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REMARKS ABOUT A SPONTANEOUS BREAKDOWN OF HADRONIC SYMMETRIES[1]

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It is shown that the use of more multiplets of scalar fields which all transform according to the complex fundamental representations of the $U(n)$ group in a plasmon generating mechanism of the $U(n)$ gauge group has to be taken with stipulation. The same holds when using (again for the $U(n)$ gauge group) a scalar multiplet transforming according to the $(n, n^*) + (n^*, n)$ representation of the $U(n)$ group.

I. INTRODUCTION

At present much attention is payed to the development of unified gauge models of weak and electromagnetic interactions [1, 2] which appeared (in the leptonic case) to be very attractive and convincing[2]. However, in the description of strongly interacting particles it is desirable to respect already existing and succesful strong interaction symmetries[3], namely the chiral $SU(3) \times SU(3)$ group. There are papers dealing with this group (and the $SU(3)$ group itself) within the framework of the plasmon generating mechanism (PGM) [3] and using it when describing the weak and electromagnetic interactions of hadrons.

It is the aim of the present note to show that the use of more complex fundamental representations ($CFRs$) as suggested in [4] is not satisfactory for giving the mass to all n^2 Yang-Mills (YM) gauge fields of the $U(n)$ group because of lacking the support of a general principle.

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[2] A comprehensive list of references on this subject is given in the paper of Lee B. W., preprint NAL-THY-92, October 1972.

[3] Lipkin H. J., preprint NAL-THY-85, September 1972.

II. METHOD

The question of a choice of the auxiliary scalar multiplet in *PGM* of the $U(n)$ group arises — how to obtain either all n^2 *YM* fields to be massive or to obtain just one massless „photon“. In [4] an answer is found in the framework of *CFR*. Since „photon“ corresponds to that generator of the group which annihilates the vacuum it is shown by construction that the use of the *CFR* leads to $(n\text{-}1)^2$ massless *YM* fields. This result is to be expected and is connected with the very important characteristics of *PGM*, namely the canonical number of Bludman and Klein [5].

Let N be the number of real scalar fields and ν (the canonical number) is the maximum number of the real scalars which may have the vacuum expectation value (*VEV*) different from zero. If $N - \nu \leqslant g$, where g is the number of generators (also the number of *YM* gauge fields), then $N - \nu$ is equal to the number of Goldstone bosons (*GB*) and simultaneously to the number of massive gauge fields having implied *PGM*. Applied to the *CFR* of the group $U(n)$ it means that $N = 2n$, $g = n^2$; therefore the number of massless gauge fields is $n^2 - (2n - \nu)$. Beacuse this number was found to be $(n-1)^2$, it follows that $\nu = 1$. Such a result is quite natural, because ν is also the number of algebraically independent invariants which can be formed out of the given representation.

Furthermore it was suggested[4] to use n *CFR* for giving a mass to all n^2 gauge particles and $n - 1$ *CFR* for giving a mass to $n^2 - 1$ ones (one „photon“). Such a construction supposes that each *CFR* can have the nonzero vacuum expectation value at its own individual position. However, with the help of a constant gauge transformation one is able to displace all the *VEVs* into the common place. It is then possible to form linear combinations of these new fields in such a way that only one multiplet will have nonzero *VEV* and will contribute to the *PGM*, so that again $(n-1)^2$ massless „photons“ remain.

Let us demostrate the sketched procedure in the case of the $U(3)$ group. The standard Lagrangian can be written as

$$-\tfrac{1}{2}\Phi^+_{(1)}\left(\partial_\mu + \frac{i}{2}\, g\lambda_k A^k_\mu + \frac{i}{2}\, g' B_\mu\right)\left(\partial_\mu - \frac{i}{2}\, g\lambda_l A^l_\mu - \frac{i}{2}\, g' B_\mu\right)\Phi_{(1)} -$$

$$- \tfrac{1}{4} F^k_{\mu\nu} F^k_{\mu\nu} - \tfrac{1}{4} F_{\mu\nu} F_{\mu\nu} + V(\Phi_{(1)}), \qquad (1)$$

where $V(\Phi_{(1)})$ is an invariant polynomial constructed out of the possible fundamental invariant $\Phi^+\Phi$ only. If we put

[4] An essentially the same mechanism as that in [6] was independently suggested by de Wit B.

$$\langle \Phi_{(1)} \rangle_0 = \begin{pmatrix} 0 \\ 0 \\ a \end{pmatrix},$$

we obtain the mass term of gauge fields in the form

$$\tfrac{1}{8} a^2 g^2 \{(A^4_\mu)^2 + (A^5_\mu)^2 + (A^6_\mu)^2 + (A^7_\mu)^2\} +$$

$$+ \tfrac{1}{2} a^2 (\tfrac{1}{4}(g')^2 + \tfrac{1}{3} g^2) \left\{ \frac{\tfrac{1}{2} g'}{\sqrt{\tfrac{1}{4}(g')^2 + \tfrac{1}{3} g^2}} B_\mu - \frac{\frac{1}{\sqrt{3}}\, g}{\sqrt{\tfrac{1}{4}(g')^2 + \tfrac{1}{3} g^2}} A^8_\mu \right\}^2. \qquad (2)$$

