DETERMINATION OF RESONANCE PARAMETERS BY THE STATISTICAL EXTRAPOLATION METHOD[1]

ANNA NOGOVÁ*, Bratislava

After a brief review we describe a new method for the determination of resonance parameters from partial wave amplitudes. In contradiction to previous methods the one described in the talk determines directly the position of the resonance pole at the second Riemann sheet. The method is based on Cutkosky's statistical approach to the represenation of data by an analytic function. The resulting value of the resonance pole position is free of the ambiguities due to specific background parametrization, which is usual in the standatd methods for the determination of resonance parameters.

I. INTRODUCTION

Everybody knows what a resonance is but nobody knows it precisely. The same situation is with the resonance parameters. In some cases it is easy to get a rough estimate of the position and the width by merely glancing through the data. It is more difficult to determine these parameters accurately. There are still cases when it is difficult to say whether there is a resonance or not. I will describe three types of methods which have been used so far for the determination of resonance parameters. In the first part of this talk I shall speak about the first two methods and in the second part I will describe a method based on Cutkosky's statistical approach and illustrate it by the case of the best known Δ (1236) resonance.

II. SOME OF THE METHODS FOR DETERMINING RESONANCE PARAMETERS

The only case which is absolutely clear is the case of a simple nonrelativistic Breit-Wigner formula

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* Fyzikálny ústav SAV, 899 30 BRATISLAVA, Dúbravská cesta

$$e^{i\delta}\sin\delta = \frac{\Gamma/2}{M_R - M - i\Gamma/2} = \frac{1}{\operatorname{cotg}\delta - i}. \tag{1}$$

If this is true, the width can be determined in three slightly different ways:

1. Width at the half maximum of the cross section. From eq. (1) it follows that

$$|\sin^2\delta| \sim \frac{(\Gamma/2)^2}{(M_R - M)^2 + (\Gamma/2)^2}. \tag{2}$$

Since the total cross section is proportional to $\sin^2\delta$, it is easy to find from (2) that the peak, which has its maximum at M_R, has the width at the half maximum equal to Γ.

2. From the phase shift. The mass of a resonance is at the energy where the phase shift $\delta(E)$ passes through 90°. If $\delta(E_1) = 45°$, $\delta(E_2) = 135°$, then the width is equal to $E_2 - E_1$.

3. From the derivative of the phase shift at the resonance. From (1) it follows that

$$\operatorname{cotg}\delta = \frac{M_R - M}{\Gamma/2}$$

and hence

$$(d\ \operatorname{cotg}\delta)/\mathrm{d}M = -2/\Gamma.$$

The trouble with the simplest Breit-Wigner given above is that it does not have the threshold behaviour built in and consequently it does not fit the data.

The parametrization which is most frequently used changes the parameter Γ from a constant to a function of energy. Various forms of $\Gamma(E)$ are currently used. With a suitable $\Gamma(E)$ one can get the correct threshold behaviour and with a sufficient number of parameters one can naturally fit the data, however, with certain drawbacks. Using the three above mentioned methods we get different resonance parameters. In this situation one should use some unambiguous definition of resonance parameters. In this situation one should use some unambiguous definition of resonance parameters. In the following we shall define the resonance as a pole on the second Riemann sheet. This definition is unambiguous but is not easy to use. If one wants to get the resonance parameters one has to get to the second sheet and to find where the pole is.

The method which is presently being used is quite simple. One applies a resonance formula with a correct analytic structure to two Riemann sheets with the pole explicitely on the second sheet and which still contains other

parameters (threshold behaviour). Then one fits the data and from the resul one immediately gets the position of the pole. This is actually the method which was recently used by Ball et al. [1]. The authors claim and prove it in a sense by calculation that the pole positions obtained with a set of reasonably chosen resonance formulae with a two sheet structure are more or less independent from the formulae used. The trouble with this approach is simple. The approach relies to a great extent on an ill-defined notion of the reasonable formula. In fact if this reasonableness assumption is abandoned one can prove that the determination of the pole position is a rather unstable problem. Now I shall show with the help of a simplified example why it is so. Since later on we shall map the second sheet onto the unit disc I shall speak about the function analytic in the unit disc.

