before the given mode of decay can occur.

This model applied to the K^0-meson decay is schematically represented in Fig. 1. For simplicity reasons the less frequent decay modes have been ommitted and also various kinds of leptons have not been distinguished. Then the subspace $\mathscr{H}_A$ can be taken as six-dimensional; it contains states with $S = \pm 1$ and $S = 0$. The decay can proceed, however, only from the states with $S = 0$. We have also assumed that direct transitions $|\Delta S| = 2$ are not possible and that transitions to semileptonic internal states are governed by the rule $\Delta S = \Delta Q$. As the semi-leptonic internal states are not eigen-states of the CP operator it is evident that the delayed 2π-decay can occur even if the CP symmetry is preserved. The decay curve depends on the transition rates between different states and a corresponding interference term can also be easily obtained. And we find that condition (6) leads to quite a new situation which gives no reason for the conclusion that the CP symmetry is really violated in the K^0-meson decay.

There is, of course, another fact which relates to the CP violation; it is the charge asymmetry of the semi-leptonic decay modes. If the CP symmetry is conserved, then the model given in Fig. 1 should be symmetrical as regards the decay into particles and antiparticles. And so in contrast to the commonly used model the charge asymmetry

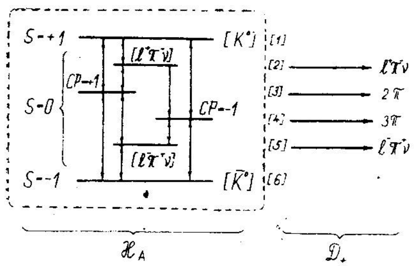

Fig. 1. Irreversible decay scheme of the K^0-meson.

[5] See the note on page xxx and Alda X. et al. *Superposition principle and quantum mechanical description of decay and collision processes.* To appear in Proc. 3. Symp. High En. Phys. Sinaia (Romania) 3. – 10. Oct. 1973.

to the semi-leptonic decay modes should change its sign according to whether the K^0 object is prepared in the state $S = +1$ or $S = -1$. All existing experiments were performed in a neutral beam of an accelerator where the states $S = +1$ prevailed. And so the measurement of this charge assymmetry for a beam prepared originally in the state $S = -1$ could bring a result important not only for the K^0 problem but also for the general problem of the decaying particles.

REFERENCES

[1] Williams D. N., Comm. Math. Phys. *21* (1971), 314.
[2] Horwitz L. P. et al., J. Mat. Phys. *12* (1971), 2537.
[3] Шапиро И. С., Яд. Физ. *16* (1972), 1318.
[4] Lax P., Phillips R. S., *Scattering theory*. New York 1967.

Received December 4th, 1973