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SOME OPEN QUESTIONS IN K DECAY[1]

MARY K. GAILLARD*, Geneva

We concentrate our attention on the decays $K_{\pi 3}$ – with respect to the origin of the $|\Delta I| = \frac{1}{2}$ rule – and $K_{\mu 3}$ – with respect to the breaking of chiral symmetry. We also comment on K_{e4} and $K_{\mu\mu}$ decays. The various isospin amplitudes are extracted from existing data on non-leptonic decays and ambiguities arising from electromagnetic mass differences are displayed. The $|\Delta I| = \frac{3}{2}$ amplitudes are compared with the pole contribution and with current algebra predictions. We discuss the extrapolation of the soft pion theorem for $K_{\mu 3}$ to the physical region. Bounds are given for the divergence form factor as a function of chiral symmetry breaking parameters under the assumption of twice subtracted scalar and pseudoscalar propagators.

I. NON-LEPTONIC DECAYS: A PROBE OF THE ORIGIN OF THE $\Delta I = \frac{1}{2}$ RULE

It is a well known fact that the usual current-current theory of weak interactions gives rise to an effective nonleptonic, strangeness changing, operator which contains $|\Delta I| = \frac{1}{2}$ and $|\Delta I| = \frac{3}{2}$ components which are a priori of a roughly equal strength. However, the observed $|\Delta I| = \frac{1}{2}$ selection rule has led theorists to construct alternative interactions which have only a component with $|\Delta I| = \frac{1}{2}$; higher transitions would then arise only at the level of electromagnetic corrections.

Thus we have on the one hand a theory which predicts

$$A(\tfrac{3}{2})/A(\tfrac{1}{2}) \sim 1$$

$$A(\tfrac{5}{2}) \sim A(\tfrac{7}{2}) \sim \alpha A(\tfrac{1}{2})$$

and on the other hand a theory which predicts

$$A(\tfrac{3}{2}) \sim A(\tfrac{5}{2}) \sim \alpha A(\tfrac{1}{2})$$

$$A(|\Delta I| > \tfrac{5}{2}) \sim \alpha^2 A(\tfrac{1}{2})$$

[1] Talk given at the Triangle Meeting on Weak Interactions, June 4–6, 1973 at SMOLENICE.

* CERN – Div. Théorique, 1211 GENEVA 23, Switzerland. and Laboratoire de Physique Théorique, ORSAY, France.

Experimentally, there is no evidence for transitions with $|\Delta I| > \frac{3}{2}$, and in non-leptonic K decay

$$A(\tfrac{3}{2})/A(\tfrac{1}{2}) \sim 5\,\%.$$

This is large in comparison with the expected ratio ($\alpha \sim 10^{-2}$), if the violation of the $|\Delta I| = \frac{1}{2}$ rule is purely electromagnetic in origin. Thus we are faced with the task of explaining why the violation is so large — and why it is predominantly $|\Delta I| = \frac{3}{2}$.

Conversely, if we assume that the origin of the $|\Delta I| = \frac{3}{2}$ transition is in the effective weak interaction itself, we must explain why it is suppressed relative to the $|\Delta I| = \frac{1}{2}$ part. (This problem is even more acute in Λ decay).

I will not try to offer any explanation for either situation. What I wish to do here is to look closely at the data and see what information can be extracted from them. We shall consider the following questions:

a) what are the relative strengths of the various amplitudes which can contribute?

b) if the violation of $|\Delta I| = \frac{1}{2}$ is of electromagnetic origin, can the pole η contribution account for the observed effect in $K \to 3\pi$;

c) if the $|\Delta I| = \frac{3}{2}$ amplitude is inherent to the weak interaction, soft pion theorems lead to definite predictions for these amplitudes.
Are these predictions satisfied?

The 3π decay is unique in that it can occur via a transition with $|\Delta I| = \frac{7}{2}$. This transition is forbidden to order α^2 if the weak interaction contains only $|\Delta I| = \frac{1}{2}$, whereas it occurs in order α if the weak interaction contains a $|\Delta I| = \frac{3}{2}$ part. It is possible that whatever dynamical mechanism suppresses $|\Delta I| = \frac{3}{2}$ relative to $|\Delta I| = \frac{1}{2}$ will not strongly suppress $|\Delta I| = \frac{7}{2}$ relative to $|\Delta I| = \frac{5}{2}$. The measurement of these amplitudes might then provide an additional clue as to the origin of the $|\Delta I| = \frac{1}{2}$ rule. Unfortunately, since these amplitudes are in any case very small, it will probably not be possible to determine them because of ambiguities arising from electromagnetic mass differences.

I.1. How do we extract the various isospin amplitudes from the data?

$K \to 2\pi$

In this case the analysis is straightforward. The matrix element is a constant, so the phase space corrections due to mass differences are easily evaluated. The matrix elements are

$$\mathscr{A}_s(+-) = 2/\sqrt{3}[A_0^{1/2} + 1/\sqrt{2}(A_2^{3/2} + A_2^{5/2})]$$
$$\mathscr{A}_s(00) = \sqrt{2/3}[A_0^{1/2} - \sqrt{2}(A_2^{3/2} + A_2^{5/2})]$$
$$\mathscr{A}_+(0+) = \sqrt{3/2}(A_2^{3/2} - \sqrt{2/3}A_2^{5/2}),$$

where

$$\mathscr{A}_i(QQ') \equiv \mathscr{A}(K_i \to \pi^Q \pi^{Q'})$$

and $A_I^{\Delta I}$ denotes the change of isospin and the total isospin in the final state. We have the phase conditions

$$A_I^{\Delta I} = |A_I^{\Delta I}|\, e^{i\delta_S^I}$$

where δ_S^I is the $\pi\pi$ phase shift in the $e = 0$ channel. Using the phase shift data[2] [1, 2]:

$$\cos(\delta_S^2 - \delta_S^0) = 0.619 \pm 0.052$$

and the decay rates, we obtain [3]:

$$A_2^{3/2}/A_1^{1/2} = (4.49 \pm 0.55) \times 10^{-2}$$
$$A_2^{5/2}/A_1^{1/2} = (0.08 \pm 0.28) \times 10^{-2}.$$

There is no evidence for a $|\Delta I| = \frac{5}{2}$ transition, but there is a significant $|\Delta I| = = \frac{3}{2}$ contribution of roughly 5 %. (These results are based on the K_s branching ratio given by the Particle Data Group [4]; they would be slightly modified if the new data [5, 6] were used).

$K \to 3\pi$

The evaluation in this case is complicated by several factors. First consider the ideal case where there are no electromagnetic mass shifts. For the K^+ and the K_2 decays the allowed final states are $I = 1, 2, 3$. Each of these can occur via two transitions: $\Delta I = I \pm \frac{1}{2}$. For a given transition to a given final state, all decay distributions are determined by a single function of the pion energies

$$f^I(y_1, y_2, y_3);\; y_i = \frac{3T_i}{Q} - 1;\; \sum y_i = 0;$$

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[2] Protopopescu S. D., Alston-Garnjost M., Barbaro-Galtieri A., Flatté S M., Friedman J. H., Lasinski T. A., Lynch G. R., Rabin M. S., Solnitz F. T. LBL preprint 787 (1972).

where T_i is a pion kinetic energy and Q is the total energy release. The projection onto a particular decay mode is a linear combination of f^I with appropriate permutations of the y_i. For $I = 1$ the squared amplitudes satisfy the integral relations

$$\int_{\text{phase space}} d^3y\, |f_{(y_i)}^{(+-0)}|^2 = \int d^3y\, |f_{(y_i)}^{(00+)}|^2$$
$$\int d^3y |f_{(y_i)}^{(000)}|^2 = \int d^3y (|f_{(y_i)}^{(+-0)}|^2 - |f_{(y_i)}^{(00+)}|^2).$$

Thus if only one transition amplitude, say $\Delta I = \frac{1}{2}$, contributes, we obtain the predictions

$$\frac{\gamma(+-0)}{2\gamma(00+)} = \frac{\gamma(000)}{\gamma(++-) - \gamma(00+)} - 1 \qquad (1)$$

where γ is the "reduced rate" (corrected for phase space). Equation (1) is the only prediction for the rates which is independent of the decay distributions. If we have only $I = 1$ final states, but both $|\Delta I| = \frac{1}{2}$ and $\frac{3}{2}$ transitions, we will have:

$$f_L^{(k)}(y_i) = f_{1/2}^{(k)}(y_i) + f_{3/2}^{(k)}(y_i)$$
$$f_+^{(k)}(y_i) = f_{1/2}^{(k)}(y_i) - \tfrac{1}{2} f_{3/2}^{(k)}(y_i)$$

where k denotes the final state charge mode. We expect that $f^{3/2}$ and $f^{1/2}$ will have different functional dependence, so that the rate ratios in Eq. (1) will not be modified in the same way.

However, if quadratic and higher terms can be neglected in the decay distributions, then since a linear energy dependence does not affect the rates, the two ratios in (1) will remain equal (but different from unity), and the (symmetric) $\Delta I = \frac{3}{2}$ amplitude will be well defined. Then we obtain the further predictions

$$\frac{3\gamma(+-0)}{2\gamma(000)} = \frac{\gamma(++-)}{4\gamma(00+)} = 1 \qquad (2)$$

in the absence of $I > 1$ final states. $I = 2$ states contribute to the rates only through the square of the amplitude which is suppressed by potential barrier effects; thus Eq. (2) is really a test for the absence of $I = 3$ final states.

One can of course remove the dependence on decay distributions by defining the phase space integral as

$$\varphi_k = \int d^3y |\mathscr{A}_i^k(y_1, y_2, y_3)|^2 / |\mathscr{A}_i^k(0, 0, 0)|^2. \qquad (3)$$

Then the "reduced rate"

$$\gamma^k = \Gamma^k/\varphi_k \equiv |\mathscr{A}_i^k(0, 0, 0)|^2 \qquad (4)$$

is simply the squared amplitude at the centre of the Dalitz plot where only totally symmetric waves contribute; in this way the various amplitudes are unambigously defined.

