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TESTING THE *CPT* IN THE K-OPTICS[1]

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The available experiments have indicated that neutral K decays violate the CP and T symmetries, but conserve the CPT symmetry, which is strictly related to local causality. The accuracy of this indication is discussed.

I. SYMMETRIES AND ASYMMETRIES IN $K^\circ - \bar{K}^\circ$ SYSTEM

The well-known CPT theorem states that in a relativistic field theory, subject to the principle of causality, the CPT must be a strict symmetry [1, 2]. As usual, C stands for charge conjugation, P for space reflection, T for time reversal and CPT is the product of the three transformations. The CPT symmetry is supported by the observed equality of the masses and lifetimes of particles and antiparticles, but in the light of the observed faint CP asymmetry it has become necessary to check both T and CPT to the same accuracy.

In the physics the most sensitive experimental method has been offered by the interference phenomena. A superposition of different CPT eigenstates is, however, in general forbidden: the states$|\rangle$ and $CPT|\rangle$ are either completely neutral and consequently identical, or they carry opposite electric, baryonic or leptonic charges and consequently a superselection destroys their coherence. The only lucky exception has been given by the $K^\circ - \bar{K}^\circ$ system. The states $|K^\circ\rangle$ and $|\bar{K}^\circ\rangle = CPT|K^\circ\rangle$ are orthogonal; they differ in the value of the hypercharge Y. The hypercharge obeys an approximate conservation law, consequently K° and $\bar{K}^\circ$ are not separated by a superselective barrier. The coherence of the K° and $\bar{K}^\circ$ states enables us to test the C, CP, CPT symmetries up to an amazing accuracy, which reminds us of the exceptional advantages of the optics of coherent light beams in understanding the wave nature of light. Let us summarize briefly the formulas on which such investigations rest [2].

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State vectors describing the unstable neutral K mesons have the following structure:

$$|K(t)\rangle = a_+(t)|K^\circ\rangle + a_-(t)|\bar{K}^\circ\rangle + \text{orthogonal decay products.} \qquad (1)$$

Here $|K^\circ\rangle$ and $|\bar{K}^\circ\rangle$ are simultaneous eigenvectors of the strong Hamiltonian H_0 and of the hypercharge Y:

$$[H_0, Y] = 0, \qquad [H_0, CP] = 0, \qquad \{Y, CP\} = 0 \qquad (2)$$

$$H_0|K^0\rangle = m_0|K^0\rangle, \qquad H_0|\bar{K}^0\rangle = m_0|\bar{K}^0\rangle \qquad (3)$$

$$Y|K^0\rangle = +|K^0\rangle, \qquad Y|\bar{K}^0\rangle = -|\bar{K}^0\rangle \qquad (4)$$

If we turn on the weak perturbation H with the properties

$$H_{total} = H_0 + H \qquad [H, Y] \neq 0, \qquad (5)$$

K^0 and $\bar{K}^0$ will not be steady state solutions any longer. The time dependence of $|K(t)\rangle$ in eq. (1) is given by the Weisskopf-Wigner theory (see e.g. [3, 4]):

$$i\frac{\partial}{\partial t}\begin{bmatrix} a_+ \\ a_- \end{bmatrix} = \mathscr{H}\begin{bmatrix} a_+ \\ a_- \end{bmatrix}, \qquad (6)$$

where $\mathscr{H}$ is the effective Hamiltonian in the subspace of neutral K mesons. This non-Hermitian two-by-two matrix is parametrized as follows:

$$\mathscr{H} = \frac{M_S + M_L}{2} + \frac{M_S - M_L}{2}\begin{bmatrix} \sin 2\delta & e^{2\varepsilon}\cos 2\delta \\ e^{-2\varepsilon}\cos 2\delta & -\sin 2\delta \end{bmatrix}. \qquad (7)$$

M_S, M_L, ε and δ are complex numbers. The components of $\mathscr{H}$ are given by the perturbation theory, e.g.

$$H_{11} = m_0 + \langle K^0|H|K^0\rangle + \sum_r \langle K^0|H|r\rangle\left[\frac{P}{m_0 - E_r} - i\pi\delta(m_0 - E_r)\right]\langle r|H|K^0\rangle + \ldots$$

$$H_{12} = \langle K^0|H|\bar{K}^0\rangle + \sum_r \langle K^0|H|r\rangle\left[\frac{P}{m_0 - E_r} - i\pi\delta(m_0 - E_r)\right]\langle r|H|\bar{K}^0\rangle + \ldots$$

The eigensolutions of (6) are:

$$|K_S\rangle e^{-iM_St} = \frac{N_S}{\sqrt{2}}\left[e^{\varepsilon}(\cos\delta + \sin\delta)|K^0\rangle + e^{-\varepsilon}(\cos\delta - \sin\delta)|\bar{K}^0\rangle\right]e^{-iM_St}, \qquad (8)$$

$$|K_L\rangle e^{-iM_Lt} = \frac{N_L}{\sqrt{2}}\, e^{\varepsilon}(\cos\delta - \sin\delta)|K^0\rangle - e^{-\varepsilon}(\cos\delta + \sin\delta)|\bar{K}^0\rangle\, e^{-iM_Lt}.$$

K_S may be identified with the observed short-lived neutral K mesons, K_L with the observed long-lived neutral K meson. The real and imaginary parts of the eigenvalues M_S and M_L give the experimental mass and lifetime of the corresponding particles:

$$M_S = m_S - \frac{i}{2\tau_S}, \qquad M_L = m_L - \frac{i}{2\tau_L}. \qquad (9)$$

It will be shown that the two other complex numbers, ε and δ, describe the CP, T and CPT asymmetries of the neutral K subspace.

