acta phys. slov. 24 (1974), No. 2

DETERMINATION OF THE DIFFERENTIAL THRESHOLD BY THE BILATERAL METHOD OF CONSTANT STIMULI

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If we want to determine the differential threshold by the doubleanswering method of constant stimuli in a given experiment we select the values of the comparative stimulus so that they are to be constantly smaller (at most equal), or constantly greater (at most equal) than the value of the standard stimulus. Thus we obtain in fact a "unilateral" differential threshold. If we take the measurement by the method of limits, we use for the determination of the differential threshold only values of 0 % and 100 % of the perception of the difference of the presented stimuli. The aim of this work is to show such a method of measurement that eliminates this deficiency and in one experiment allows to obtain a differential threshold in the sense of the statistical definition for greater and smaller stimuli than the standard one and also the differential threshold in the sense of the definition of the method of limits. With regard to this property we have denoted it as the bilateral method of constant stimuli. Then we can deside if the differential threshold does or does not depend on the relation of the comparative and the standard stimuli.

I. INTRODUCTION

If we want to describe the dynamic properties of the given sensor we must know its discrimination ability. Therefore it is necessary to deal in more detail with the study of the discrimination ability. The discrimination ability of the given sensor is properly expressed by the psychophysical variable called the differential threshold *DL*. Several methods of measurement of *DL* are known at present. According to Guilford [1] there are three fundamental, mutually different and independent methods of the measurement of *DL*. One of them is the so-called doubleanswering method of constant stimuli. The basic idea of this method is the following. If the difference of the values of the considered parameter of the constant stimulus *St* and of the comparative stimulus *Ko* is lying somewhere in the transient region between the stimuli which can be

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constantly perceived and those which can never be perceived, this difference definitely can be perceived sometimes, even if not always. Consequently this difference can be perceived in a certain percentage of experiments. The threshold then is the value of difference perceived by the subject in 50 % of cases. The test itself consists of the following procedure. In the preliminary test the limit value Ko^+ of the comparative stimulus Ko is determined. The subject will always observe the difference of the stimuli, if Ko^+ is given to it for consideration in a pair with St. The interval between the value Ko^+ and $Ko = St$ will be equally divided (uniformity of the division is not a necessary condition, but will considerably simplify the mathematical calculation of the experiment) into 5—10 parts [2]. Values thus divided are presented to the subject together with St, and the percentage of perception of the difference of the values of the individual types of the pairs of stimuli is found out.

During this method an experimentator has a chance to select the value of Ko^+ from two alternatives: $Ko^+ > St$ or $Ko^+ < St$. If he takes $Ko^+ > St$, then $Ko \geq St$ will hold true in pairs of presented stimuli and vice versa, if he selects $Ko^+ < St$, then in all pairs $Ko \leq St$. If in the whole experiment only one kind of stimuli is used — e.g. $Ko \leq St$, we will obtain values of DL only for this kind of stimuli. As there is no reason to assert that this value of DL is valid also for $Ko \geq St$, we consider the obtained differential threshold as unilateral. To determine DL for the values of the stimuli $Ko \geq St$ another experiment must be carried out.

The gain of a unilateral DL can be considered (besides other difficulties, like e.g. the sensibility of the method to the size of the change of Ko and the number of the presentations of this individual Ko) as the principal difficulty of the method of constant stimuli.

The second fundamental method of the measurement of DL is the method of limits. The basic idea of this method is as follows. The pairs of stimuli are arranged in increasing and decreasing series. In the increasing series St initially has a higher value of the considered parameter than Ko, then $St = Ko$, and lastly St has a lesser value of the parameter than Ko. The arrangement in the decreasing series is exactly the opposite to that of the increasing one. The subject can answer in three alternatives:

a) if Ko is perceived bigger than St, he will answer (+),
b) if Ko is perceived the same as St, he will answer (=),
c) if Ko is perceived smaller than St, he will answer (—).

Because the subject can answer in three ways, in each series we shall have two limits separating the different categories of the answers. The distance of these limits will determine the interval of uncertainty. Then the value of DL is half of the interval of uncertainty.

For the determination of DL only values of the perception of 0 % and 100 % of the difference of Ko and St are used and from these values the value of 50 % is interpolated, so that this value corresponds to the definition of the differential threshold. Generally we can presume that the DL for $Ko < St$ is different from that for $Ko > St$. If DL is determined as one half of the interval of uncertainty such a suggestion will be a priori excluded.