However, at this point, mass differences become important. The centre of the Dalitz plot is the point where all energies are equal:

$$y_i = 0,\ T_i = Q/3.$$

However, if we use instead the variables

$$s_i = (p_k - p_i)^2, \qquad s_0 \equiv \sum s_i/3,$$

the centre of the Dalitz plot is defined by

$$s_i - s_0 = 0$$

$$T_i = Q/3 - \left(m_i - \frac{\sum m_j}{3}\right) + m_i^2 - \frac{\sum m_j^2}{3}.$$

Thus even the definition (4) of the reduced rates is ambiguous. This problem already arises at the level of linear distributions; because the Dalitz plot is distorted by mass differences, the linear term in fact contributes to the integral over phase space. A similar problem appears in the determination of the various transition amplitudes for the linear terms (nonsymmetric waves); the results depend on the parametrization chosen. We shall not advocate a particular parametrization, but we shall compare different parametrizations commonly used. Bose symmetry requires that the matrix elements for K^+ and K_L decay be symmetric in two energies. We may take as variables

$$y = y_3 = -(y_1 + y_2)$$

$$x = (y_1 - y_2)/\sqrt{3}$$

$$x^2 + y^2 = \tfrac{2}{3}\sum y_i^2$$

where 3 refers to the odd pion in the K^+ decay and π^0 in the K_L decay. Thus

$$\mathscr{A} = \mathscr{A}(0)\left(1 + \frac{\sigma}{2}y + \frac{\alpha}{2}y^2 + \frac{\beta}{2}x^2 + \ldots\right). \qquad (5)$$

For the $3\pi^0$ mode total symmetry requires

$$\sigma = 0,\ \alpha = \beta.$$

I.1.1. Violation of $|\Delta I| = \frac{1}{2}$ in the symmetric waves

We normalize the isospin amplitudes by

$$\mathscr{A}_L^{(+-0)}(0) \equiv A_1^{1/2} + A_1^{3/2} + A_3^{5/2} + A_3^{7/2}$$

Table 1
Partial rates and reduced rates (Coulomb corrections included)

	$\Gamma \times 10^{-6}$ sec		φ (norm. arbitrary)	
Mode	*PDG*	Kellett	*PDG*	Kellett
$+-\bigcirc$	2.43 ± 0.05	2.35 ± 0.10	1.279	1.19 ± 0.05
$\bigcirc\bigcirc\bigcirc$	4.15 ± 0.16	3.99 ± 0.20	1.444	1.44 ± 0.07
$\bigcirc\bigcirc+$	1.40 ± 0.04	1.38 ± 0.04	1.147	1.16 ± 0.06
$++-$	4.52 ± 0.02	4.51 ± 0.03	1.000	1.00

where the notation is the same as for $K \to 2\pi$. The Particle Data Group (4) (PDG) evaluates $|\mathscr{A}(0)|$ using a linear parametrization

$$|\mathscr{A}|^2 = |\mathscr{A}(0)|^2[1 + \sigma(s_3 - s_0)]$$

and Kellett [7] has used an expansion in QY/M_K and $(QX/M_K)^2$ up to cubic terms; he has increased errors to allow for experimental uncertainties in the quadratic terms. His input (1971) is not up-to-date, but we are using his reduced rates as a comparison to illustrate the effects of uncertainties. The input partial rates and phase space factors are shown in Table 1. In Table 2 and 3 we show the amplitudes extracted from both sets of reduced rates. The ratios A_3/A_1 are defined to be equal if $\Delta I = \frac{3}{2}$ and $\Delta I = \frac{7}{2}$ are negligible in the $I = 1$ and the $I = 3$ states, respectively.

There is no evidence for $I = 3$ final states; although there is a three strandard deviation effect in K^+ with the PDG rates, this can easily be due to phase space ambiguities. On the other hand, there is a significant $\Delta I = \frac{3}{2}$ amplitude, comparable to that in $K \to 2\pi$.

I.1.2. Slopes: the non-symmetric amplitudes

In a linear approximation, the amplitudes are of the form

$$\mathscr{A}^{+-0} = A_1^{1/2}(1 + \sigma_1^{1/2}y) + A_1^{3/2}(1 + \sigma_1^{3/2}y) + A_3^{(L)}$$

Table 2
$I = 3$ contribution

	$(A_3/A_1)_{K_L}$	$(A_3/A_1)_{K_+}$	average: $A_3^{5/2}/A_1^{1/2}$
PDG	−0.003 ± 0.014	−0.020 ± 0.006	−0.012 ± 0.007
Kellett	0.019 ± 0.028	−0.013 ± 0.023	0.003 ± 0.018

Table 3
Evaluation of the transition amplitude strengths

	$A_1^{3/2}/A_1^{1/2}$	$A_3^{5/2}/A_1^{1/2}$	$A_3^{7/2}/A_1^{1/2}$
PDG	-0.061 ± 0.010	-0.013 ± 0.005	0.010 ± 0.006
	-0.064 ± 0.008	assumed absent	
Kellett	-0.059 ± 0.037	0.0 ± 0.012	0.018 ± 0.014
	-0.059 ± 0.034	assumed absent	

$$\mathscr{A}^{++-}/\sqrt{2} = A_1^{1/2}(1 - \tfrac{1}{2}\sigma_1^{1/2}y) - \tfrac{1}{2}A_1^{3/2}(1 - \tfrac{1}{2}\sigma_1^{3/2}y) + + \tfrac{3}{4}A_2\sigma_2 y + \tfrac{1}{3}A_3^{(+)}$$

$$\sqrt{2}\,\mathscr{A}^{00+} = A_1^{1/2}(1 + \sigma_1^{1/2}y) - \tfrac{1}{2}A_1^{3/2}(1 + \sigma_1^{1/2}y) + + \tfrac{3}{2}A_2\sigma_2 y - \tfrac{4}{3}A_3^{(+)}.$$

The $I = 2$ amplitude arises from both $|\Delta I| = \frac{3}{2}$ and $\frac{5}{2}$ transitions, but they cannot be distinguished except in comparison with the K_s decay. The $I = 3$ final state affects the slope only through the normalization factor. In the following analysis we assume $I = 3$ waves to be absent.

In Table 6 we compare the slopes using three parametrizations. We have used PDG data [4]; a new result [8] presented at NAL is indicated in parentheses. The $\Delta I = \frac{1}{2}$ rule requires

$$\sigma^{+-0} = \sigma^{00+} = -2\sigma^{++-}. \tag{6}$$

The absence of $I = 2$ waves requires

$$\sigma^{00+} = -2\sigma^{++-} \tag{7}$$

Table 4
Extracted values of the slopes

$d\Gamma$ \ σ	σ^{+-0}	σ^{00+}	$-2\sigma^{++-}$
$1 + \sigma y$	-0.804 ± 0.027 (-0.859 ± 0.005)	-0.791 ± 0.037	-0.542 ± 0.010
$1 + \sigma y \frac{Q}{M}$	-4.79 ± 0.17 (-5.12 ± 0.09)	-4.62 ± 0.22	-3.57 ± 0.033
$1 + \sigma(s_3 - s_0)$	0.604 ± 0.023 (0.648 ± 0.009)	0.523 ± 0.023	0.428 ± 0.008

Table 5
Strengths of the transition amplitudes for non-symmetric waves;
$\delta_I^{\Delta I} \equiv (A_I^{\Delta I}\sigma_I^{\Delta I})/(A_1^{1/2}\sigma_1^{1/2})$

δ \ $d\Gamma$	$1 + \sigma y$	$1 + \sigma y \frac{Q}{M}$	$1 + \sigma(s_3 - s_0)$
$\delta_1^{3/2}$	0.061 ± 0.033	0.039 ± 0.031	0.097 ± 0.033
δ_2	0.240 ± 0.042	0.248 ± 0.048	0.192 ± 0.042

and the current-current theory (to be discussed below) requires

$$\sigma^{+-0} = \sigma^{00+}. \tag{8}$$

The ratios

$$\delta_I^{\Delta I} \equiv A_I^{\Delta I}\sigma_I^{\Delta I}/A_1^{1/2}\sigma_1^{1/2} \tag{9}$$

give a measure of the relative transition strengths in the non-symmetric waves. They are shown in Table 5 where the PDG slopes have been used. If the new value for σ^{+-0} were used, the violation in the $I = 1$ state would be increased by about 0.04. At this level the uncertainties in the reduced rates are negligible (we used PDG), but the results clearly depend on the slope parametrization. However, there appears to be a violation of the $|\Delta I| = \frac{1}{2}$ rule in the $I = 1$ non-symmetric waves of the same order as in the symmetric waves. The presence of an $I = 2$ final state is particularly indicated, and its strength is apparently enchanced relative to the violation in the $I = 1$ state. (Our amplitudes are all normalized to contribute with the unit relative coefficient in $K^0 \to \pi^+\pi^-\pi^0$; the K_s amplitude for $I = 2$ is $\mathscr{A}_s(+-0) = -A_2\sigma_2(y_+ - y_-)$ if $\Delta I = \frac{5}{2}$ is absent). However, we should perhaps make the comparison

Table 6
Data on quadratic terms

	Expansion in (x, y)		Expansion in $\frac{Q}{M} \times (x, y)$	
Mode	α	$\alpha + \beta$	α	$\alpha + \beta$
○+−	-0.266 ± 0.034 (-0.060 ± 0.010)	—	-9.4 ± 1.2 (-2.12 ± 0.36)	—
○○+	-0.396 ± 0.122	—	-14 ± 4	—
○○○	—	0.12 ± 0.14	—	3.5 ± 4
++−	-0.011 ± 0.014 (0.066 ± 0.010)	-0.041 ± 0.023	-0.46 ± 0.60 (0.29 ± 0.45)	-1.76 ± 1.00
	$(3\beta - \alpha)/2 = -0.05 \pm 0.04$		$(3\beta - \alpha)/2 = -2.2 \pm 1.6$	

in a different way. The leading term for $I = 1$ is a pure s wave and the leading term for $I = 2$ is a combination of p waves, i.e., of the form

$$\mathscr{A}_2 = A_2\sigma_2(y_1 - y_2) = \tfrac{3}{2}\,\frac{A_2\sigma_2}{QM}\,(\mathbf{q}_1 - \mathbf{q}_2)\mathbf{q}_3.$$

If $\Delta I = 3/2$ contributes a priori with equal strength to both waves we would expect

$$|\mathscr{A}_2^{3/2}/\mathscr{A}_1^{3/2}| \sim |\mathbf{q}_1 - \mathbf{q}_2|\,|\mathbf{q}_3|\,R^2,$$

where R is the interaction radius. Taking $A_1^{3/2}/A_1^{1/2} = -0.06$ and $\delta_2^{3/2} = \cdot 0.24$ we find

$$R \approx 3.6/M_k \approx 1/m_\pi.$$

It is a question of intuition as to whether this is an unreasonably large value for R.