If Nature were described exactly by the Hamiltonian H_0 (possessing the properties (2)), $e^{-icY}H_0e^{icY}$ would evidently be identical with H_0. If we turn on the small perturbation H with the property (5), this will no longer be true:

$$H_{new}(c) = e^{-icY}H_{tot}e^{icY} = H_0 + H(c), \qquad H(c) = e^{-icY}H_0e^{icY}. \qquad (10)$$

It is still true, however, that the matrix elements of $H(c)$ are identical with those of H up to some phase factor if they are taken between two eigenstates of Y:

$$\langle y_1|H(c)|y_2\rangle = e^{-ic(y_1-y_2)}\langle y_1|H|y_2\rangle.$$

The hypercharge Y is almost always accompanied by electric or baryonic charges. The latter generate superselection, and consequently the physical states are in most cases eigenstates. This means that for such states H_{tot} and $H_{new}(c)$ are physically equivalent. The only important exceptions are the K_S and K_L states, these being superpositions of the two Y eigenstates K^0 and $\bar{K}^0$:

$$\langle K^0|H(c)|K^0\rangle = \langle K^0|H|K^0\rangle, \qquad \langle K^0|H(c)|\bar{K}^0\rangle = e^{-2ic}\langle K^0|H|\bar{K}^0\rangle,$$

$$\langle \bar{K}^0|H(c)|\bar{K}^0\rangle = \langle \bar{K}^0|H|\bar{K}^0\rangle, \qquad \langle \bar{K}^0|H(c)|K^0\rangle = e^{2ic}\langle \bar{K}^0|H|K^0\rangle.$$

Consequently replacing H_{tot} by $H(c)$ entails the replacement

$$\varepsilon \to \varepsilon - ic \qquad (c = \text{arbitrary real value}).$$

By exploiting this freedom the imaginary part of ε can always be modified arbitrarily, for instance with the appropriate c value a replacement $H_{tot} \to H_{new}(c)$ can make Imε even zero.

The CP and CPT transformations produce $\bar{K}^0$ from K. The time reversal T does not affect a K^0 or $\bar{K}^0$ at rest, as both are spinless particles:

$$CP|K^0\rangle = e^{ia}|\bar{K}^0\rangle \qquad CP|\bar{K}^0\rangle = e^{i\bar{a}}|K^0\rangle$$

$$CTP|K^0\rangle = e^{ib}|\bar{K}^0\rangle \qquad CPT|\bar{K}^0\rangle = e^{i\bar{b}}|K^0\rangle$$

$$T|K^0\rangle = e^{i(b-\bar{a})}|K^0\rangle \qquad T|\bar{K}^0\rangle = e^{i(\bar{b}-a)}|\bar{K}^0\rangle.$$

Here CP is unitary, while CPT and T are antiunitary operators. From the conditions

$$(CP)^2 = (CPT)^2 = 1 \tag{11}$$

one has $\bar{a} = -a$, $\bar{b} = b$. Using the combinations

$$e^{iaY}CP \text{ and } e^{-ib}CPT$$

as new CP and CPT operators, which also obey eqs. (2) and (11), one has simply

$$CP|K^0\rangle = CPT|K^0\rangle = |\bar{K}^0\rangle, \qquad T|K^0\rangle = |K^0\rangle. \tag{12}$$

Now, by making use of eqs. (8) and (12) it is easy to verify that

i) if $[H, CP] = 0$ then $\mathscr{H}_{11} = \mathscr{H}_{22}, \mathscr{H}_{12} = \mathscr{H}_{21}$, ie. $\varepsilon = \delta = 0$, (13)
ii) if $[H, CPT] = 0$ then $\mathscr{H}_{11} = H_{22}$, ie. $\delta = 0$,
iii) if $[H, T] = 0$ then $\mathscr{H}_{12} = \mathscr{H}_{21}$, ie. $\varepsilon = 0$.

Parameter ε is the measure of the CPT asymmetry, while δ the measure of the T asymmetry in the neutral K subspace.