Besides errors resulting from habitude (the property to answer for a very long time with one category of answers) and anticipation (just the opposite property), these are the serious deficiences of the method of limits. But in this method it is not possible to proceed in a different way, because other points of the psychometric function are not present.

There is the question if with a suitable change of the method of measurement we could determine DL also for $Ko \leq St$ and $Ko \geq St$. It was shown that with a suitable modification of the method of constant stimuli this can be done. Owing to the character of this modified method, the detailed description of which can be found in section II., we have called it a bilateral method of constant stimuli. The method of constant stimuli modified in such a way allows to obtain DL values for both types of stimuli in one experiment and at the same time it retains all characteristic features of the methods of constant stimuli as well as of the method of limits.

II. MEASUREMENT PROCEDURE

The bilateral method of constant stimuli contains characteristic elements of the method of constant stimuli and of the method of limits. First of all, according to the way of calculation we will be able to consider the method on the one hand as the modification of the method of constant stimuli and on the other hand as the modification of the method of limits.

Measurements according to the experimentator's instructions proceeds in the following way. The stimuli St and Ko are presented to the subject. The subject has evaluated the pair of stimuli with the answer ($\neq$) not equal if St and Ko evoke in him different sensations and with the answer (=) equal if St and Ko evoke equal sensations. The pair of the stimuli are arranged in increasing i and decreasing d series. At the beginning of the i series Ko has such a value of Ko_1 ($Ko_1 < St$) which clearly always evokes the subject's answer ($\neq$). Let us denote $St - Ko_1 = \Delta$. The value of Ko begins to grow about an suitably selected step k.* The subject's answers will alternate irregularly between the

* Step k will be selected according to [2] so that the series includes 5—10 pairs of the stimuli.

values (≠) and (=). After certain steps the answer (=) will predominate. The subject will practically always evaluate the pair of the stimuli St and $Ko = St$ with the answer (=). If the value Ko increases further, the answer of the subject (=) and (≠) will begin to alternate irregularly, until after certain steps the answer (≠) will predominate. The last value of Ko in the series i is $Ko_{\bar{n}} = St + \Delta$. This value will always be evaluated by the subject with the answer (≠). The series d begins with the pair of stimuli St, $Ko_{\bar{n}}$ and ends with the pair of St and Ko_1. Thus each series has an equal range.

The measurement is carried out alternately with the series i and d, so as to minimize or to eliminate the errors arising from habit and anticipation, respectively. During the experiment equal numbers of the i and the d series are measured. The number of the series is unlimited. When choosing the experiment it is important to choose one that is not lasting for a long time, not longer than 30 minutes [3], as the concentration and the attention of the subject, owing to fatigue are rapidly decreasing.

From the above description it can be seen that the subject must evaluate his sensation by an exact answer. The answer "I do not know how to decide" is not allowed. Therefore we do not obtain the class of answers including such uncertain answers. According to our experience, the stimuli evoke uncertain sensations mainly in the second half of the experiment when the subject begins to show signs of fatigue. Therefore it is convenient to shorten the duration of experiment proposed in [3] to a maximum of 20 minutes.

Owing to the fact that each series contains an equal number of stimuli each value of Ko_j ($j = 1, 2, \ldots, \bar{n}$) is presented by an equal number of times.

III. THE MEASUREMENT COMPUTATION

Considering the fact that the described method includes not only elements characteristic for the traditional method of constant stimuli, but also elements which are characteristic for the method of limits, the measurement can be computed in two ways typical for the two methods.

The determination of DL will be demonstrated — for reasons of a better illustration and lucidity — by a particular example of the given experiment.

Let us change Ko by the step $k = 1$ (in arbitrasy units). Let us suppose that the number of the measured values Ko in the series is $\bar{n} = 9$, and the sum of the increasing and the decreasing series is $N = 14$. Then the answers of the subject can be recorded in Table 1, from which we can quickly compute the data necessary for the determination of DL. The data from Table 1 can be illustrated by a series of points, as shown in Figure 1. Owing to the arrangement of the experiment it is reasonable to expect no systematic deviation from the normal distribution. Thus we can suppose that the points $A_j(a_j, b_j)$,

