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THE $SU(3)$ BREAKING AND CABIBBO ANGLE[1]

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A theory of the Cabibbo angle ϑ is suggested within the framework of the $SU(3)$ symmetry breaking. In this theory the parameter ϑ is fixed and computed by linking together the Cabibbo rotation and the full strong interaction $SU(3)$ breaking, and by using a subsidiary condition that the value of ϑ is zero in the world with zero pion masses. The computed value $\vartheta = 0.27 \pm 0.05$ is in a good agreement with experiment.

I. INTRODUCTION

After the discovery of the approximate $SU(3)$ symmetry of strong interactions (see for example [1]) it become apparent [2, 3] that the $SU(3)$ furnishes a unified basis for describing both the electromagnetic and the weak interactions using the notion of currents. In the $SU(3)$ scheme, the electromagnetic and weak interactions of hadrons are described by octets of vector $J_i^\mu(x)$ and axial-vector $J_i^{5\mu}(x)$ $(i = 1, \ldots, 8)$, $(\mu = 0, 1, 2, 3)$, currents.

The hadronic weak current $J_h^\mu(x)$ has been suggested to be of the form [3]

$$J_h^\mu(x) = (J_1^\mu + iJ_2^\mu + J_1^{5\mu} + iJ_2^{5\mu}) \cos\vartheta + (J_4^\mu + iJ_5^\mu + J_4^{5\mu} + iJ_5^{5\mu}) \sin\vartheta, \tag{1a}$$

or, in another representation

$$J_h^\mu(x) = e^{-2i\vartheta F_7}[J_1^\mu + iJ_2^\mu + J_1^{5\mu} + iJ_2^{5\mu}]e^{2i\vartheta F_7}, \tag{1b}$$

where F_7 is the $SU(3)$ generator, and where the phenomenological parameter (the Cabibbo angle) ϑ is used to describe the relations between processes with $\Delta S = 0$ and processes with $\Delta S = 1$. As Cabibbo [3] has shown this current gives a good description of leptonic weak interactions with ϑ of the order of 15°. This value of ϑ accounts, through the factor tg $\vartheta \approx 0.26$, for the suppression of strangeness-changing decays relative to decays which conserve strangeness.

It is easily seen from relations (1) that the following hadronic charges J_+, J_- and J_0, defined as follows

$$J_+ = \tfrac{1}{2}\int d^3x J_h^0(x),\ J_- = J_+^+,\ J_0 = \tfrac{1}{2}[J_+, J_-], \tag{2}$$

form an algebra of the $SU(2)$ group. This represents the Gell-Mann's version of the universality of strength of weak interactions [4].

The universality requires that the parameter ϑ should be the same for the vector and axial-vector currents, but evidently the universality does not fix this parameter. We see from (1) and (2) that the angle ϑ measures the difference between the two $SU(2)$ subgroups of the $SU(3)$ group, namely, between the subgroup $SU(2)$ generated by the weak hadronic vector charges (the vector part of generators (2)) and the isotopic $SU(2)$ subgroup which remains invariant after the breaking of $SU(3)$ strong interactions [5]. So, in order to construct a theory which fixes the parameter ϑ, one has to link together the strong interaction breaking of the $SU(3)$ symmetry and the weak current, or the Cabibbo rotation angle as it is seen from eq. (1b). This has been attempted in a number of papers [6—16] using various kinds of assumptions.

In our previous paper [16] we have suggested a new theory for the computation of the Cabibbo angle within the $(3, \bar{3}) + (\bar{3}, 3)$ chiral symmetry breaking model. It is the purpose of the present paper to apply this theory of the Cabibbo angle to more general models of the chiral symmetry breaking, namely for those consisting of only $SU(3)$ singlet and octet parts but with otherwise arbitrary chiral transformation properties.