In an exactly CP, T and CPT-symmetric world we would have

$$|K_1\rangle = \frac{1}{\sqrt{2}}[|K^0\rangle + |\bar{K}^0\rangle], \qquad CP|K_1\rangle = +|K_1\rangle \tag{14}$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}[|K^0\rangle - |\bar{K}^0\rangle], \qquad CP|K_2\rangle = -|K_2\rangle.$$

K_L decays indicate a faint violation of the CP symmetry, so the parameters ε and δ cannot be large. In the whole discussion we shall restrict ourselves to the terms of first order in the parameters, which describe the CP, T and CPT asymmetries. For small values of ε and δ the effective Hamiltonian may be written as

$$\mathscr{H} = \frac{M_S + M_L}{2} + \frac{M_S - M_L}{2}(\sigma_1 + 2i\varepsilon\sigma_2 + 2\delta\sigma_3) \tag{15}$$

and the eigenvectors are

$$|K_S\rangle = |K_1\rangle + (\varepsilon + \delta)|K_2\rangle, \; |K_L\rangle = |K_2\rangle + (\varepsilon - \delta)|K_1\rangle. \tag{16}$$

II. EXPERIMENTAL FACTS

If the CP were an exact symmetry, K_1 and K_2 would be observable particles. By conserving the CP quantum number the K_1 meson would decay into $\pi\pi$,

the K_2 meson into three particles. The fact that both $K_S \to \pi\pi$ and $K_L \to \pi\pi$ decays have been observed means a breakdown of the CP symmetry. This breakdown is characterized by the complex numbers

$$\eta_{+-} = \frac{\langle\pi^+\pi^-|H+\ldots|K_L\rangle}{\langle\pi^+\pi^-|H+\ldots|K_S\rangle}, \quad \eta_{00} = \frac{\langle\pi^0\pi^0|H+\ldots|K_L\rangle}{\langle\pi^0\pi^0|H+\ldots|K_S\rangle}. \tag{17}$$

The experimental decay rates and the phases of the interference terms in the time dependence of $K \to \pi\pi$ events enable us to compute the empirical values of these asymmetry parameters. Let us quote the world averages, prior to the recent Steinberger experiment[2], just to give an impression about the present accuracy:

$$|\eta_{+-}| = (1.98 \pm 0.04) \times 10^{-3}, \qquad |\eta_{00}| = (2.09 \pm 0.10) \times 10^{-3}, \tag{18}$$
$$\arg \eta_{+-} = 41.8^0 \pm 2.8^0, \qquad \arg \eta_{00} = 43^0 \pm 19^0.$$

Similar parameters of the $K \to \pi\pi$ decays are:

$$\eta_{+-0} = \frac{\langle\pi^+\pi^-\pi^0|H+\ldots|K_S\rangle}{\langle\pi^+\pi^-\pi^0|H+\ldots|K_L\rangle}, \quad \eta_{000} = \frac{\langle\pi^0\pi^0\pi^0|H+\ldots|K_S\rangle}{\langle\pi^0\pi^0\pi^0|H+\ldots|K_L\rangle}. \tag{19}$$

The observed time dependence of the neutral $K \to \pi^+\pi^-\pi^0$ and $K \to \pi^0\pi^0\pi^0$ gives[2]

$$\eta_{+-0}\,(0.14 \pm 0.17) + i\,(-0.12 \pm 0.30) \tag{20}$$
$$\eta_{000} = (0.04 \pm 0.45) + i(0.45 \pm 0.60)$$

(If the dominating $\pi\pi\pi$ final state is characterized with the isospin $I = 1$, one may expect

$$\eta_{000} = \eta_{+-0}. \tag{21}$$

Otherwise η_{+-0} may come also from a isospin mixture but is necessarily related to the CP violation.) For leptonic decays let

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[2] The experimental data are world averages given in the Review of Particle Properties, *Rev. Mod. Phys. April 1973* and the values presented at the *16th International Conference on High Energy Physics, Batavia, September 1972*. We have deliberately not taken into account the new measurement of the K_S life time, presented at the Batavia Conference by O. Skjeggestad et al. (Paper No. 267) and by C. Geweniger, J. Steinberger et al: $\tau_s = (0.8958 \pm 0.0048) \times 10^{-10}$. This value is significantly higher than the combined results of the previous experiments and has a number of implications for the other parameters discussed in the report. (E. g. $|\eta_{\pm}| = 2.3 \times 10^{-3}$). This change has been discussed in details by J. Steinberger at the Smolenice Conference.

$$\eta_{\pi^-e^+\nu} = \frac{\langle \pi^- e^+ \nu | H + \ldots | K_S \rangle}{\langle \pi^- e^+ \nu | H + \ldots | K_L \rangle}, \quad \eta_{\pi^+e^-\nu} = \frac{\langle \pi^+ e^- \nu | H + \ldots | K_S \rangle}{\langle \pi^+ e^- \nu | H + \ldots | K_L \rangle}. \tag{22}$$

The nonvanishing value of these parameters does not necessarily indicate a CP breakdown, because the final states are not CP eigenstates. These parameters can be expressed in terms of other parameters, which have more direct physical meanings.

$$x = \frac{\langle \pi^- e^+ \nu | H + \ldots | K^0 \rangle}{\langle \pi^+ e^- \nu | H + \ldots | K^0 \rangle}, \quad \bar{x} = \frac{\langle \pi^+ e^- \nu | H + \ldots | \bar{K}^0 \rangle}{\langle \pi^+ e^- \nu | H + \ldots | \bar{K}^0 \rangle}, \tag{23}$$

characterize the violation of the $\Delta Y = \Delta Q_{hadron}$ selection rule. It is easy to show that

$$\eta_{\pi^-e^+\nu} = \frac{1 + x'}{1 - x'}, \quad \eta_{\pi^+e^-\nu} = \frac{1 + \bar{x}'}{1 - \bar{x}'} \tag{24}$$

with

$$x' = x e^{-2\varepsilon}(1 + \delta) + \delta, \quad \bar{x}' = \bar{x} e^{2\varepsilon}(1 - \delta) - \delta. \tag{25}$$