Table 1

Determination of DL by the bilateral method of constant stimuli

series a_j	−4.0	−3.0	−2.0	−1.0	0	0	2.0	3.0	4.0	$-P(-)_l$	$P(+)_l$	IN_l
i	≠	≠	≠	=	=	≠	≠	≠	≠	1.5	0.5	2.0
d	≠	≠	≠	≠	=	=	≠	≠	≠	0.5	1.5	2.0
i	≠	≠	≠	=	=	=	=	≠	≠	1.5	2.5	4.0
d	≠	=	=	=	=	≠	≠	≠	≠	3.5	0.5	4.0
i	≠	≠	≠	=	=	=	=	=	≠	1.5	3.5	5.0
d	≠	≠	=	≠	=	=	=	≠	≠	2.5	2.5	5.0
i	≠	≠	=	≠	=	=	≠	=	≠	2.5	3.5	6.0
d	≠	≠	≠	=	=	≠	≠	≠	≠	1.5	0.5	2.0
i	≠	≠	≠	≠	=	=	=	≠	≠	0.5	2.5	3.0
d	≠	≠	≠	=	=	=	≠	=	≠	1.5	3.5	5.0
i	≠	≠	=	≠	=	=	≠	≠	≠	2.5	1.5	4.0
d	≠	≠	=	=	=	≠	≠	≠	≠	2.5	0.5	3.0
i	≠	=	=	=	=	=	=	≠	≠	3.5	2.5	6.0
d	≠	≠	≠	=	=	=	≠	≠	≠	1.5	1.5	3.0
b_j	14	12	8	5	0	4	9	11	14			

the abscissa of which has the values of 0, k, $2k$, $3k$, $4k$ ($Ko \geq St$,) have a more or less good approximation to the integral curve of the normal distribution. Points, whose abscissa has the values 0, $-k$, $-2k$, $-3k$, $-4k$ ($Ko \leq St$) approximate to curve which is the mirror image of the empiric integral curve of the normal distribution. Of course the number of the specific points is very low and therefore we obtain only a very gross approximation of the curve.

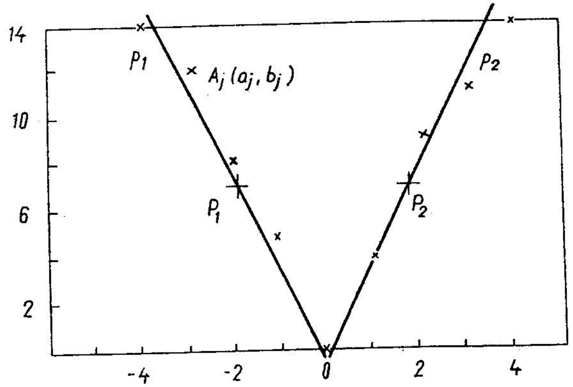

Fig. 1. Determination of DL by the method of the least squares from the data given in Table 1. $A_j(a_j, b_j)$ are the individual measured points of the empirical integral curve, a_j is the value of increment of Ko_j, b_j indicates the number of the subject's answers ($\neq$) to the pair of stimuli St, Ko_j; p_1 is the straight line approximating the mirror image of the empirical integral curve of the normal distribution ($Ko \leq St$), p_2 is the straight line approximating the empirical integral curve of the normal distribution ($Ko \geq St$).

The value of DL from the mesaured data will be determined by the method of the least squares in the following way. We approximate the above mentio- ned curves by the method of the least squares by the straight lines $p_1 \equiv y = k_1x + q_1$ and $p_2 \equiv y = k_2x + q_2$. For the approximation of the straight lines p_1 and p_2, the squares of the distance are counted in the direction of the axis y, hence not perpendicularly to the straight lines p_1 and p_2, because we consider the abscicca values to be accurate (these are the data read on the scale of the experimental device) and loaded with error are only the values of the ordinate, which, is the carrier of the random errors.*

* Under the term of random errors we understand two kinds of errors:
a) the random errors to which even the most perfectly carried out experiment is subjected. They are normally distributed and the mean of the distribution equals zero [4].
b) Errors caused by the time variation of the threshold values [5]. We suppose that these errors are normally distributed. This assumption cannot be experimentally verified Under these assumptions errors of both kinds could be summarized into one column as "under random errors".

If we know the straight lines p_1 and p_2, then the task has been essentially accomplished. DL is in fact the abscissa of the point of intersection of the straight line corresponding to the probably level of 0.5 and of the straight lines p_1 or p_2, respectively, because the extreme values of $Ko_1 = St - \Delta$ and $Ko_{\bar{n}} = St + \Delta$ were chosen so as to correspond to a 100 % evaluation of the stimuli ($\neq$).