I.1.3. Quadratic terms

Since there has been some evidence for quadratic terms in the matrix elements we shall briefly discuss them. If we parametrize the amplitudes as in Eq. (5), the decay distribution is

$$|\mathscr{A}|^2 = \mathscr{A}|(0)|^2\left(1 + \sigma y + \left(\alpha + \frac{\sigma^2}{4}\right)y^2 + \beta x^2 + \dots\right).$$

If the quadratic term in y differs significantly from $\sigma^2/4$, this is evidence of a quadratic term in the matrix element. If these terms are important, comparison with soft pion predictions becomes very ambiguous because the expansion will not converge outside the physical region ($Q/M \sim 1$).

If there are no $I = 3$ final states one has the predictions

$$\alpha^{+-0} + \beta^{+-0} = \alpha^{000} + \beta^{000} \qquad (10)$$

$$\alpha^{++-} + \beta^{++-} = \alpha^{00+} + \beta^{00+}. \qquad (11)$$

The $\Delta I = \frac{1}{2}$ rule requires the equality of (10) and (11) as well as the conditions

$$\alpha^{+-0} = \alpha^{00+} = (3\beta^{++-} - \alpha^{++-})/2. \qquad (12)$$

We do not propose looking at quadratic terms to test the presence of higher isospin transitions. However, given the suppression of these transitions in lower order terms, it seems fair to suppose that if the above conditions are not roughly satisfied, the extracted values for higher order coefficients can be considered as spurious. In Table 6 we have collected some data on quadratic

terms. The results in parentheses are recent results (for (+−0), Ref. [8], for (++−), Ref. [9]) presented at NAL; the other values are from an analysis by Kellett [7] of earlier data (for (++−) Ref. [10, 11][3], for (00+) Ref. [12], for (+−0) Ref. [13], for (000) Ref. [14]).

The most compelling evidence for a quadratic term appears to be in the (+−0) mode. In this respect it is interesting that there is also evidence for a quadratic term in the η decay. It has been suggested [15] that an electromagnetic violation of the $\Delta I = \frac{1}{2}$ rule in $K \to 3\pi$ might be enhanced by the proximity of the η pole (Fig. 1). As η decays to 3π via a $\Delta I = 1$ transition, this would explain the absence of a comparable $\Delta I = \frac{5}{2}$ amplitude. (However, no similar argument is readily found for $K \to 2\pi$). The η pole contributes only to the K_L decay and could also account for a quadratic energy dependence which would be absent in the K^+ decay.

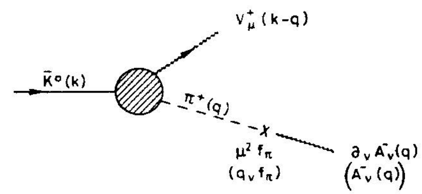

Fig. 1. The η pole contribution to $K_L \to 3\pi$.

I.2. Can the η pole explain the violation of $|\Delta I| = \frac{1}{2}$ in $K \to 3\pi$?

It can certainly not account for the $I = 2$ waves in the K^+ decay. Let us nevertheless consider in detail its contribution to the $I = 1$ amplitude. As it does not contribute to the K^+ decay, the η amplitude must contain $\Delta I = \frac{1}{2}$ and $\frac{3}{2}$ parts in the ratio

$$\mathscr{A}_\eta^{1/2} = \tfrac{1}{2}\,\mathscr{A}_\eta^{3/2}.$$

The contribution to the K_L decay is then

$$\mathscr{A}_\eta = \mathscr{A}_\eta^{1/2} + \mathscr{A}_\eta^{3/2} = \tfrac{3}{2}\,\mathscr{A}_\eta^{3/2}.$$

If A is the weak amplitude ($\Delta I = \frac{1}{2}$) in the absence of the η pole, the relative $|\Delta I| = \frac{3}{2}$ amplitude will be

$$A^{3/2}/A^{1/2} = \tfrac{2}{3}\mathscr{A}_\eta/(A + \tfrac{1}{3}\,\mathscr{A}_\eta)$$

Using the results of Table 3, we find

$$\mathscr{A}_\eta/A = \begin{cases} -0.093 \pm 0.011 \ \text{(PDG)} \\ -0.086 \pm 0.047 \ \text{(Kellett)} \end{cases}$$

[3] Grauman et al., Stevens Institute of Technology, preprint SIT-P256 (1970).

Including linear terms in the matrix element

$$\mathscr{A}(+-0) = A\left(1 + \frac{\sigma}{2}y\right) + \mathscr{A}_\eta\left(1 + \frac{\sigma_\eta}{2}y\right)$$

we obtain an effective slope for the decay distribution

$$\sigma^{+-0} = (A\sigma + \mathscr{A}_\eta\sigma_\eta)/(A + \mathscr{A}_\eta)$$

where $\sigma = \sigma^{00+} = -2\sigma^{++-}$ is the slope in the absence of the η pole. Thus we obtain a prediction

$$\sigma^{00+} = -2\sigma^{++-} = \sigma^{+-0} - \frac{\mathscr{A}_\eta}{A}(\sigma_\eta - \sigma^{+-0}). \tag{13}$$

Before using Eq. (13) we must evaluate the measured slope in η decay in terms of K decay variables. Here again we run into an ambiguity.

Technically, the η pole is evaluated by writing a dispersion relation in the variable $(\sum p_i)^2$, keeping other variables fixed. The choice of these variables is not unique. If we keep $E_3 = E_{\pi^0}$ fixed, we may parametrize the η decay by

$$d\Gamma_\eta \sim 1 + y_\eta\sigma_\eta/2$$

$$y_\eta = 3T_3/Q_\eta - 1 = \frac{Q_K}{Q_\eta}(y + 1) - 1$$

where Q_K and Q_η are the ener gy releases in K_L and η decay, respectively Then the effective slope to be used in Eq. (13) is

$$\sigma_\eta^{\text{eff}} = \sigma_\eta \frac{Q_K}{Q_\eta}\left[1 - \frac{\sigma_\eta}{2Q_\eta}(Q_\eta - Q_K)\right]^{-1}.$$

If instead we keep the fixed invariant

$$s_3 = (p_1 + p_2)^2$$

and parametrize η decay by

$$d\Gamma_\eta \sim 1 + \frac{\sigma_\eta}{2}(s_3 - s_0)$$

$$s_0^\eta = (m_\eta^2 - \sum m_i^2)/3 = s_0^K + (m_\eta^2 - m_K^2)/3,$$

the effective slope in (13) is

$$\sigma_\eta^{\text{eff}} = \sigma_\eta[1 - \sigma_\eta(m_\eta^2 - m_K^2)/6]^{-1}.$$

Table 7
Predictions for slopes in the K^+ decay if $|\Delta I| = > 1/2$ is due to the η pole:
$a_\eta/A = -0.093 \pm 0.011$

Variable fixed		$d\Gamma$	σ_η	σ_η^{eff}	$\sigma^{00+} = -2\sigma^{+-0}$
	E_3	$1 + \sigma y$	-1.08 ± 0.01	-0.555 ± 0.004	-0.781 ± 0.026 (-0.831 ± 0.005)
	E_3	$1 + \sigma y \frac{Q}{M}$	-4.40 ± 0.04	-3.31 ± 0.025	-4.65 ± 0.156 (-4.95 ± 0.032)
	S_3	$1 + \sigma(s_3 - s_0)$	0.452 ± 0.004	0.359 ± 0.006	0.581 ± 0.021 (0.622 ± 0.005)

The predictions for both cases are shown in Table 7 where we have taken[4]

$$\sigma_\eta = -1.08 \pm 0.01 \qquad \text{for } d\Gamma \sim 1 + y\sigma_\eta.$$

The predictions in parentheses obtain if the new value [8] for σ^{+-0} is used. In all cases agreement with the experimental slopes is improved (see Table 4) with respect to the $\Delta I = \frac{1}{2}$ prediction (6) (but a discrepancy remains if the new value of σ^{+-0} is used).

A recent experiment indicates that the η decay amplitude requires a quadratic term[4]

$$d\Gamma(\eta \to 3\pi) \sim 1 + (-1.08 \pm 0.01)y + (0.03 \pm 0.03)y^2$$

giving (see Eq. (5))

$$\alpha_\eta = -0.26 \pm 0.03.$$

The effect of the quadratic term on the predicted slopes in K decay is shown in Table 8 along with the η contribution to the quadratic term in $K_L \to (+-0)$. The η contribution clearly cannot account for the quadratic term (Table 6) observed in this decay.

I.3. Current algebra and the $\Delta I = 3/2$ amplitude

It is well known that the amplitudes for $K \to 3\pi$ can be related to those for $K \to 2\pi$ by means of soft pion theorems; the theorems are of the form [16, 17]:

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[4] Layter J. G., Appel J. A., Kotlewski A., Lee W., Stein S., Thaler J. J., Columbia University preprint (1972).

Table 8
Predictions from η pole contribution with quadratic decay distribution (E_3 fixed)

Parametrization	$\sigma^{00+} = -2\sigma^{++-}$	η contribution to $\alpha(+-\bigcirc)$
x, y	-0.776 ± 0.029 (-0.826 ± 0.006)	0.025 ± 0.003
$\frac{Q}{M} \times (x, y)$	-4.53 ± 0.158 (-4.83 ± 0.043)	0.589 ± 0.070

$$\lim_{p_i \to 0} \langle \pi_i \pi_j \pi_k | H^{\text{weak}} | K \rangle = \frac{i}{f_\pi} \langle \pi_j \pi_k | [F_i^{(5)}, H^{\text{weak}}] | K \rangle \tag{14}$$

where if A_μ^i is a component of the $I = 1$ axial current,

$$F_i^{(5)} = \int d^3x A_0^i(x, o) \tag{15}$$

is a generator of chiral $SU_2 \otimes SU_2$. The chiral properties of the current-current interaction are such that

$$[F_i^{(5)}, H^{\text{weak}}] = [T_i, H^{\text{weak}}],$$

where T_i is a generator of isotopic spin. Since commutation with T_i cannot change the isospin representation of an operator, we obtain separate theorems for the $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ amplitudes.