The consequences of the symmetry assumptions are the following:

$$\bar{x} = x^* \text{ if } [H, CPT] = 0$$

$$x = x^* \text{ and } \bar{x} = \bar{x}^* \text{ if } [H, T] = 0.$$

A numerical analysis of the $K \to \pi e \nu$ experiments [3] makes use of an assumed CPT symmetry:

$$\eta_{\pi^-e^+\nu} = \frac{1 + x}{1 - x} = -\eta^*_{\pi^+e^-\nu} \text{ for } [H, CPT] = 0. \tag{26}$$

The observed value is [3]

$$x = (-3 \pm 27) \times 10^{-3} + i(-5 \pm 28) \times 10^{-3}. \tag{27}$$

Evidently $\xi = \mathrm{Im}x$ is a measure of the CP and T breakdown in the $\Delta Y = \Delta Q_{hadron}$ violating leptonic decay.

The charge asymmetry of the $K_L \to \pi^\mp e^\pm \nu$ decays is an easily observable quantity:

$$\alpha = \frac{\Gamma(K_L \to \pi^- e^+ \nu) - \Gamma(K_L \to \pi^+ e^- \nu)}{\Gamma(K_L \to \pi^- e^+ \nu) + \Gamma(K_L \to \pi^+ e^- \nu)} = \frac{|\langle \pi^- e^+ \nu | H + \ldots | K_L \rangle : \langle \pi^+ e^- \nu | H + \ldots | K_L \rangle|^2 - 1}{|\langle \pi^- e^+ \nu | H + \ldots | K_L \rangle : \langle \pi^+ e^- \nu | H + \ldots | K_L \rangle|^2 + 1},$$

which can be written in the simple form

$$\alpha = 2\mathrm{Re}\varepsilon \frac{1 + \mathrm{Re}x}{1 - \mathrm{Re}x} + \beta, \tag{28}$$

where $\beta = 0$ in the case of the CPT symmetry. The experimental value is (see the note[2])

$$\alpha = (3.27 \pm 0.42) \times 10^{-3}. \tag{29}$$

The empirical parameters η_{+-}, η_{00}, α indicate a violation of the CP symmetry, but the T and CPT asymmetric effects are mixed in them. Our main task will be the separation of these effects.

III. NUMERICAL ANALYSIS OF THE K ASYMMETRIES

In order to learn from these data as much as possible we shall first exploit the unitarity. From eq. (6) one can deduce

$$-\frac{d}{dt} \langle K(t) | K(t) \rangle = \langle K(t) | \Gamma | K(t) \rangle. \tag{30}$$

Here $\Gamma = i(\mathscr{H} - \mathscr{H}^+)$ is the anti-hermitian part of $\mathscr{H}$, while according to the rules of the perturbation theory it is given by the formula

$$\langle K | \Gamma | K' \rangle = 2\pi \sum_r \langle K | H + \ldots | r \rangle \delta(m_0 - E_r) \langle r | H + \ldots | K' \rangle.$$

Let us substitute the expression

$$|K(t)\rangle = u|K_S\rangle e^{-iM_S t} + v|K_L\rangle e^{-iM_L t}$$

into eq. (30). Since u and v are arbitrary constants,

$$\langle K_S | \Gamma | K_L \rangle = \tfrac{1}{2} \langle K_S | K_L \rangle \left[\frac{1}{\tau_S} + \frac{1}{\tau_L} + 2i\Delta m \right]$$

or, because $\tau_S = (0.862 \pm 0.006) \times 10^{-10}\,\mathrm{s} \ll \tau_L = (5.181 \pm 0.042) \times 10^{-8}\mathrm{s}$, we have

$$\langle K_S | \Gamma | K_L \rangle = \frac{1}{2\tau_S} \langle K_S | K_L \rangle (1 + i \tan z) \tag{31}$$

where according to the experimental data (see the note[2])

$$z = \tan^{-1}(2\tau_S \Delta m) = 42.96^0 \pm 0.23^0. \tag{32}$$

Only the final states with branching ratios above 1 % will be taken into account:

$$\langle K_S|\Gamma|K_L\rangle = \frac{1}{\tau_S}\{\eta_{+-}B(K_S\to\pi^+\pi^-) + \eta_{00}B(K_L\to\pi^0\pi^0)\} +$$

$$+\frac{1}{\tau_L}\{\eta^*_{+-0}B(K_L\to\pi^+\pi^-\pi^0) + \eta^*_{000}B(K_L\to\pi^0\pi^0\pi^0) +$$

$$+\eta^*_{\pi^-e^+\nu}B(K_L\to\pi^-e^+\nu) + \eta^*_{\pi^+e^-\nu}B(K_L\to\pi^+e^-\nu) +$$

$$+\eta^*_{\pi^-\mu^+\nu}B(K_L\to\pi^-\mu^+\nu) + \eta^*_{\pi^+\mu^-\nu}B(K_L\to\pi^+\mu^-\nu)\}.$$