There remain $(3-1)^2 = 4$ massless *YM* fields A^1_μ, A^2_μ, A^3_μ and

$$\frac{\frac{1}{\sqrt{3}}\, g}{\sqrt{\tfrac{1}{4}(g')^2 + \tfrac{1}{3} g^2}} B_\mu + \frac{\tfrac{1}{2} g'}{\sqrt{\tfrac{1}{4}(g')^2 + \tfrac{1}{3} g^2}} A^8_\mu .$$

Suppose further there exists also a multiplet $\Phi_{(2)}$ for which

$$\langle \Phi_{(2)} \rangle_0 = \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} \qquad (3)$$

The Lagrangian is the same as (1), only due to the form of the *VEV* (3) the mass term of *YM* fields becomes

$$\tfrac{1}{8} b^2 g^2 \{(A^1_\mu)^2 + (A^2_\mu)^2 + (A^6_\mu)^2 + (A^7_\mu)^2\} + \tfrac{1}{8} b^2 ((g')^2 + \tfrac{4}{3} g^2) \left\{ \frac{g'}{\sqrt{(g')^2 + \tfrac{4}{3} g^2}} B_\mu - \right.$$

$$\left. - \frac{g}{\sqrt{(g')^2 + \tfrac{4}{3} g^2}} A^3_\mu + \frac{\frac{1}{\sqrt{3}}\, g}{\sqrt{(g')^2 + \tfrac{4}{3} g^2}} A^8_\mu \right\}^2. \qquad (4)$$

Indeed, formal unification of the mass terms (2) and (4) results in 8 massive gauge fields (one „photon“, so called $n - 1$ scheme). However, the linear combinations of the fields A^3_μ, A^6_μ and B_μ are not orthogonal; moreover, we can perform a constant rotation

$$\Phi'_{(2)} = \exp\left(i \frac{\pi}{2} \lambda_7\right) \Phi_{(2)}$$

(and the corresponding redefinition of the gauge fields), which leaves the Lagrangian invariant and

$$\langle \Phi'_{(2)} \rangle_0 = \begin{pmatrix} 0 \\ 0 \\ -b \end{pmatrix},$$

which means that the structure of the *PGM* for $\Phi'_{(2)}$ is the same as that of $\Phi_{(1)}$. Therefore the resulting mass term (using both multiplets $\Phi_{(1)}$ and $\Phi'_{(2)}$ is the same as (2) with the substitution $a^2 \to a^2 + b^2$. But this clearly corresponds to the use of the *PGM* for one scalar multiplet

$$\Psi_{(1)} = \frac{1}{\sqrt{a^2 + b^2}} (a\Phi_{(1)} - b\Phi'_{(2)})$$

with

$$\langle \Psi_{(1)} \rangle_0 = \begin{pmatrix} 0 \\ 0 \\ \sqrt{a^2 + b^2} \end{pmatrix}$$

while an orthogonal linear combination

$$\Psi_{(2)} = \frac{1}{\sqrt{a^2 + b^2}} (b\Phi_{(1)} + a\Phi'_{(2)})$$

has $\langle \Psi_{(2)} \rangle_0 = 0$ and does not contribute to the *PGM*.

For a more detailed analysis the n *CFR* are written in the form (see [4]) of the $n \times n$ complex matrix Φ, but it is possible to conclude from the form of the gauge transformations that the representation in question is rather $(n, n^*) + (n^*, n)$ of the group $U(n) \times U(n)$. Here, only one $U(n)$ group is treated locally (together with $n^2 Ym$ fields), the other $U(n)$ group is only global (constant group parameters):

$$\Phi \to (SS_B)^{-1}\Phi; \qquad \Phi^+ \to \Phi^+(SS_B)$$
$$\Phi \to \Phi(SS_B); \qquad \Phi^+ \to (SS_B)^{-1}\Phi^+,$$

where

$$S = \exp (i \sum_{\alpha=1}^{n^2-1} k^\alpha \lambda^\alpha), \qquad S_B = \exp ik^{(n^2)},$$

λ^α are the generators of the $SU(n)$ group in the *CFR*.