An ambiguity arises from the fact that for $\Delta I = \frac{1}{2}$, the two pions are in an $I = 0$ S-wave state and the amplitude on the right-hand side of (14) has a significant phase [1, 2][2] ($\delta_s^0 = 44° \pm 5°$). In the physical region for 3π decay phase shifts are expected to be small; thus the amplitude should be nearly real. This problem is diminished when we study $\Delta I = \frac{3}{2}$ transitions since the $I = 2$ phase shift is small [1, 2][2] ($\delta_s^2 = -7.7° \pm 1.2°$). To the lowest order in $A^{3/2}/A^{1/2}$ one obtains the predictions [18–20]

a) equality of the expressions (which vanish for $\Delta I = \frac{1}{2}$):

$$V_1 = \frac{2\gamma(00+)}{\gamma(+-0)} - 1, \qquad V_2 = \frac{\gamma(++-) - \gamma(00+)}{\gamma(000)} - 1,$$

$$V_3 = \left(\frac{\gamma(+-)}{2\gamma(00)} - 1\right) / \cos(\delta_s^1 - \delta_s^2), \qquad V_4 = 2\sqrt{\frac{6\gamma(0+)}{\gamma(+-) - \gamma(00)}}.$$

b) $\sigma^{+-0} = \sigma^{00+} = -2\sigma^{++-}(1 - \frac{9}{4}V_3)$.

The comparison is shown in Tables 9 and 10. In Table 10 we have used the

Table 9
Comparison of $\Delta I = 3/2$ amplitudes with soft pion predictions: rates

	$K \to 3\pi$		$K \to 2\pi$	
	V_1	V_2	V_3	V_4
PDG	0.288 ± 0.045	0.146 ± 0.038	0.195 ± 0.030	0.187 ± 0.001
	ave: $V_{3\pi} = 0.217 \pm 0.030$			
Kellett	0.20 ± 0.16	0.20 ± 0.19	ave: $V_{2\pi} = 0.191 \pm 0.015$	
	ave: $V_{3\pi} = 0.20 \pm 0.12$			

average value of V_4 and V_3 and the experimental value for σ^{++-} (Table 4) as input.

Recall that the absence of $\Delta I > \frac{5}{2}$ requires $V_1 = V_2$ and $V_4 = V_3$. If the average values, $V_{2\pi}$ and $V_{3\pi}$, are used, the agreement with the rate predictions is good. Comparison with Table 4 shows that the slope prediction is also reasonable, especially considering the uncertainties associated with the choice of parametrization, and the 10 % uncertainty inherent in soft pion predictions. However, current algebra cannot explain a significant discrepancy between σ^{+-0} and σ^{0++} if it is real.

Of course, the η contribution can be expected to be present in any case. If A represents, say, the amplitude for $(-+0)$ in the absence of the η contribution, we have

$$2(\mathscr{A}^{(00+)}/A)^2 = \gamma(+-)/2\gamma(00) = 1 + V(2\pi).$$

With the η pole included we obtain

$$2\left|\frac{\mathscr{A}^{(00+)}}{A + \mathscr{A}_\eta}\right|^2 = \frac{1 + V(2\pi)}{(1 + \mathscr{A}_\eta/\mathscr{A})^2} = 1 + V(3\pi)$$

so that

$$\mathscr{A}_\eta/A = \left(\frac{V(2\pi) + 1}{V(3\pi) + 1}\right)^{1/2} - 1.$$

Table 10
Slope prediction for $\sigma^{+-0} = \sigma^{00+}$

$d\Gamma$	$1 + \sigma y$	$1 + \sigma y \frac{Q}{M}$	$1 + \sigma(s_3 - s_0)$
σ	-0.738 ± 0.016	-4.86 ± 0.10	0.582 ± 0.012

The slope correction can then be calculated as in Section 2. However, the present agreement between $V(2\pi)$ and $V(3\pi)$ only allows for an η contribution of about 1 % — the expected order of electromagnetic corrections — and the effect on the slopes is negligible.

We are thus left with a rather unsatisfactory situation. If the η pole predictions were satisfied, we would tend to favour the idea that the violation of the $\Delta I = \frac{1}{2}$ rule is electromagnetic; agreement with current algebra predictions would favour the opposite conclusion. Either hypothesis is partially, but not entirely succesful. The second may be favoured since the discrepancy $\sigma^{+-0} \neq \sigma^{00+}$ is less well established than the discrepancy $\sigma^{00+} \neq -2\sigma^{++-}$. However, it is difficult to draw a firm conclusion at present.

II. LEPTONIC DECAYS: A PROBE OF CHIRAL SYMMETRY BREAKING

We shall mainly consider the $K_{\mu 3}$ decay and more particularly the scalar, or divergence, form factor. The decay amplitude is given by

$$M = \frac{G}{\sqrt{2}} \sin \Theta < \pi | V_\mu^\pm | K > \overline{U}_e \gamma_\mu (1 + \gamma_5) U_\nu \tag{16}$$

where G is the Fermi constant, Θ the Cabibbo angle and $V_\mu^+ = (V_\mu^-)^+$ the hadronic strangeness changing current. Its matrix element is traditionally written in terms of two form factors, for example

$$\langle \pi^+(q) | V_\mu^+ | \bar{K}^\circ(k) \rangle = (k+q)_\mu f_+(t) + (k-q)_\mu f_-(t). \tag{17}$$

The normalization is chosen such that, assuming the Cabibbo theory to be correct, $f_+(0) = 1$ in the limit of SU_3 symmetry. The matrix elements of V_μ^+ between other charge states are determined from (16) by charge conjugation and/or the $\Delta I = \frac{1}{2}$ rule. I would like to stress here that the semi-leptonic $|\Delta I| = \frac{1}{2}$ rule and the Cabibbo theory of currents which incorporates it are basic to the entire subsequent discussion which will centre on the soft pion theorem in the $K_{\mu 3}$ decay. The failure of the soft pion theorem might be accommodated by changing a parameter in the theory, whereas the failure of the $\Delta I = \frac{1}{2}$ rule (and consequently of Cabibbo theory) would require a re-writing of the whole theory — and in particular would invalidate the soft pion theorem.

The scalar form factor is a particaular combination of f_+ and f_- which is proportional to the matrix element of the current divergence

$$f(t) \equiv f_+(t) + \frac{t}{M^2 - \mu^2} f_-(t) = \frac{i}{M^2 - \mu^2} \langle \pi^+ | \partial_\mu V_\mu^+ | \bar{K}^0 \rangle \tag{18}$$

where M and μ are the K and π masses, respectively. While $f_+(t)$ corresponds to the pure spin one exchange, $f(t)$ corresponds to the spin zero exchange. Thus $f_+(t)$ and $f(t)$ are dynamically independent amplitudes and more suitable for theoretical analysis than f_+, f_-.

Let us recall the well-known soft pion theorem [21—23] for the K_{13} form factors

$$f_+(M^2) + f_-(M^2) = f_K/f_\pi \tag{19}$$

where f_K and f_π are the decay constants for K_{l2} and π_{l2}, respectively. Equation (19) may readily be expressed as a theorem for the divergence form factor

$$f(M^2) = f_+(M^2) + f_-(M^2) + \frac{\mu^2}{M^2 - \mu^2} f_-(M^2) = \frac{f_K}{f_\pi} + 0\left(\frac{\mu^2}{M^2}\right). \tag{20}$$

Note that: ***a***) one expects in any case corrections of order μ^2; ***b***) from what we know about $f_+(t)$ (it is probably compatible with K^* dominance in the physical region) extrapolation to $t = M^2$ implies that $f_-(M^2) \ll 1$ if Eq. (19) holds; in that case the difference between (19) and (20) should be much less than 10 %.

Thus we shall hereafter refer to the prediction

$$f(M^2) = f_K/f_\pi \tag{21}$$

as the soft pion theorem. The bulk of our discussion will be devoted to two questions: ***a***) why is it important to test the soft pion theorem? ***b***) how can it be tested, as the prediction is for a t value outside the range accessible to experiment?

II.1. Why?

The validity of Eq. (21) is closely related to the validity of chiral $SU_2 \otimes SU_2$ as an approximate symmetry of the Lagrangian. To see this it is instructive to recall the derivation of Eq. (19). It is based on the Ward identity

$$iM_\mu(q) = q_\nu M_{\mu\nu}(q) - i \int d^4x e^{iqx} \delta(x_0) \langle | [A_0^-(x), V_\mu^+(0)] | \bar{K}^0(k) \rangle. \tag{22}$$

The amplitudes M_μ and $M_{\mu\nu}$ are illustrated in Fig. 2 and A_μ^- is the charge lowering $\Delta S = 0$ axial current. In the limit $q_\mu \to 0$, the first member on the right in (22) vanishes and the second member reduces to (see Eq. (15))

$$-i \langle | [F_-^{(5)}(0), V_\mu^+(0)] | \bar{K}^0(k) \rangle = -i \langle | A_\mu^{6+i7}(0) | \bar{K}^0 \rangle = k_\mu f_K. \tag{23}$$

The commutator (23) has been evaluated using the $SU_3 \otimes SU_3$ algebra of currents which is derived from Cabibbo theory on the one hand, and abstraction from a free quark model on the other hand (it is of course valid in many other models).

The soft pion prediction is obtained under the assumption that for $0 \lesssim q^2 \lesssim \mu^2$, the amplitude $M_\mu(q)$ is dominated by the pion pole (Fig. 3)

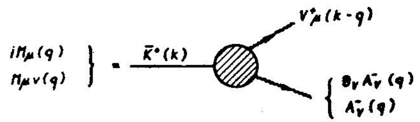

Fig. 2. Diagrammatic representation of the amplitudes M_μ and $M_{\mu\nu}$ (Eq. (22)).