As a consequence of the smallness of $\tau_S/\tau_L = (1.66 \pm 0.016)\times 10^{-3}$ our formula is insensitive to the data to be substituted into the second paranthesis. So we write

$$B(K_S\to\pi^+\pi^-) = 1 - B(K_S\to\pi^0\pi^0)]$$

$$B(K_L\to\pi^+\pi^-\pi^0) + B(K_L\to\pi^0\pi^0\pi^0) = B(K_L\to\pi\pi\pi)$$

$$B(K_L\to\pi^-e^+\nu) + B(K_L\to\pi^+e^-\nu) + B(K_L\to\pi^+e^-\nu) +$$

$$+ B(K_L\to\pi^+\mu^-\nu) = 1 - B(K_L\to 3\pi)$$

$$B(K_L\to\pi^-e^+\nu) - B(K_L\to\pi^+e^-\nu) = B(K_L\to\pi^-\mu^+\nu) -$$

$$- B(K_L\to\pi^+\mu^-\nu).$$

We arrive at the result

$$\tfrac{1}{2}\langle K_S|K_L\rangle(1 + i\tan z) = B(K_S\to\pi^+\pi^-)\eta_{+-} + B(K_S\to\pi^0\pi^0)\eta_{00} + \frac{\tau_S}{\tau_L}\eta_3,$$

where

$$\eta_3 = B(K_L\to\pi^+\pi^-\pi^0)\eta_{+-0} + B(K_L\to\pi^0\pi^0\pi^0)\eta_{000} +$$

$$+ [1 - B(K_L\to 3\pi)]\,[\alpha(1 + 2\,\mathrm{Re}x) + x - x'].$$

The experimental branching ratios are (see the note[2])

$$B(K_S\to\pi^0\pi^0) = 0.312 \pm 0.003 \qquad (33)$$

$$B(K_L\to 3\pi^0) = 0.214 \pm 0.007$$

$$B(K_L\to\pi^+\pi^-\pi^0) = 0.126 \pm 0.003$$

If the smallness of x' is taken into account,

$$\tfrac{1}{2}(\eta_{\pi^-e^+\nu} - \eta_{\pi^+e^-\nu}) \simeq 1,$$

$$\tfrac{1}{2}(\eta_{\pi^-e^+\nu} + \eta_{\pi^+e^-\nu}) \simeq (x - \bar{x}) + 2\delta,$$

but if the CPT symmetry is assumed on the right-hand side of the last equation, we can put simply $2i\xi$. Finally, for numerical calculations one can use the simple formula

$$\tfrac{1}{2}\langle K_S|K_L\rangle = \mathrm{Re}\varepsilon - i\mathrm{Im}\delta = (1 + i\tan\delta)^{-1}\Big[B(K_S\to\pi^+\pi^-)\eta_{+-} +$$

$$+ B(K_S\to\pi^0\pi^0)\eta_{00} + \frac{\tau_S}{\tau_L}B(K_L\to\pi^+\pi^-\pi^0)\eta^*_{+-0} +$$

$$+\frac{\tau_S}{\tau_L}B(K_L\to\pi^0\pi^0\pi^0)\eta^*_{000} + \dots\Big]. \qquad (34)$$

All numbers on the right-hand side of the unitarity equation have been measured and are quoted above, the others are negligible, so we can compute $\mathrm{Re}\varepsilon$ and $\mathrm{Im}\delta$.

$$\mathrm{Re}\varepsilon = (1.47 \pm 0.15)\times 10^{-3},\quad \mathrm{Im}\delta = (0.09 \pm 0.19)\times 10^{-3}. \qquad (35)$$

Knowing $\mathrm{Re}\varepsilon$, the value of β can be obtained from eq. (28):

$$\beta = (0.35 \pm 0.46)\times 10^{-3}. \qquad (36)$$

A more detailed knowledge of the asymmetry parameters is offered by a detailed analysis of the $K\to\pi\pi$ decays, for which the most complete set of experimental data is available.

As the spin of K is zero, the orbital angular momentum of the $\pi\pi$ final state is also zero. The Bose statistics allow $I = 0$ and $I = 2$ in this final state. The K_S branching ratio gives

$$\left|\frac{\langle\pi^0\pi^0|H+\dots|K_S\rangle}{\langle\pi^+\pi^-|H+\dots|K_S\rangle}\right|^2 = \tfrac{1}{2}\left|\frac{1-\sqrt{2}\,p}{1+p/\sqrt{2}}\right|^2$$

where

$$p = \frac{\langle\pi\pi, I=2|H+\dots|K_S\rangle}{\langle\pi\pi, I=0|H+\dots|K_S\rangle} \qquad (37)$$