Thus the abscissa $\bar{a}_1$ of the point P_1, $P_1 \equiv p_1 \times v$ ($v \equiv y = 0.5$) is [2]

$$\bar{a}_1 = \frac{n\sum_j a_j^2 - \sum_j a_j \sum_j a_j}{n\sum_j a_j b_j - \sum_j a_j \sum_j b_j} \left(\frac{b_0 + b_{-(n-1)}}{2} - \frac{\sum_j b_j}{n} \right) + \frac{\sum_j a_j}{n}; \tag{1}$$

$\bar{a}_1$ is a negative number because with the straight line p_1 we have approximated the mirror image of the empirical integral curve of the normal distribution and because the values of the "increments" a_j Ko_j were negative numbers. b_j indicates how many times the pair St, Ko_j was evaluated by the subject's answer (=), $j = 0, -1, \ldots, -(n-1)$, where $n = 5$ is the number of the specific points of half of the series.

The abscissa $\bar{a}_2$ of the point P_2, $P_2 \equiv p_2 \times v$, is

$$\bar{a}_2 = \frac{n\sum_j a_j^2 - \sum_j a_j \sum_j a_j}{n\sum_j a_j b_j - \sum_j a_j \sum_j b_j} \left(\frac{b_0 + b_{n-1}}{2} - \frac{\sum_j b_j}{n} \right) + \frac{\sum_j a_j}{n}; \tag{2}$$

where $j = 0, 1, \ldots, n$-1 and the rest of the symbols has the same significance as in (1).

The numerical value of the abscissa, or its absolute value, respectively, is the value of DL. Thus $DL_1 = |\bar{a}_1|$ is the differential threshold for the stimuli if $Ko \leq St$; and $DL_2 = \bar{a}_2$ is the differential threshold for the stimuli if $Ko \geq St$.

If we prove (e.g. by testing with the t-test) that the differential thresholds for $Ko \leq St$ and $Ko \geq St$ on the chosen level of significance do not change, the differential thresholds of both halves of the experiment can be determined. Then the differential threshold DL_K determined by the method of the least squares for the whole expriment is

$$DL_K = (DL_1 + DL_2)/2. \tag{3}$$

In addition to the determination of DL in the manner characteristic for the method of the constant stimuli, we shall determine DL also in the manner typical for the method of limits.

In Table 1 the intercategory limits are determined. We suppose the limit to lie in the middle between two different answers. The lower limit P $(-)_l$

(the low limit in the l-series is lying between the first transfer of the answer($\neq$) to answer($=$). The upper limit in the l-series $P(+)_l$ will be determined from the other end of the series, it lies in the middle between the firts transfer of the answer ($\neq$) to answer ($=$). After the determination of the intervals of the answer ($\neq$) to answer ($=$). After the determination of the intervals of uncertainty $IN_l = P(+)_l - P(-)_l$ (IN_l is the interval of the uncertainty of the l-series), DL will be determined from the following relation

$$DL_{IN} = \sum_{l=1}^{N} IN_l/2N, \quad (4)$$

N is the number of series in the experiment, DL_{IN} is the differential threshold established from the interval of uncertainty. From the data given in Table 1 $DL_K = 1.80$, $DL_{IN} = 1.93$.

IV. CONCLUSION

With the help of the bilateral method of constant stimuli we determine the differential threshold for $Ko \geq St$ as well as for $Ko \leq St$. The number of the various values of Ko is taken from the interval of 5—10 values. This number is not very large, especially if we realize that by these points we approximate two different curves. Because the measurement is carried out with $Ko \leq St$ and $Ko \geq St$, the calculation of the experiment by the method of the least squares (typical for the method of constant stimuli) can be divided into two parts. 9 different values of Ko seem to be the optimal range of series, because by dividing the experiment into two parts, we have 5 specific points, that is the minimal number of points by which we can approximate the empirical integral curve.

As the number of the specific points is strictly limited, it is necessary to choose properly the step k of the change Ko. It is just the suitable choice of step k and of the corresponding number of the specific points Ko that makes the greatest difficulty by testing with the bilateral method of constant stimuli. The selection of k and that of the suitable number of specific values of Ko can only be attained by patient testing in the preliminary experiment. In spite of mentioned difficulty one can suppose that the bilateral method of constant stimuli is suitable for the measurement of the differential threshold in psychophysics. It can be used everywhere where the value of Ko can be changed in small steps. Another advantage of this method is the fact that according to the experimentator's instructions, the subject can respond precisely to his sensation.

The main advantage of this method is that the calculation of DL can be carried out in two ways, which are characteristic for two different measuring methods. In addition to the determination of DL from the interval of uncertainty we can also determine DL exactly on the sence of the statistical definition of thresholds. The results obtained in this way can then be compared with the results obtained either by the traditional method of constant stimuli or by the method of limits.

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Received October 24th, 1973