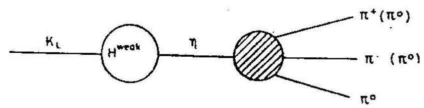

Fig. 3.

$$M_\mu(q) \approx \frac{f_\pi \mu^2}{\mu^2 - q^2} \{f_+(t)\,(k+q)_\mu + f_-(t)\,(k-q)_\mu\} \tag{24}$$

and by equating (24) with (23) for $q_\mu \to 0$. Thus the validity of (19) (or of (21)) appears to depend only on the nearness of the pion pole, that is, on the smallness of the pion mass. Consider, however, the limit of the zero pion mass. Then the pion decouples from the axial divergence

$$\langle|\partial_\mu A|\pi(q)\rangle = -iq_\mu\langle|A_\mu|\pi(q)\rangle = q^2 f_\pi \xrightarrow[\mu^2\to 0]{} 0. \tag{25}$$

However, for $\mu^2 = 0$ the amplitude $M_{\mu\nu}(q)$ has a pole (Fig. 3) of the form

$$\frac{q_\mu f_\pi}{-q^2} \{f_+(t)\,(k+q)_\mu + f_-(t)\,(k-q)_\mu\}.$$

Then in the limit $q_\mu \to 0$, Eq. (22) becomes

$$q_\nu M_{\mu\nu}(q) \xrightarrow[q_\nu\to 0]{} -f_\pi[f_+(M^2) + f_-(M^2)]k_\mu = iM_\mu(0) - f_K k_\mu.$$

The soft pion theorem is satisfied if and only if

$$M_\mu(0) \approx 0,$$

which will hold if the axial current is nearly conserved

$$\partial_\nu A_\nu(x) \approx 0.$$

If the smallness of the pion mass is a dynamical accident, we cannot generally expect soft pion results to be good; on the other hand, if the smallness of the pion mass reflects an approximate chiral symmetry of the Lagrangian we indeed expect soft pion theorems to work.

The above considerations hold in the general case where V^+_μ is replaced by

any local operator. The interesting point in the K_{l3} decay is that we can derive an alternative soft pion theorem which conflicts with Eq. (21) if the chiral SU_2 symmetry is badly broken. Consider the matrix element of the current divergence. We obtain

$$i\langle\pi^+|\partial_\mu V^+_\mu|\bar{K}^0\rangle \xrightarrow[q_\mu\to 0]{} \frac{1}{f_\pi}\langle|[F^{(5)}_-,\ \partial_\mu V^+_\mu]|\bar{K}^0\rangle = \frac{1}{f_\pi}\partial_\mu\langle|[F^{(4)}_-,\ V^+_\mu]|\bar{K}^0\rangle - \frac{1}{f_\pi}\langle|[\dot{F}^{(5)}_-,\ V^+_\mu]|\bar{K}^0\rangle. \tag{26}$$

The first term on the right in (26) is given by Eq. (23); thus

$$(M^2 - \mu^2)f(M^2) = M^2\frac{f_K}{f_\pi} - \frac{1}{f_\pi}\langle|[\dot{F}^{(5)}_-,\ V^+_\mu]|K^0\rangle. \tag{27}$$

This agrees with Eq. (21) up to the order μ^2/M^2 only if the axial generator is quasi-time independent

$$\dot{F}^{(5)}_- = 0 + O(\mu^2/M^2),$$

i.e. if the current is nearly conserved.

Let us now specialize to the most popular model of symmetry breaking, the $(3, \bar{3}) \oplus (\bar{3}, 3)$ model, abstracted from free quark theory. The strong interaction Hamiltonian is

$$\mathscr{H} = \mathscr{H}_{\text{invariant}} + \varepsilon_8 U_8 + \varepsilon_0 U_0. \tag{28}$$

The terms which break chiral $SU_3 \otimes SU_3$ transform like quark mass terms

$$U_i = \bar{q}\lambda_i q.$$

In this model the correction to Eq. (27) may be explicitly evaluated

$$f(M^2) = \frac{M^2}{M^2 - \mu^2}\frac{f_K}{f_\pi}\left(1 - \frac{\varepsilon_\pi}{\varepsilon_K}\right), \tag{29}$$

where the parameters

$$\varepsilon_\pi = -(\sqrt{2}\,\varepsilon_0 + \varepsilon_8)/\sqrt{3} \tag{30}$$
$$\varepsilon_K = -(\sqrt{2}\,\varepsilon_0 - \varepsilon_8/2)/\sqrt{3}$$

measure the breaking of the chiral symmetry in the Hamiltonian. Ordinary SU_3 requires $\varepsilon_\pi/\varepsilon_K \sim 1$, while the chiral SU_2 requires $\varepsilon_\pi \sim 0$. If the latter situation does not hold, there is a conflict between (29) and (21) and the soft pion technique is clearly not a valid procedure. This is what was meant by its failure requiring the change of a parameter: the ratio $\varepsilon_\pi/\varepsilon_K$.

II.2. How?

In order to test the prediction (21) we must be able to extrapolate from $t = M^2$ to the experimentally accessible region $m_\mu^2 \leqslant t \leqslant (M - \mu)^2$. First we note that the K_{l3} data, which measure the vector form factor $f_+(t)$, determine the ratio (by comparison with the K_{l2} and the π_{l2} decay rates)

$$\frac{f_K}{f_\pi f_+(0)} = 1.27 \pm 0.03\,. \tag{31}$$

Since from the definition (18)

$$f(0) = f_+(0), \tag{32}$$

we find that

$$\frac{f(M^2)}{f(0)} = 1.27 \pm 0.03 \tag{33}$$

thus the scalar form factor is expected to increase with t; in a linear approximation

$$f(t) = f(0)[1 + \lambda_0 t/\mu^2] \tag{34}$$

we find

$$\lambda_0 \approx 0.02. \tag{35}$$

However, if this simple parametrization should fail, we have not shown that the soft pion theorem fails.

II.2.1. Ward identities

In order to learn more about the extrapolation to the physical region, many authors[5] have studied the three point function $F(q^2, k^2, t)$ together with the two point (spectral) functions $\Delta_i(p)$, which are defined in Fig. 4, where

$$M(q^2, k^2, t) = \mu^2 f_\pi M^2 f_K F(q^2, k^2, t)/(\mu^2 - q^2)(M^2 - k^2),$$
$$J_\mu^\pi \equiv A_\mu^-,\; J_\mu^K \equiv A_\mu^6 - iA_\mu^7,\; J_\mu^\varkappa \equiv V_\mu^+. \tag{36}$$

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[5] For a review of K_{l3} and an extensive list of references, see [24]. For quadratic expansions see [25], also Kang K., Pham X. Y., Pond P., CERN preprint TH. 1655 (1973). For more recent work on bounds see references listed in Proceedings of the XVI International Conference on High Energy Physics. Vol. 2 (1972), 237; Okubo S., Ueda Y., Göteborg preprint (1973) and [26].

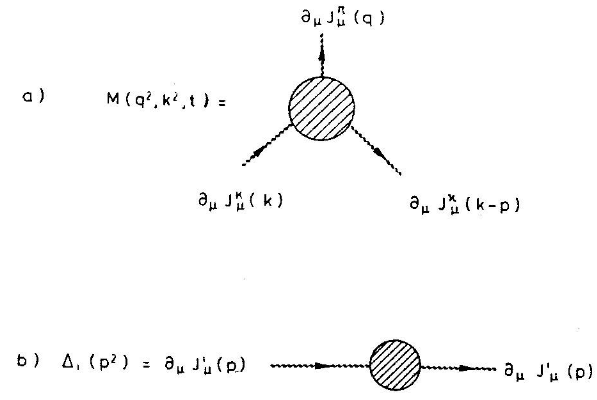

Fig. 4 Diagrammatic representation of the three-point function (*a*) and of the two-point (spectral) functions (*b*) for divergence operators.

Chiral symmetry is assumed to be broken as in Eq. (28). Then the two point functions satisfy the low energy theorems

$$\Delta_i(0) = 2\varepsilon_i \langle U_i \rangle, \tag{37}$$

where see Eq. (30)

$$U_\pi = \sqrt{2}\,U_0 + U_8$$
$$U_K = \sqrt{2}\,U_0 - U_8/2 \tag{38}$$
$$U_\varkappa = U_\pi - U_K,\; \varepsilon_\varkappa = \varepsilon_\pi - \varepsilon_K.$$

Now the three point function of Fig. 4 reduces to the divergence form factor when we take q^2 and k^2 to the pion and kaon mass shells, respectively

$$F(\mu^2, M^2, t) = (M^2 - \mu^2) f(t). \tag{39}$$

Furthermore it is related to the spectral functions by the Ward identities; if we take the various momenta in Fig. 4a to zero, we obtain the low energy theorems

$$q_\mu \to 0:\; F(0, t, t) = -\frac{(t - M^2)}{f_\pi f_K M^2}\left\{\frac{\varepsilon_K}{\varepsilon_\varkappa}\Delta_\varkappa(t) - \frac{\varepsilon_\varkappa}{\varepsilon_\varkappa}\Delta_K(t)\right\}, \tag{40a}$$

$$k_\mu \to 0:\; F(t, 0, t) = \frac{-(t - \mu^2)}{f_K f_\pi \mu^2}\left\{\frac{\varepsilon_\pi}{\varepsilon_\varkappa}\Delta_\varkappa(t) - \frac{\varepsilon_\varkappa}{\varepsilon_K}\Delta_\pi(t)\right\}, \tag{40b}$$

$$(k-q)_\mu \to 0:\; F(t,t,0) = \frac{-(t-M^2)(t-\mu^2)}{\mu^2 M^2 f_\pi f_K}\left\{\frac{\varepsilon_\pi}{\varepsilon_K}\Delta_K(t) - \frac{\varepsilon_K}{\varepsilon_\pi}\Delta_\pi(t)\right\}. \quad (40c)$$