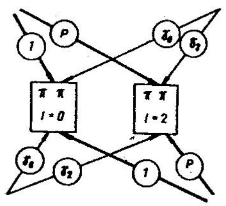
is the ratio of the $\Delta I > \frac{1}{2}$ amplitude to the $\Delta I = \frac{1}{2}$ amplitude in the CP allowed $K_S\to\pi\pi$ decay. From the experimental value (33) one gets

$$\mathrm{Re}p = 0.0251 \pm 0.0028. \qquad (38)$$

The CP-forbidden $K_L\to\pi\pi$ transitions may have two different sources:

either the CP impurity of the K_L eigenstate (tke presence of K_1 being characterized by $\varepsilon - \delta$) or a direct jump of CP in the $K_2 \to \pi\pi$ transition (to be characterized by new parameters γ_0 and γ_2, see Fig. 1). We define

$$K_s = K_1 + (\varepsilon - \delta) K_2$$



$$K_L = K_2 + (\varepsilon - \delta) K_1$$

Fig. 1. $K \to \pi\pi$ descay amplitudes.

$$\varepsilon_0 = \frac{\langle \pi\pi, I = 0 | H + \ldots | K_L \rangle}{\langle \pi\pi, I = 0 | H + \ldots | K_S \rangle}, \quad \varepsilon_2 = \frac{\langle \pi\pi, I = 2 | H + \ldots | K_L \rangle}{\langle \pi\pi, I = 0 | H + \ldots | K_S \rangle} \tag{39}$$

In accordance with the isospin decomposition of the $\pi^+\pi^-$ and $\pi^0\pi^0$ states one can write

$$\eta_{+-} = \frac{\varepsilon_0 + \varepsilon_2/\sqrt{2}}{1 + p/\sqrt{2}}, \qquad \eta_{00} = \frac{\varepsilon_0 - \varepsilon_2\sqrt{2}}{1 - p\sqrt{2}} \tag{40}$$

If the only source of the $K_L \to \pi\pi$ transitions were the K_1 impurity, one would have

$$\frac{\varepsilon_2}{\varepsilon_0} = \frac{\langle \pi\pi, I = 2 | H + \ldots | K_L \rangle}{\langle \pi\pi, I = 0 | H + \ldots K_L \rangle} = \frac{\langle \pi\pi, I = 2 | H + \ldots | K_S \rangle}{\langle \pi\pi, I = 0 | H + \ldots | K_S \rangle} = p.$$

The other source might be the CP-breaking $K_2 \to \pi\pi$ transition. Let us write

$$\langle \pi\pi, I | H + \ldots | K^0 \rangle = A_I e^{i\delta_I}, \ \langle \pi\pi, I | H + \ldots | \bar{K}^0 \rangle = \bar{A}_I e^{i\delta_I}. \tag{41}$$

Here δ_I is the scattering phase shift of the strong $\pi\pi$ final state interaction at $E = m_0c^2$ with the isospin I. With a certain amount of hesitation π-meson physics says that [5]

$$\delta_2 - \delta_0 = -51.7^0 \pm 5^0. \tag{42}$$

In the case of the CP symmetry one would have $A_I = \bar{A}_I$, thus the direct CP breakdown in the $K_2 \to \pi\pi$ transition is characterized by the two complex parameters

$$\gamma_0 = \frac{A_0 - \bar{A}_0}{A_0 + \bar{A}_0}, \quad \gamma_2 = \frac{A_2 - \bar{A}_2}{A_2 + \bar{A}_2}. \tag{43}$$

(Up to the first order of the perturbation theory $\mathrm{Re}\gamma_I \neq 0$ indicates the CP and the CPT breakdowns; $\mathrm{Im}\gamma_I \neq 0$ indicates the CP and the T breakdowns. By making use of eq. (14), (16), (39), (40), (41) and (43) one arrives at the following relations:

$$\varepsilon_0 = \gamma_0 + \varepsilon - \delta = \frac{2\eta_{+-} + \eta_{00}}{3} + \tfrac{2}{3}p(\eta_{+-} - \eta_{00}), \tag{44}$$

$$\gamma_2 - \gamma_1 = \frac{\varepsilon_2}{p} - \varepsilon_0 = \frac{\sqrt{2}}{3}\frac{\eta_{+-} - \eta_{00}}{p}\left(1 - \frac{p}{\sqrt{2}} + p^2\right). \tag{45}$$

(Here all the CP asymmetry parameters have been taken into account in the first order; no other parameters have been neglected.) $\mathrm{Re}\, p$ is known experimentally, and it is easy to show that

$$p = \frac{A_2 + \bar{A}_2}{A_0 + \bar{A}_0} e^{i(\delta_2 - \delta_0)} \frac{1 + \gamma_2(\varepsilon + \delta)}{1 + \gamma_0(\varepsilon + \delta)}.$$

Now if $K \to \pi\pi$ is dominated by the CP symmetric weak interaction for both the $I = 0$ and the $I = 2$ final states, one can write

$$p = \pm\left|\frac{A_2}{A_0}\right| e^{i(\delta_2 - \delta_0)} + CP \text{ asymmetric terms} =$$

$$= \pm(0.040 \pm 0.007)e^{i(-52^\circ \pm 5^\circ)} + CP \text{ asymmetric terms}. \tag{46}$$

Substituting the experimental values (18) and (46) into the formulas (44) and (45) one obtains

$$|\varepsilon_0| = (1.99 \pm 0.15) \times 10^{-3} \arg \varepsilon = 42.2^0 \pm 7.2^0, \tag{47}$$

$$|\gamma_2 - \gamma_0| < 12 \times 10^{-3} \arg (\gamma_2 - \gamma_0) = \text{unknown}. \tag{48}$$

($|\varepsilon_0|$ is given essentially by $|\eta_{+-}|$ and $|\eta_{00}|$ accurately, but $|\gamma_2 - \gamma_0|$ is sensitive to the inaccuracy of arg η_{+-} and arg η_{00}.) The separated information available on the CP-breaking parameters of the neutral K meson system is summarized in the following Table 1. (All values are given in 10^{-3} units.) The Table shows that the CP and the T symmetries are definitely broken in neutral K meson decays, but there is no indication of any CPT breaking of a comparable strength. Nature thus reveals itself to be microscopically irreversible, although even here it behaves causally (Fig. 2).