II. THE CONNECTION OF THE CABIBBO ANGLE WITH THE STRONG INTERACTION BREAKING OF THE $SU(3)$ SYMMETRY

In the following we shall assume the usually accepted strong interaction Hamiltonian of the form

$$H = H_0 + gH_8, \tag{3}$$

where H_0 is the $SU(3)$ symmetric part and H_8 represents the eighth component of the $SU(3)$ octet. The parameter g describes the portion of H_8 which is needed for the Hamiltonian (3) to be the actual strong interaction Hamiltonian. So, another portion of H_8, for example g_0H_8 (where $g_0 \neq g$) instead of gH_8 in (3) and with the same portion of H_0 should represent the different, non-physical Hamiltonian of the following form

$$\mathscr{H} = H_0 + g_0H_8. \tag{4}$$

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Now in analogy with the considerations of ref. [16] we shall define the connection between the Cabibbo angle ϑ and the realistic $SU(3)$ noninvariant strong interaction Hamiltonian (3) to be as follows:

$$\langle 0|e^{-2i\vartheta F_7}\mathscr{H}e^{2i\vartheta F_7}|0\rangle = \langle 0|H|0\rangle. \tag{5}$$

Because the portion of the $SU(3)$ singlet (i.e. H_0) is the same in both Hamiltonians (3) and (4) by construction and since, F_7 commutes with H_0, we have from equation (5)

$$\langle 0|e^{-2i\vartheta F_7}H_8 e^{2i\vartheta F_7}|0\rangle = \frac{g}{g_0}\langle 0|H_8|0\rangle. \tag{6}$$

For further considerations we shall also assume the following relations concerning the vacuum expectation values of the members H_i $(i = 1, 2, \ldots, 8)$ of the octet

$$\langle 0|H_i|0\rangle = 0, \text{ where } i = 1, 2, \ldots, 7 \tag{7}$$

and

$$\langle 0|H_8|0\rangle \neq 0$$

to be valid.

It can be easily seen that the relations (7) are very closely related to the invariance of the vacuum under the isotopic $SU(2)$ subgroup of $SU(3)$, but to the noninvariance of the vacuum under the full $SU(3)$ group. In fact, since H_i $(i = 1, \ldots, 8)$ form the octet, we have immediately

$$\langle 0|[F_i, H_j]|0\rangle = i f_{ijk}\langle H_k\rangle_0, \tag{8}$$

where F_i $(i = 1, 2, \ldots, 8)$ are the generators of the $SU(3)$ group. Due to the isotopic $SU(2)$ invariance of the vacuum, that is $F_i|0\rangle = 0$, for $i = 1, 2, 3$, it necessarily follows from eq. (8) that $\langle H_i\rangle_0 = 0$ for $i = 1, 2, \ldots, 7$ and that $\langle H_8\rangle_0$ is arbitrary. If the vacuum is invariant under the full $SU(3)$ group, *i.e.* $F_i|0\rangle = 0$ for any $i = 1, 2, \ldots, 8$, then also $\langle H_8\rangle_0 = 0$. Thus, eqs. (7) imply that the vacuum can only be symmetric under the isotopic group but not under the full $SU(3)$ group. In other words the physical state of strongly interacting particles can form only the isotopic exactly degenerated multiplets but not fully and exactly degenerated $SU(3)$ multiplets. In such a way the requirements (7) should be understood as the ones fixing a certain portion of the $SU(3)$ breaking of strong interactions, namely that $SU(3)$ breaking which concerns the approximate $SU(3)$ supermultiplet structure of the hadronci states. So, eq. (5) (or eq. (6)) only together with conditions (7) can represent a theory of the Cabibbo angle in the sense that they link together the Cabibbo

rotation and the full strong interaction $SU(3)$ breaking consisting partly of the $SU(3)$ breaking of the Hamiltonian (if $g \neq 0$) and partly of the $SU(3)$ noninvariance of hadronic states. It is easily seen that in fact, within the framework of such a theory the Cabibbo angle is unambigously fixed by the value of the ratio g/g_0 of parameters.