If we now assume that ***a***) the amplitude $F(q^2, k^2, t)$ is dominated by the π and K poles in the q^2 and k^2 channels, and further that ***b***) the propagators Δ_π and Δ_K are pole dominated, we may solve the Ward identities (40) for $f(t)$ and $\Delta_\varkappa(t)$ over the range of t for which pole dominance is expected to be valid (say, $0 \lesssim t \lesssim M^2$). We obtain

$$\Delta_\varkappa(t) \approx (M^2 f_K - \mu^2 f_\pi)(f_K - f_\pi)/(1-\alpha t) \quad (41)$$

$$f(t) \approx (1-\alpha t)^{-1} \quad (42)$$

for $0 \lesssim t \lesssim M^2$, and with

$$\alpha = (f_K - f_\pi)/(M^2 f_K - \mu^2 f_\pi) \approx (1\ \mathrm{GeV})^{-1} \quad (43)$$

and the further conditions

$$f(0) = x = 1 \quad (44)$$

where

$$x \equiv \frac{\varepsilon_\pi}{\varepsilon_K}\,\frac{M^2 f_K}{\mu^2 f_\pi}. \quad (45)$$

Thus with no a priori assumptions regarding the t dependence we have obtained from pole dominance

a) agreement with the soft pion theorem; from (42)

$$(M^2-\mu^2) f(M^2) = M^2 \frac{f_K}{f_\pi} - \mu^2,$$

b) $\varkappa$ dominance of $f(t)$ with a reasonable $\varkappa$ mass (cf. (42—43)).

c) the Gell—Mann—Oakes—Renner [27] (GMOR) symmetry breaking solution $x = 1$, which implies a near chiral SU_2 symmetry

$$\varepsilon_\pi/\varepsilon_K \ll 1.$$

However, if we also consider Ward identities involving V_μ rather than $\partial_\mu V_\mu$, π and K pole dominances require

$$f_+(t) = 1, \qquad 0 \lesssim t \lesssim M^2,$$

which is violated by about 30%. Since this is just the size of the effect we have determined in Eq. (42), the result may be meaningless. Our assumptions

are clearly too strong. What is usually done is a linear or quadratic expansion of the three point function in the variables k^2, q^2, $t = (k-q)^2$, with the pole terms (including a $\varkappa$ pole) separated out. This procedure is unsatisfactory for several reasons

a) one does not know where to stop the expansion; this problem is particularly acute in the pion channel, since the 3π cut is reached as $t \to M^2$ in Eqs. (40b) and (40c);

b) some *a priori* assumption about the t channel behaviour is necessary in order to obtain predictions. One would like to have predictions which depend only on the chiral symmetry breaking, as this is what we are trying to test; K pole dominance as well as π pole dominance is related to chiral symmetry breaking; pole dominance in the t channel is not.

Ecker [28, 29] has removed the first difficulty by using dispersion relations — and arguing that certain cut contributions are small — rather than by cutting off the momentum expansion at an arbitrary order. He must assume unsubtracted spectral functions, and problem ***b***) remains.

II.2.2. Bounds

A recent development[5] which may help to resolve the above difficulties is the derivation of rigorous bounds on $f(t)$ under a few well defined assumptions. Li and Pagels first pointed out that the positive definite property of the absorptive part $\varrho_\varkappa(t)$ of the spectral function $\Delta_\varkappa(t)$ allows one to bound $\varrho_\varkappa(t)$ in terms of its lowest lying contribution, which is the $K\pi$ state

$$\varrho_\varkappa(t) \geqslant K(t)|f(t)|^2,\; t \geqslant (M+\mu)^2, \quad (46)$$

where $K(t)$ is a kinematic factor. Then if $\Delta_\varkappa(t)$ is unsubtracted and $\Delta_\varkappa(0)$ is known or bounded from above, Eq. (46) can be used to bound $f(t)$ in the physical region in terms of

$$\Delta_\varkappa(0) = \int \frac{dt}{t}\,\varrho_\varkappa(t).$$

Now if we assume pole dominance for the pion and the kaon spectral functions (cf. Eq. (37))

$$\Delta_i(0) = m_i^2 f_i^2 = 2\varepsilon_i \langle u_i \rangle \qquad i = \pi, K,$$

we may determine the ratio

$$\frac{\langle u_K \rangle}{\langle u_\pi \rangle} = \frac{f_K}{f_\pi}\,x$$

which may be used in (38) to express $\Delta_\varkappa(0)$ as a function of x. If we take the GMOR solution (44) for x we have determined $\Delta_\varkappa(0)$. Or, assuming only $\varepsilon_\pi/\varepsilon_K > 0$ (recall that SU_3 requires $\varepsilon_\pi = \varepsilon_K$; here we assume a common sign, we obtain an upper bound for $\Delta_\varkappa(0)$.

Using techniques developed by Okubo and Shih, Bourrely [30] derived the bounds shown in Fig. 6 under the above assumptions (with $\varepsilon_\pi/\varepsilon_K > 0$ only) and the additional constraint that $f(t)$ must satisfy the soft pion theorem. If this constraint is dropped, the bounds are weakened, and $f(t)$ may have a slightly negative slope. However, if the $K\pi$ phase shift information is included, the slope is again constrained to be positive.

II.2.3. Subtracted spectral functions

It has been objected that the assumption of unsubtracted spectral functions is too strong. Again, abstracting from a free quark model[6], we expect two subtractions in the Δ_i. Shih and Okubo have considered the case of one or two subtractions and, assuming only that $\Delta_\varkappa(t)$ has a Breit-Wigner form in the region of the $\varkappa$ mass, have found again that a positive slope is required if the soft pion theorem is to be satisfied. Similar results are obtained under the assumption that the absorptive part of $f(t)$ changes sign no more than once on the cut. No assumption of the type of symmetry breaking is needed.

Here again we have resorted to some a priori assumptions — however, weak — of the t channel behaviour. Can we replace these with assumptions directly related to chiral symmetry breaking? The answer is yes — with the help of the Ward identities (40). Let us first state our assumptions. We abstract the high energy behaviour as well as the chiral symmetry breaking from free quark theory and assume

a) $(3, \bar{3}) \oplus (\bar{3}, 3)$ symmetry breaking (Eq. (28));

b) Δ_i requires at most two subtractions; then we may write for the pion and kaon spectral functions

$$\Delta_i = \frac{f_i^2 m_i^4}{m_i^2 - t} + a_i + b_i t + t^2 c_i(t) \quad \text{for } i = \pi, K, \tag{47}$$

where a_i and b_i are arbitrary subtraction constants and $c_i(t) = \int \frac{dt'}{t'^2} \times$

$\times \frac{\varrho_i(t')}{t' - t}$ is the contribution from the cut;

[6] See, for example: Fritzsch H., Gell-Mann M., Proceedings of the International Conference on Duality and Symmetry. Tel Aviv 1971.

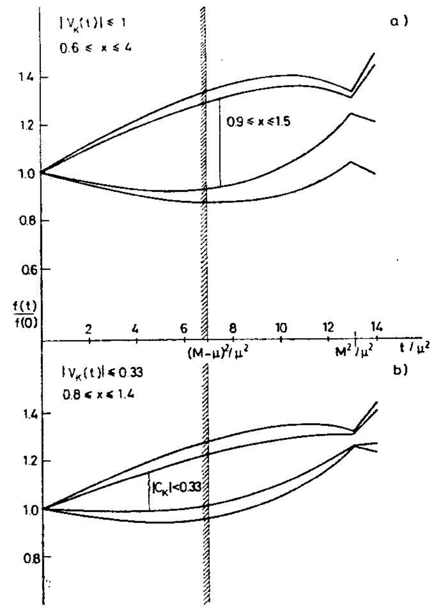

Fig. 5. Bounds for the divergence form factor in $K_{\mu 3}$ assuming two subtractions for the spectral functions, pion pole dominance at the zero pion mass and, at most 100 % (*a*) or 30 % (*b*) violation of kaon pole dominance at zero kaon mass. The cut contributions are bounded by $|C_\pi(t)| \leq |C_K f_K f(0)$ and $|C_\pi| \leq (C_K f_K M_R)^2$, with $|C_K| = 1$ or 0.33(b).

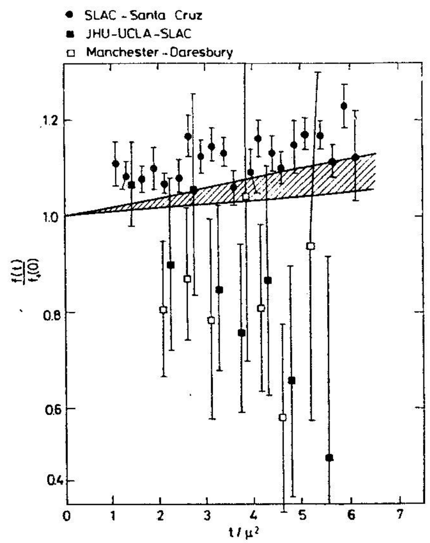

Fig. 6. Dalitz plot results [31, 32][7] for the divergence form factor in $K_{\mu 3}$; the shaded area [30] covers the allowed area, assuming $\Delta_\varkappa$ unsubtracted, Δ_π and Δ_K pole dominated, $(3,\bar{3}) \oplus (\bar{3},3)$ symmetry breaking and the validity of the soft pion theorem.

c) the three point function requires at most one subtraction in q^2 and in k^2; then we may write

$$F(q^2, k^2, t) = (M^2 - \mu^2) f(t) + \frac{(\mu^2 - q^2)}{\mu^2 f_\pi} D_\pi(q^2, t) + \frac{(M^2 - k^2)}{M^2 f_K} D_K(k^2, t) +$$
$$+ (\mu^2 - q^2)(M^2 - k^2) D(q^2, k^2, t)/M^2 f_K \mu^2 f_\pi \tag{48}$$

with
$$D_i(s, t) \equiv (M^2 - \mu^2)[V_i(t) f_i + s C_i(s, t)], \quad i = \pi, K \tag{49}$$

$V_i(t)$ measures the violation of the pole dominance in the s channel at $s = 0$ when the other particle is on shell, and

$$C_i(t) = \int \frac{A b D(s', t) ds'}{s'(s' - s)} \tag{50}$$

is a cut contribution.