Table 1

Source of information	Irreversibility (*T* asymmetry)	Acausality (*CPT* asymmetry)
Unitarity $K \to \pi\pi,\ I = 0$ $K \to \pi\pi,\ I = 2$ $K \to \pi e\nu$	$\mathrm{Re}\,\varepsilon = 1.47 \pm 0.15$ $\mathrm{Im}\,(\varepsilon + \gamma_0) = 1.43 \pm 0.26$ $\lvert\mathrm{Im}\,(\gamma_2 - \gamma_0)\rvert < 12$ $\xi = -5 \pm 38$	$\mathrm{Im}\,\delta = 0.09 \pm 0.19$ $\mathrm{Re}\,(\delta - \gamma_0) = 0.01 \pm 0.19$ $\lvert\mathrm{Re}\,(\gamma_2 - \gamma_0)\rvert < 12$ $\beta = 0.35 \pm 0.46$ (all values in 10^{-3} units)

IV. THE MINIMUM MODEL

The only asymmetry parameter significantly differing from zero is ε. By exploiting the possibility of the $H_{tot} \to H_{new}(c)$ replacement (10) we can put $\mathrm{Im}\gamma_0 = 0$ (Wu-Yang convention [6]), which gives

$$\varepsilon = (1.47 \pm 0.15) \times 10^{-3} + i(1.43 \pm 0.26) + 10^{-3}. \tag{49}$$

It is very tempting to assume that all the observed asymmetries are explained by this single parameter, which characterizes the time-odd part of the effective Hamiltonian $\mathscr{H}$ [7]:

$$\mathscr{H} = \frac{M_S + M_L}{2} + \sigma_1 \frac{M_S - M_L}{2} + i\varepsilon\sigma_2(M_S - M_L). \tag{50}$$

Let us check the confidency of this „minimum model". By assuming that $\beta = \mathrm{Re}\,\gamma_I = \delta = 0$ (exact *CPT* symmetry) and $\mathrm{Im}\,\gamma_2 = \xi = 0$ (the microscopic irreversibility is concentrated into the neutral *K* eigenstates produced by the effective Hamiltonian (49) and no further irreversibility is to be found in the decay matrix elements), we obtain the unique relations

$$\eta_{+-} = \eta_{00} = \varepsilon = \varepsilon_0 = p^{-1}\varepsilon_2 \text{ for the minimum model.} \tag{51}$$

The unitarity relation tells us that the phase of this complex number is equal to the z quoted in eq. (32). Its real part can be obtained also from the charge asymmetry of the leptonic decays. Thus by writing

$$\lambda = (1 + i \tan z) \frac{1 - \mathrm{Re}x}{1 + \mathrm{Re}x} \frac{\alpha}{2} \text{ for the minimum model} \tag{52}$$

we can collect three independent pieces of experimental information about the strength of the time-reversal asymmetry:

$$\varepsilon = \begin{cases} \eta_{+-}, & |\eta_{+-}| = (1.98 \pm 0.04) \times 10^{-3}, \quad \arg \eta_{+-} = 41.8° \pm 28° \\ \eta_{00}, & |\eta_{00}| = (2.09 \pm 0.10) \times 10^{-3}, \quad \arg \eta_{00} = 43.2° \pm 19° \\ \lambda, & |\lambda| = (2.25 \pm 0.29) \times 10^{-3}, \quad \arg \lambda = 43.0° \pm 0.13° \end{cases} \tag{53}$$

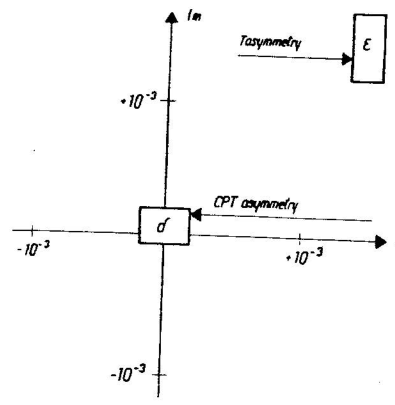


Fig. 2. Strength pf the T and the CTP violations in the neutral K eigenstates (empirical values of ε and δ).

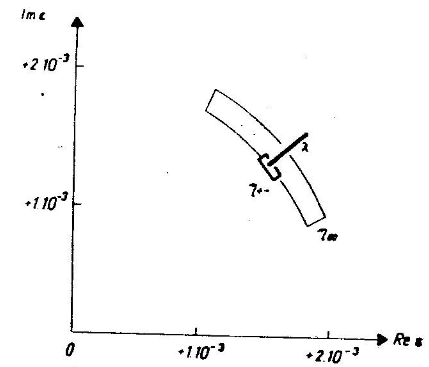


Fig. 3.