The canonical number and therefore the number of Goldstone bosons can (and in fact must) be determined regardless of gauge transformations being global or local. According to Cabibbo, Gatto and Zemach [7] every complex matrix can be written as

$$\langle \Phi \rangle_0 = U^+ \langle \Phi_D \rangle_0 V,$$

where U and V are suitable constant unitary matrices and $\langle \Phi_D \rangle_0$ is a diagonal positive definite matrix. From this it is clear that the canonical form of the *VEV* of the multiplet Φ can be chosen as $\langle \Phi_D \rangle_0$ and therefore $\nu = n$[5]. Since the number of all real scalar fields is $2n^2$, we may conclude (because $2n^2 - n \leqslant 2n^2$) that the number of *GB* is $2n^2 - n$. When treating the entire group $U(n) \times U(n)$ locally (it means together with $2n^2$ *YM* fields), the *PGM* will guarantee that all $2n^2 - n$ *GB* will be absorbed by gauge fields, $2n^2 - (2n^2 - n) = n$ of them remaining massless.

If we assume only half of the symmetry group (i.e. $U(n)$) to be local (only n^2 *YM* particles are present), it is not surprising that all n^2 gauge fields become massive; however, there remain $(2n^2 - n) - n^2 = n(n-1)$ Goldstone bosons.

Let us illustrate the situation briefly by the group $U(3) \times U(3)$. The number of *YM* fields is 18, the number of real scalar fields of the $(3,3^*) + (3^*, 3)$ representation is 18 as well. According to the general discussion $\nu = 3$, i.e. $\langle \Phi \rangle_0$ has the form of a real diagonal matrix and can be written as

$$\langle \Phi \rangle_0 = \varepsilon_0 \lambda_0 + \varepsilon_3 \lambda_3 + \varepsilon_8 \lambda_8,$$

where λ_k are the usual Gell-Mann matrices, $\lambda_0 = \sqrt{2/3}\, 1$. Such a form corresponds to three independent invariants, which can be formed out of Φ [8]:

$$i_1 = \mathrm{tr}\Phi^+\Phi$$
$$i_2 = \mathrm{tr}\Phi^+\Phi\Phi^+\Phi \qquad (5)$$
$$i_3 = \mathrm{tr}\Phi^+\Phi\Phi^+\Phi\Phi^+\Phi.$$

The number of *GB* is clearly 15 and according to the *PGM* there should exist 15 massive *YM* fields and three massless ones. The initial Lagrangian can be written as follows:

$$- \tfrac{1}{2}\mathrm{tr}(\partial_\mu \Phi^+ - \tfrac{1}{2}ig\Phi^+\lambda_k C^k_\mu + \tfrac{1}{2}ig\lambda_k\Phi^+B^k_\mu - \tfrac{1}{2}ig(C^0_\mu - B^0_\mu)\lambda_0\Phi^+) \times$$
$$\times (\partial_\mu \Phi + \tfrac{1}{2}ig\lambda_l\Phi C^l_\mu - \tfrac{1}{2}ig\Phi\lambda_l B^l_\mu + \tfrac{1}{2}ig(C^0_\mu - B^0_\mu)\lambda_0\Phi) +$$
$$+ V(\Phi) + L_{free}(B^k_\mu) + L_{free}(C^k_\mu) + L_{free}(C^0_\mu) + L_{free}(B^0_\mu).$$

[5] For the group $SU(n) \times SU(n)$ the canonical number should be $\nu = n + 1$, since $\langle \Phi \rangle_0 = \exp(i\varphi) U^+ \langle \Phi_D \rangle_0 V$, where U and V are unitary and unimodular. The remaining independent invariant stems from the complex phase.