Finally, we make the basic assumption which we started out to test; we assume

d) the validity of the soft pion theorem for the three point function

$$F(0, k^2, t) \approx F(\mu^2, k^2, t), \tag{51}$$

which implies in Eq. (48)

$$D_\pi(0, t) = V_\pi(t) = D(0, k^2, t) = 0.$$

A number of subtraction terms are thus eliminated; we eliminate others by solving the Ward identities Eq. (40) for the expression:

$$\bar{\Delta}(t) \equiv \{\mu^2[\Delta_\varkappa(t) - \Delta_\varkappa(0)] - t[\Delta_\varkappa(\mu^2) - \Delta_\varkappa(0)]\}/t\mu^2(t - \mu^2).$$

Since $\bar{\Delta}(t)$ does not depend on the subtraction constants

$$\bar{\Delta}(t) = \int \frac{\varrho_\varkappa(t')\,dt'}{t'(t' - \mu^2)(t' - t)}$$

it is suitable for bounding $f(t)$ with the help of the inequality (46). The explicit solution of the Ward identities allows us to express $\bar{\Delta}(t)$ in terms of $f(t)$, $V_K(t)$, $f(\mu^2)$, the symmetry breaking parameter x, and the cut contributions $C_K(k^2, t)$ and $C_K(t)$, Eqs. (47), (49). Let us first assume that only $f(t)$ is unknown.

The methods developed by Bourely [30] allow us to bound $f(t)$ in terms of $\bar{\Delta}(t')$

$$F_1[\bar{\Delta}(t'), f(M^2)] \leqslant f(t) \leqslant F_2[\bar{\Delta}(t'), f(M^2)]. \tag{52}$$

$f(M^2)$ is given as a function of x by the soft pion theorem, Eq. (29). Since $\bar{\Delta}(t')$ depends on $f(t')$, we first take $t' = t$ and solve (52) for bounds on $f(t)$. Now $\bar{\Delta}(t')$ decreases as $f(t')$ increases. So we next take $t' = t_0$ in Eq. (52), where the lower bound on $f(t_0)$ is the one which minimizes $\bar{\Delta}(t_0)$ and consequently minimizes F_2 and maximizes F_1. This procedure may be iterated until the bounds obtained for $f(t)$ converge.

What should we use as input for the other parameters?

a) $f(\mu^2)$: the dependence is very weak; we take $f(\mu^2)/f(0) \leqslant 2$ as a starting assumption (there is also a very weak dependence on $f(0)$; we take $f(0) = 1$);

b) cut contributions: below the threshold the cut may be expanded in a polynomial

$$C(k^2) = \sum C_n(k^2/M_R^2)^n; \tag{53}$$

we take $M_R = 1$ GeV and $|C_n(t)| \leqslant f(0) f_K$ for the three point function, $|C_n| \leqslant f_K^2 M_R^2$ for the two point function. This is simply the statement that if the cut contribution is dominated by a resonance, its coupling to the axial current is not enhanced relative to the kaon coupling; if there is no resonance we expect three-body phase space effects to suppress the cut contribution further;

c) $V_K(t)$: we assume that kaon pole dominance is not violated by more than 100%; then

$$|V_K(t)| \leqslant f(0)$$

d) the symmetry breaking parameter x: assumption ***c)***, together with the Ward identities implies

$$x/f(0) > 0.5.$$

A further constraint from the Ward identities is

$$x = f(0) - \frac{M^2}{\mu^2 f_\pi} C_\pi(M^2, 0).$$

If we reason as in ***b)***, namely if we assume that the cut contribution in the pion channel is not enhanced, we obtain

$$x f(0) < 4$$

(recall that the GMOR solution is $x = 1$).

With the above assumptions we obtain the bounds of Fig. 5a, where we also show the improvement obtained if we restrict x to values closer to the GMOR value. However, $V_K(0)$ and $V_K(\mu^2)$ are also determined by the Ward identities as a function of x. In the range $0.9 \leqslant x \leqslant 1.5$ we obtain $V(0) \approx \approx V(\mu^2)$, $|V(0)| < 0.4 f(0)$. Thus, saturation of the corresponding bounds in Fig. 2.4a would require $V_K(t)$ to be flat near the origin and rapidly rise by a factor of two or more in the physical region. In other words, unless the amplitude which violates kaon pole dominance has an anomalous behaviour as a function of the momentum carried by the vector current, near chiral symmetry requires that it be considerably less than 100 %.

In Fig. 5b, we show bounds obtained if a 30 % violation is assumed: $|V_K(t)| < 0.33 f(0)$. This assumption restricts (via the Ward identities) x to the range $0.8 < x < 1.4$.

Finally we note that the GMOR solution $x \approx 1$ implies that the breaking of the full chiral SU_3 symmetry in the Lagrangian is not worse than the breaking of ordinary SU_3 ($|\varepsilon_K| \approx |\varepsilon_\pi|$). If we assume, say, 30 % symmetry breaking effects, we should have a corresponding suppression of the cut contribution in the K channel (since only the K is coupled in the symmetry limit). We take in Eq. (53)

$$|C_n(t)| \leqslant 0.33 f_K f(0) \text{ for the three point function}$$

Table 11

Bounds with two substractions; $|C_K|$ is defined as in Fig 5.

Assumptions									
$V_\pi(t) = 0,\ f(0) = 1$			x		$\bar{\lambda}_0$				
$\|V_K(t)\|$	$\|C_K\|$	x	max	min	max	min	$\|V_K(0)\|$	$\|V_K(\mu^2)\|$
$\leqslant 1$	$\leqslant 1$	—	3	0.5	0.045	−0.018	< 0.83	< 0.81
$\leqslant 1$	$\leqslant 1$	0.9−1.5	—	—	0.037	−0.008	< 0.38	< 0.39
$\leqslant 0.33$	$\leqslant 1$	—	0.8	1.4	0.038	−0.004	< 0.33	< 0.33
$\leqslant 0.33$	$\leqslant 0.33$	—	0.8	1.4	0.033	0.002	< 0.32	< 0.32

$$|C_n| \leqslant (0.33 f_K)^2 M_R^2 \text{ for the two point function}$$

and obtain the tinghter bounds shown in Fig. 5b.

Our results are summarized in Table 11, where we have indicated bounds on the average slope over the physical region (not the slope at the origin). We should also remark that if we allow a 10% violation of the pion pole dominance (Eq. (51)) the effect on the bounds is negligible.

To conclude it appears that an observed slope as low as $\lambda_0 = -0.01$ would be difficult to reconcile with near chiral SU_2 symmetry ($x \approx 1$). If the breaking of chiral $SU_3 \otimes SU_3$ has any meaning as an approximate symmetry of strong interactions, a positive slope is required for the divergence form factor.

II.3. The K_{l3} data

Determination of $f(t)$ from three recent experiments [31, 32][7] are shown in Fig. 6. The newest results from the SLAC—SANTA Cruz Collaboration[7] appear compatible with a smooth extrapolation to the soft pion point, although they are systematically higher than the upper bound based on unsubtracted spectral functions. These bounds are quite stringent ($\lambda_0 \approx 0.015 \pm 0.005$); the SLAC data are probably consistent with the bounds of Fig. 5, in contrast with the earlier data. In Table 12 [33, 34] we list values of λ_0 extracted from Dalitz plot analyses under the constraint $f(0) \equiv f_+(0)$.

[7] Donaldson G., Fryberger D., Hitlin D., Lin J., Meyer B., Piccioni R., Rothenberg A., Uggla D., Wojcicki S., Dorfan D., SLAC—PUB-1254 (T-E) 1973. SLAC—Santa Cruz Collaboration. The results are modified relative to those presented [3] at NAL mainly by the inclusion of radiative corrections which were found to be important at a low t. In this respect, it is interesting to note that many polarization experiments, especially for K_L decay, were weighted towards low t values (cf. Ref. [14]).

Table 12

Dalitz plot results

		$\bar{\lambda}_0$	$\bar{\lambda}_+$
K_+	Previous world ave. [24]	−0.038 ± 0.020	0 .043 ± 0.017
	Ankenbrandt et al. [33]	−0.026 ± 0.014	0.024 ± 0.019
	Chiang et al. [34]	+0.02 ± 0.02	$\lambda_+ = \lambda_- = 0.03$
K_L	Albrow et al. [31]	−0.043 ± 0.039	0.085 ± 0.015
	Donaldson et al. (see the note 7)	+0.020 ± 0.003	0.030 ± 0.003

Polarization measurements have consistently given large negative values [3, 24—26] of $\xi(t)$ (around −1) which implies $\lambda_0 < 0$ for $\lambda_+ \lesssim 0.05$. A recent K_L experiment gives a somewhat higher result [35]

$$\xi(0) = -0.385 - 6.0\lambda_+ \pm 0.105 \tag{54}$$

if f and f_+ are assumed linear. However, a negative slope for f is still preferred. From (54) and (18) we obtain

$$\lambda_0 = -0.0329 + 0.488\lambda_+ \pm 0.0090. \tag{55}$$

The values of λ_+ determined from K_{l3} decay have varied [3, 24—26] between 0.017 and 0.055 but the world averages [3] $\lambda_+ = 0.031 \pm 0.004$ from K_{l3}^+, $\lambda_+ = 0.033 \pm 0.003$ from K_{Ll3} are compatible with a new precise result from the CERN-Heidelberg collaboration[8]

$$0.031 \pm 0.0025. \tag{56}$$

For comparison, we have listed in Table 12 values of λ_+ evaluated together with λ_0 in the $K_{\mu 3}$ Dalitz plot analyses.

Using the value (56) in Eq. (55) we obtain for the slope of the divergence form factor $\lambda_0 = -0.018 \pm 0.001$.

II.4. K_{e4} form factors

Another test of the chiral symmetry is in the study of the K_{e4} form factors. In this case there are three observable form factors which the soft pion theorems relate to the K_{l3} form factors at points outside the K_{e4} decay region. Extrapolation to the physical region is more complicated here because the

[8] Eisele F., private communication and Steinberger J., preceding lecture.

form factors dependen on three kinematic variables. However, if constant form factors are assumed, the predictions are satisfied to within twenty or thirty per cent.

Another feature of K_{e4} is that it allows the determination of the $\pi\pi$ phase shift very near threshold. This permits another test of chiral symmetry: the Weinberg prediction [36] for the $I = 0$ s wave scattering length $a_0 = 0.02 \varkappa_\pi$.