The coincidence of these three values makes it understandable that the „minimum model" is very popular among theoriticians (Fig. 3): is is characterized by a single asymmetry parameter appearing in the effective Hamiltonian (15):

$$[\ \delta = 0, \qquad \varepsilon = (2.00 \pm 0.04) \times 10^{-3} \exp i[43.0° \pm 0.1°].$$

V. CONCLUSION AND OUTLOOK

Let us try to formulate the conclusions of our numerical results. What can we say about the *CP*-odd Hamiltonian

$$H' = \tfrac{1}{2}[H_{tot} - (CP)^{-1}H_{tot}(CP)]? \tag{54}$$

The experimental indication that $\varepsilon \neq 0$ but $\delta = \gamma_I = \mathrm{Im}x = 0$ says that the matrix element $\langle K^0|H'|\bar{K}^0\rangle \neq 0$, but he matrix elements $\langle K^0|H'|$ decay states$\rangle \sim 0$, i.e. H' „works" within the neutral K subspace, but does not connect the K-states with states outside the K subspace. From our previous formulas it follows

$$|\varepsilon| = \frac{2\tau_S}{1 - i \tan z} \mathrm{Im}\langle \bar{K}^0|H' + \ldots|K^0\rangle| = 2 \times 10^{-3}, \tag{55}$$

thus ε can be used to estimate the strength g' of the *CP* violating H'. The predictions for the measurable quantity

$$\omega = \left| \frac{\eta_{+-} - \eta_{00}}{\eta_{+-} + \eta_{00}} \right|$$

for the case when H' is built up purely from hadronic operators, and is characterized by a definite $SU(3)$ property are summarized in Table 2 [8].

The experimental finding $\omega < \frac{1}{2}$ can be considered to be a hint for $\omega \ll 1$. This strong inequality can be explained in the three different ways: with superweak, milliweak or millistrong realizations of the millistrong model. A choice among these possibilities would be possible only with asymmetry experiments performed outside neutral K-meson physics.

What about the coupling constant of hypothetical CPT-violating interactions? We know that

Table 2

ΔY	ΔI	g'	ω	
0	0	10^{-3}	137^{-1}	millistrong
	1	10^{-3}	1	
	2	10^{-3}	1	
	3	10^{-1}	137	
1	1/2	10^{-8}	137^{-1}	milliweak
	3/2	10^{-8}	1	
	5/2	10^{-6}	1	
2	0	10^{-11}	10^{-10}	superweak
	1	10^{-13}	10^{-10}	
	2	10^{-11}	10^{-8}	
$n > 2$		10^{-23+5n}	10^{-10}	

$$|\delta| = \left| \frac{i\tau_S}{1 - i\tan z} \langle K^0 \right| H-(CPT)^{-1}H(CPT) + \text{higher order terms } |K^0\rangle| < $$
$$< 2 \times 10^{-4}.$$

If the CPT-odd part of the Hamiltonian were characterized by the selection rule $\Delta Y = 0$ or 1 or 2, the corresponding upper limits on its coupling constant would be 10^{-14}, 10^{-9}, 10^{-4}. We can state that a CPT asymmetry with a strength comparable to the T asymmetry can be excluded experimentally only for $\Delta Y = 0$ transitions.

Coming back to the definitely observed T asymmetry, we still have to answer the puzzling question: If Nature is irreversible even microscopically, why does it hide this property so well? Is it a primary fact that the coupling constant g' in the odd coupling H' is small, or may we ask for an explanation of the weakness of the T asymmetry?

Well, it is known that no exotic particles can be found in the recent Table of Particle Properties. For this reason we are allowed to think in terms of quarks: only such particles and such vertices are of importance for nature, which can be built up simply by quarks. On the other hand, it is exceedingly difficult to write down a coupling which is invariant against all transformations but C and T. One is forced to use the scalar product of γ_μ and ∂_μ (with current odd and the momentum even with respect to C). Examples are:

$\bar{\psi}\gamma\psi_\mu \cdot \partial_\mu\bar{\psi}\psi$ corresponding to $\omega_\mu\partial_\mu\varepsilon$
$\bar{\psi}\gamma_5\partial_\mu\psi \cdot \bar{\psi}\gamma_5\gamma_\mu\psi$ corresponding to $A_{1\mu}B_\mu$
$\bar{\psi}\gamma_\mu\psi \cdot \bar{\psi}\gamma_5\psi \cdot \partial_\mu\bar{\psi}\gamma_5\psi$ corresponding to $(\pi\varrho_\mu)\partial_\mu\eta$

(Here ω, ε, A_1, B, π, ϱ and η stand for the field operators of the corresponding mesons.) These structures are realized by rare particles (excited bound quark pairs) or by sophisticated centrifugal barriers. This offers an explanation of why it is so hard to observe the T asymmetry in Nature, but we have still not succeeded in clarifying the CP puzzle. We have shown only that it may be related to another unsolved problem of nature: to the quark puzzle.

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