To show this we combine eqs. (6) and (7), and after some calculation we obtain the following equation (see Appendix)

$$\left(1 - \frac{g}{g_0} - \tfrac{3}{2}\sin^2\vartheta\right)\langle H_8\rangle_0 = 0. \tag{9}$$

If the vacuum is fully $SU(3)$ invariant, then due to Coleman's theorem [17] there is no $SU(3)$ breaking at all and so, the value of the Cabibbo angle should be arbitrary. In other words one can perform any rotations in the $SU(3)$ space without changing the physics. We see that eq. (9) also describes this phenomenon because in the case of the exact $SU(3)$ invariance of the vacuum $\langle H_8\rangle_0 = 0$ should be valid and so it follows from eq. (9) that the parameter ϑ can be arbitrary. Thus the conditions (7) are the necessary requirements to obtain the definite value for the Cabibbo angle from eq. (9). Assuming the validity of eqs. (7) (i.e. $\langle H_8\rangle_0 \neq 0$ in eq. (9)) we have

$$\sin^2\vartheta = \tfrac{2}{3}\left(1 - \frac{g}{g_0}\right). \tag{10}$$

The present theory of the Cabibbo angle accounts naturally for the $SU(3)$ limiting value of the parameter ϑ. This value can only be fixed if $\langle H_8\rangle_0 \neq 0$ and $g = 0$, that is, only if the $SU(3)$ symmetry of the Hamiltonian exists but not that of the vacuum. In this case the $SU(3)$ noninvariance of the vacuum should be realized through the existence of the spinless scalar Nambu-Goldstone particle $\varkappa$ with the isospin $I = \frac{1}{2}$. Then from eq. (10) we obtain the following $SU(3)$-limiting value of the Cabibbo angle $\mathrm{tg}^2\,\vartheta = 2$, which is also in agreement with the results of refs. [13, 16].

In order to calculate ϑ for the realistic world (i.e. if $g \neq g_0$) we have to calculate the ratio g/g_0 of the symmetry breaking parameters. For this one extra condition concerning the property of the non-realistic world with the Hamiltonian (4) (i.e. when $g = g_0$) must be added. Consistently with considerations in previous papers [9, 12—16] we shall assume that in the non-realistic world described by the Hamiltonian (4) pions have zero masses, or mathematically:

$$\langle\pi|H_0|\pi\rangle + g_0\langle\pi|H_8|\pi\rangle = 0. \tag{11}$$

It means that we fix the zero value of the Cabibbo angle in the case of zero pion masses. Thus it is, in fact, proposed that there exists the same mechanism which introduces both the non-conservation of the strange-preserving axial currents (i.e. $m^2 \neq 0$ due to PCAC) and the strangeness non-conservation in weak processes (i.e. $\vartheta \neq 0$). In such a way the condition (11) should be understood as a subsidiary condition containing an extra information about the possible dynamical origin of the Cabibbo parameter ϑ.

Using an appropriate normalization, the realistic Hamiltonian (3) gives the physical masses for pseudoscalar mesons in the following form

$$m_\pi^2 = \langle\pi|H_0|\pi\rangle + g\langle\pi|H_8|\pi\rangle, \qquad (12)$$
$$m_K^2 = \langle K|H_0|K\rangle + g\langle K|H_8\rangle K\rangle,$$

and the Wigner-Eckert theorem gives the conditions

$$\langle\pi|H_0|\pi\rangle = \langle K|H_0|K\rangle, \qquad (13)$$
$$\langle\pi|H_8|\pi\rangle = -2\langle K|H_8|H\rangle.$$

Combining eqs. (11), (12) and (13) we get

$$\frac{g}{g_0} = 1 - \tfrac{3}{2}\,\frac{m_\pi^2}{m_K^2 + \dfrac{m_\pi^2}{2}}\,. \qquad (14)$$

It is worthwhile to note here that this ratio g/g_0 does not depend on the normalization conditions, as it should be. Also, because of using the Wigner-Eckart theorem in the computation of value (14) and because the physical hadron states are only the approximate $SU(3)$ multiplets the realistic value of g/g_0 which should be used in eq. (10) is expected to differ from the value (14) within approximately 10 %—20 %, and so, we expect the approximately equal error in the calculation of ϑ when using eq. (14) in relation (10). We have then

$$\sin^2\vartheta = \frac{m_\pi^2}{m_K^2 + \dfrac{m_\pi^2}{2}}$$

from which it follows that $\vartheta \approx 0.27$. We see that the agreement with the experiment is good within the expected error.