For the sake of simplicity we assume only one common coupling constant

$$g_{SU(3)_L} = g_{SU(3)_R} = g_{U(1)_L} = g_{U(1)_R} = g.$$

The resulting mass term of the gauge fields reads as

$$\begin{aligned}
&\tfrac{1}{4}g^2\varepsilon_3^2\{(V^1_\mu)^2 + (V^2_\mu)^2\} + \tfrac{1}{4}g^2\left(\sqrt{\frac{2}{3}}\,\varepsilon_0 + \frac{1}{\sqrt{3}}\,\varepsilon_8\right)^2 \{(A^1_\mu)^2 + (A^2_\mu)^2\} + \\
&+ \tfrac{1}{4}g^2\left(\tfrac{1}{2}\varepsilon_3 + \frac{\sqrt{3}}{2}\,\varepsilon_8\right)^2 \{(V^4_\mu)^2 + (V^5_\mu)^2\} + \tfrac{1}{4}g^2\left(\sqrt{\frac{2}{3}}\,\varepsilon_0 + \tfrac{1}{2}\varepsilon_3 - \frac{1}{2\sqrt{3}}\,\varepsilon_8\right)^2 \times \\
&\times \{(A^4_\mu)^2 + (A^5_\mu)^2\} + \tfrac{1}{4}g^2\left(\tfrac{1}{2}\varepsilon_3 - \frac{\sqrt{3}}{2}\,\varepsilon_8\right)^2 \{(V^6_\mu)^2 + (V^7_\mu)^2\} + \\
&+ \tfrac{1}{4}g^2\left(\sqrt{\frac{2}{3}}\,\varepsilon_0 - \tfrac{1}{2}\varepsilon_3 - \frac{1}{2\sqrt{3}}\,\varepsilon_8\right)^2 \{(A^6_\mu)^2 + (A^7_\mu)^2\} + \\
&+ \frac{1}{12}g^2\left(\sqrt{\frac{2}{3}}\,\varepsilon_0 + \frac{1}{\sqrt{3}}\,\varepsilon_8 + \varepsilon_3\right)^2 \left\{A^0_\mu + \sqrt{\frac{3}{2}}\,A^3_\mu + \frac{1}{\sqrt{2}}A^8_\mu\right\}^2 + \\
&+ \frac{1}{12}g^2\left(\sqrt{\frac{2}{3}}\,\varepsilon_0 + \frac{1}{\sqrt{3}}\,\varepsilon_8 - \varepsilon_3\right)^2 \left\{A^0_\mu - \sqrt{\frac{3}{2}}\,A^3_\mu + \frac{1}{\sqrt{2}}A^8_\mu\right\}^2 + \\
&+ \frac{1}{12}g^2\left(\sqrt{\frac{2}{3}}\,\varepsilon_0 - \frac{2}{\sqrt{3}}\,\varepsilon_8\right)^2 \{A^0_\mu - \sqrt{2}\,A^8_\mu\}^2,
\end{aligned} \tag{6}$$

where we have introduced

$$V^k_\mu = C^k_\mu + B^k_\mu$$
$$A^k_\mu = C^k_\mu - B^k_\mu.$$

It is seen that the fields V^0_μ, V^3_μ and V^8_μ remain massless. For the expression (6) it is also immediately visible what it means (as in [4]) to treat only one $U(3)$ group locally (only A^k_μ exists) and to asusme only $\langle\Phi\rangle_0 = \eta 1$ (i.e. $\varepsilon_0 = \sqrt{\frac{3}{2}}\,\eta$, $\varepsilon_3 = \varepsilon_8 = 0$. Then the mass term (6) is of the simple form $\frac{1}{4}g^2\eta^2 [(A^0_\mu)^2 + (A^k_\mu)^2]$. Since the number of GB is given only by a representation, there must be 6 GB-s which remain in the model. On principle we could visualize them by performing the „polar decomposition" according to Higgs [9] and Kibble [10][6].

Moreover, the form of invariants (5) prompts to be reserved since due to the renormalizability $V(\Phi)$ (constructed out of i_1, i_2 and i_3) can be at most quartic in Φ. Fortunately, the recently suggested mechanism [11] leaves room for attractive speculations.

Let us deal yet for a moment with the (3,3*) + (3*, 3) representation of the chiral $SU(3) \times SU(3)$ group and let us wish to get a mass to all YM fields except the electromagnetic field $V^3_\mu + \frac{1}{\sqrt{3}}V^8_\mu$. For such a task it is indispensable to put $\langle\Phi\rangle_0$ into the form[7]

$$\begin{pmatrix} a & 0 & 0 \\ 0 & b & d \\ 0 & ie & c \end{pmatrix}$$

which is just the assumption of the tadpole model of Coleman and Glashow [12]. Unfortunately, such an assumption is not justified since the tadpoles d and e, which correspond to the apparent parity and hypercharge violation, can be transformed away by a suitable constant gauge transformation [13].

Nevertheless such an attempt suggests to find other representations, which would also contain parity and hypercharge violating, but non-rotatory tadpoles.

III. CONCLUSION

The use of scalar multiplets in the *PGM* should be assumed only as a tool for giving a mass to gauge fields in such a way as to preserve the renormalizability of a theory. It is even an emergency tool, since for the remaining massive scalar fields one hardly finds an interpretation. From this we conclude that although the use of the discussed approaches to the *PGM* (more the *CFR* and the (3, 3*) + (3*, 3) representation for the group $U(n)$) does not respect all the requirements of a general formalism, it may have an informative worth like the tadpole model quoted (up to the present).

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[6] This is possible if we rewrite the multiplet Φ into a nine-row column, the generators of the $SU(3)_L$ and $SU(3)_R$ groups being $(J^L_m)_{ij} = \frac{1}{2}(if_{imj} - \alpha_{imj})$, $(J^R_m) = \frac{1}{2}(if_{imj} + \alpha_{imj})$, $m = 1, \ldots, 8$, $i, j = 0, \ldots, 8$.

[7] The complex phase is disregarded since it corresponds to the CP violation (Maiani L., preprint ISS 71/21).

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