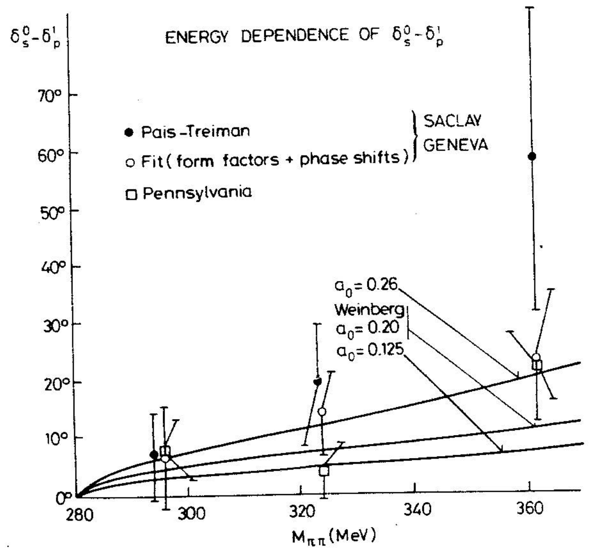

Fig. 7. Experimental determinations [37, 38] and theoretical predictions- based on the Weinberg low energy theorem [36].

Results of phase shift measurements [37, 38] are shown in Fig. 7. The central curve corresponds to a simple parametrization satisfying the Weinberg contraints; the upper and lover curves are the extrema of recent theoretical predictions[9].

One should be cautious, however, in drawing conclusions since the real test of chiral symmetry again lies not in a particular parametrization. Any scattering amplitude which is consistent with the Weiberg constraints (again at non-physical points) is consistent with chiral symmetry. Furthermore, at the present level of the data, a reliable value for the scattering length cannot be extracted.

[9] See, Schmid Ch., Proceedings Amsterdam Conference on Elementary Particles, (1971), 265.

III. RARE LEPTONIC DECAY MODES: $\Delta S = 1, \quad \Delta Q = 0$

The interest in leptonic K decays with no charge exchange to the leptons lies in the extremely low limits that are being found for these rates. The best example is $K_L \to \mu\mu$.

There is a controversy over the actual value for this decay rate; two conflicting experiments give for the branching ratio [39]:

$$\Gamma_L(\mu\mu)/\Gamma_L < 1.8 \times 10^{-9} \; (2.5 \times 10^{-9}) \tag{57}$$

and [10]

$$\Gamma_L(\mu\mu)/\Gamma_L = (1.1^{+1}_{-.5}) \times 10^{-8} \; (1.5 \times 10^{-8}) \tag{58}$$

at the 90 % confidence level. The values in parentheses obtain if the new value [40, 41] for $|\eta^{+-}|$, significantly higher than the previous world average, is used.

The first result is in conflict with the well-known unitarity limit. The decay $K_L \to \mu\mu$ can proceed electromagnetically via a two photon exchange. Since the rate for $K_L \to \gamma\gamma$ is known, and the amplitude for $\gamma\gamma \to \mu\mu$ is determined by quantum electrodynamics, this contribution to the absorptive part (Fig. 8)

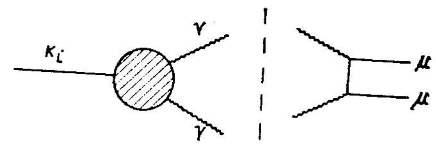

Fig. 8. Dominant contribution to the absorptive amplitude for $K_L \to \mu\mu$.

is completely determined. Other contributions to the absorptive part have been carefully studied and found to be negligible. Since there is no interference with the dispersive part of the amplitude, this sets a lower bound

$$\Gamma_L(\mu\mu)/\Gamma_L \geqslant (6 \pm 0.4) \times 10^{-9} \tag{59}$$

in conflict with (57).

In the light of this conflict, it has been conjectured [42] that a very strong amplitude for $K_1 \to \mu\mu$, together with a maximal violation of CP in this mode and/or $K \to \gamma\gamma$, could provide a cancellation of the absorptive part of $K_2 \to \mu\mu$ via the small admixture of K_1 in K_L. If this were the case, one would observe

[10] Caritheres W. C., Modis T., Nygren D. R., Pun T. P., Schwatz E. L., Sticker H., Steinberger J., Weilhammer P., Christenson J. H., preprint Columbia University 1973.

a very large partial rate for $K_s \to \mu\mu$. If the above mechanism is to account for a $K_L(\mu\mu)$ branching ratio as low as 1.8×10^{-9}, we must have, on the grounds of unitarity alone [43]

$$\Gamma_S(\mu\mu)/\Gamma_S \geqslant 2 \times 10^{-7}. \tag{60}$$

From dynamical considerations, however, we expect it more likely that [42, 44]

$$\Gamma_S(\mu\mu)/\Gamma_S \geqslant 5 \times 10^{-7} \tag{61}$$

while the simplest interaction proposed to account for the above mechanism requires [44][11]

$$\Gamma_S(\mu\mu)/\Gamma_S \geqslant 10 \times 10^{-7}. \tag{62}$$

In a recent experiment [45] the decay $K_s \to \mu\mu$ was unsuccesfully searched for, giving an upper bound

$$\Gamma_S(\mu\mu)/\Gamma_S \leqslant 3.1 \times 10^{-7} \tag{63}$$

which all but rules out the above mechanism.

In view of the more recent result (58) on $K_L \to \mu\mu$, it may well be that there is no conflict with the unitarity limit. I would like to point out nevertheless that the lower bounds (60—62) are extremely sensitive to the exact value of the K_L branching ratio. If the value in parantheses in (57) is taken, the bounds (60—62) become respectively

$$\Gamma_S(\mu\mu)/\Gamma_S \geqslant \begin{cases} 0.5 \times 10^{-7} \\ 3 \times 10^{-7} \\ 6 \times 10^{-7}. \end{cases}$$

If instead we have[12]

$$\Gamma_L(\mu\mu)/\Gamma_S \leqslant 3.5 \times 10^{-9}$$

the bounds are lowered to

$$\Gamma_S(\mu\mu)/\Gamma_S \geqslant \begin{cases} < 10^{-8} \\ 0.9 \times 10^{-7} \\ 3 \times 10^{-7}. \end{cases}$$

[11] Wolfenstein L., University of Michigan preprint 1971, unpublished.

[12] The authors of Ref. [39] are re-examining their data and expect a final limit (to be published) somewhat higher than those in Eq. (57). Wenzel A., private communication.

The point is that if the final answer should turn out to be larger than the original Berkeley result, but still in conflict with unitarity, the question of the K_1, K_2 cancellation mechanism would have to be reconsidered.

In any case the $K_L \to \mu\mu$ branching ratio is very small, and in particular the real or dispersive amplitude must be small. This result places considerable constraints on the gauge theories which unify weak and electromagnetic interactions and allow for the possibllity of renormalization. For these theories to work the photon and the intermediate bosons must couple to currents which have a group structure; the current defined by the commutator of two coupled currents

$$[J_0^1(x),\ J_\mu^2(y)]\delta(x_0 - y_0) \equiv J_\mu^3\delta^4(x - y)$$

must also couple. The commutator of the usual charged Cabibbo currents is a neutral current containing a $\Delta S = 1$ part. Various models have been proposed to eliminate this piece so as to avoid the $\Delta S = 1$, $\Delta Q = 0$ currents in the lowest order. However, the problem does not end here; in these theories, higher order corrections to weak processes are of the order αG, not G^2. This is unacceptable for $K_L \to \mu\mu$ where the amplitude is at most of the order $\alpha^2 G$ with respect to, say, $K^+ \to l\nu$.

To eliminate the lowest order couplings with $\Delta S = 1$, $\Delta Q = 0$, Weinberg [46] invoked the Glashow-Iliopoulos-Maiani mechanism [47]. The usual Cabibbo current is of the form

$$J^+ \sim \bar{p}(\cos\Theta\, n + \sin\Theta\, \lambda) \tag{64}$$

and the commutator of J^+ with J^- contains the term

$$J_0 \sim (\bar{n}\lambda + \bar{\lambda}n)\sin\Theta\cos\Theta. \tag{65}$$

The trick is to add an extra "charmed" quark with the coupling

$$J^{+\prime} = \bar{p}'(\lambda\cos\Theta - n\sin\Theta),$$

then the $\bar{n}\lambda + \bar{\lambda}n$ term in the commutator of $J^{+\prime}$ with its Hermitian conjugate just cancels the contribution (65).

This cancellation would work all orders if the p and p' masses were degenerate. We know that this cannot be the case; the observed hadron spectrum requires that the p'mass be considerably higher than the (p, n, λ) masses. However, in order not to have a $K_L \to \mu\mu$ amplitude of the order αG, we must have at least a partial concellation; this causes the p, p' mass difference not to be too large.

An explicit calculation of the graph of Fig. 9 has been done by Lee, Primack and Trieman[13] in an eight-quark version of the Georgi-Glashow model [48] in which the charged currents are constructed so that their commutator is just the electromagnetic current. Taking the branching ratio to be the unitarity limit, Eq. (59), they find

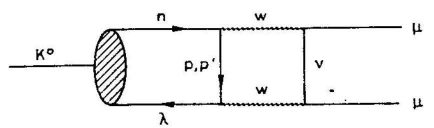

Fig. 9. Diagrams contributing to the amplitude for $K_L \to \mu\mu$ in the order $G\alpha$.

$$+ \frac{(m_p^2 - m_{p'}^2)}{M_W^2} \ln \frac{M_W^2}{m_{p'}^2} < 5 \times 10^{-3},$$

where M_W is the mass of the intermediate boson. A similar result is expected in the Weinberg model.

Of course, once the $K_L(\mu\mu)$ branching ratio is established — and provided it satisfies the unitarity limit — one must really subtract off the absorptive contribution before comparison with higher order effects of this type. We do not know what the quark mass is (perhaps a mass as low as 300 MeV is acceptable for p) but we do know that if the charmed quarks are forced to be nearly degenerate with the usual ones, we will have a hard time understanding the hadron spectrum. Therefore a value of the branching ratio much lower than that in Eq. (58) would be a source of trouble for the gauge models.

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