III. CONCLUSION

In the present paper a theory of the Cabibbo angle ϑ has been suggested within the framework of the $SU(3)$ symmetry breaking by strong interactions.

This theory fixes the parameter by linking together the Cabibbo rotation and the full strong interaction $SU(3)$ breaking, consisting of both the $SU(3)$ Hamiltonian and the $SU(3)$ vacuum noninvariances (eqs. (5—7)). One subsidiary condition (eq. (11)) has been shown necessary in order to present the complete theory of the Cabibbo angle. This condition introduces into the theory an information about the possible dynamical origin of the Cabibbo angle. The computed value of the Cabibbo angle $\vartheta = 0.27 \pm 0.05$ is in good agreement with the experiment.

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APPENDIX

To derive eq. (9) from (6) and (7) we have, first, to rewrite the left-hand side of eq. (6). If we assign

$$H_8(\vartheta) = e^{-2i\vartheta F_7} H_8 e^{2i\vartheta F_7}, \qquad (A1)$$

where $H_8 \equiv H_8(0)$ is the eighth component of an octet H_i ($i = 1, 2, \ldots, 8$), which fulfils the following commutation relations

$$[F_i, H_j] = i f_{ijk} H_k, \quad i, j, k = 1, 2, \ldots, 8. \qquad (A2)$$

It can be easily proved that eq. (A1) can also be written in the equivalent Taylor's series form

$$H_8(\vartheta) = H_8 - 2i\vartheta[F_7, H_8] + \frac{(-2i\vartheta)^2}{2!}[F_7, [F_7, H_8]] + $$
$$+ \frac{(-2i\vartheta)^3}{3!}[F_7, [F_7, [F_7, H_8]]] + \ldots\,. \qquad (A3)$$

Combining (A2) and (A3) the following most general form of $H_8(\vartheta)$ can be found

$$H_8(\vartheta) = a_1(\vartheta)H_8 + a_2(\vartheta)H_6 + a_3(\vartheta)H_3, \qquad (A4)$$

where $a_i(\vartheta)$ ($i = 1, 2, 3$) are unknown functions, evidently satisfying the conditions:

$$a_1(0) = 1, \; a_2(0) = a_3(0) = 0 \qquad (A5)$$

because of $H_8(0) = H_8$. The derivative of the relation (A1) gives

$$\frac{dH_8(\vartheta)}{d\vartheta} = -2i[F_7, H_8(\vartheta)],$$

and combining this equation with (A4) and (A2), one obtains the differential equations for the functions $a_i(\vartheta)$ as follows:

$$a_1'(\vartheta) = -\sqrt{3}a_2(\vartheta), \tag{A6}$$
$$a_2'(\vartheta) = \sqrt{3}a_1(\vartheta) - a_3(\vartheta),$$
$$a_3'(\vartheta) = a_2(\vartheta).$$

After some manipulations and with respect to conditions (A5) the eqs. (A6) can be rewritten in the following system of algebraic equations

$$a_1(\vartheta) = -\sqrt{3}a_3(\vartheta) + 1, \tag{A7}$$
$$\sqrt{3}a_1(\vartheta) - a_3(\vartheta) = \sqrt{3}\cos 2\vartheta,$$
$$a_2(\vartheta) = \frac{\sqrt{3}}{2}\sin 2\vartheta.$$

From (A7) and (A4) we get

$$H_8(\vartheta) = e^{-2i\vartheta F_7}H_8e^{2i\vartheta F_7} = \tfrac{1}{4}(1 + 3\cos 2\vartheta)H_8 + $$
$$+\frac{\sqrt{3}}{2}(\sin 2\vartheta)H_6 + \frac{\sqrt{3}}{4}(1 - \cos 2\vartheta)H_3.$$

Now introducing this relation into eq. (6) and imposing conditions (7) on the result one arrives at eq. (9) after some manipulations.

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