Let $f(z)$ be a function analytic in the unit disc apart from a single pole:

$$f(z) = \frac{\alpha_1}{z - \lambda_1} + \sum_0^\infty a_n z^n \, .$$

Let the data be given on an arc L with an error ε.

Let us now construct the function

$$f(z) - \frac{\alpha_2}{z - \lambda_2} = g(z),$$

where α_2, λ_2 are two numbers selected beforehand and arbitrarily.

The function $g(z)$ is known also with an error of the order ε. Now, according to a well-known mathematical theorem the function $g(z)$ can be fitted by some suitable polynomial to the desirable accuracy ε. Hence it follows that we have two functions with different pole positions

$$f(z) = \frac{\alpha_1}{z - \lambda_1} + \sum a_n z^n,$$

$$\tilde{f}(z) = \frac{\alpha_2}{z - \lambda_2} + \sum b_n z^n = \frac{\alpha_2}{z - \lambda_2} + g(z),$$

which fit the data equally well. The two functions will be, of course, significantly different on the rest of the unit circle.

This shows immediately that one has to make a tacit assumption about the behaviour of the amplitude on the boundary, where no data are known. If one wants to have reasonable estimates of the errors of the parametrization found, one shlould use a method which is truly statistical. In the following I shall describe one of these methods.

III. THE STATISTICAL EXTRAPOLATION METHOD

The method which we have used for the determination of Δ (1236) resonance parameters is based on Cutkosky's statistical approach [2] and was recently elaborated by Pázman et. al. [3]

I am not going to give the general description of the method, but I shall demonstrate it directly for the case of the Δ (1236) resonance.

The analytic structure of the partial wave πN amplitude in the s-plane is well known. It has the physical right-hand cut starting at $s = (M + \mu)^2$, (M, μ are nucleon and pion masses) and left-hand cuts consisting of the cut along the circle of the radius $M^2 - \mu^2$ and two separate cuts along the real axis. We can conformally map the whole s-plane aparat from the inner part of the circle onto a unit disc. Naturally, the unit circle of this new plane (let us call it the z plane) will correspond to all cuts of the s-plane (except those cuts in the s-plane which are inside the circle). In our case the mapping was done in such a way that the right-hand cut of the s-plane was mapped onto the arcs (0°, 97.4°) and (−97.4°, 0°) on the unit circle. The left-hand cuts were mapped onto the arcs (97.4°, 180°) and (−180°, −97.4°). The data which we have used (CERN 67 analysis) are given on a part of the right-hand cut. This corresponds to the arcs (52.6°, 97.4°) and (−97.4°, −52.6°) in the z-plane. In the part of the unit circle (−52.5°, 52.6°) corresponding to the low energy region we can use scattering length parametrization. We do not know the values of the amplitude in the remaining part of the circle corresponding to the left-hand cuts. Here we shall use some hypothesis with a great error.

The same is true for the amplitude on the second Riemann sheet, because its analytic structure is the same as for the amplitude on the first Riemann sheet. We use the same mapping and calculate the values of the second sheet amplitude f^{II} from the equation

$$f^{II} = \frac{f^I}{1 + 2iqf^I}$$

where f^{II} is the amplitude on the first Riemann sheet and q is the CMS momentum.

Now we can say that we have a function f^{II} which is analytic (except for possible poles) inside a unit disc and we know its values and errors along the boundary. We want to test whwther such a function has some singularities or not.

If there are no poles inside, we can expand the function into the Laurent series, without singular terms:

$$f^{II}(z) = \sum_{0}^{\infty} a_n z^n .$$

If there is a pair of complex conjugated poles (which would correspond to the presence of a resonance), we can write

$$f^{II}(z) = \frac{\alpha}{z - \lambda} + \frac{\alpha^*}{z - \lambda^*} + \sum_{0}^{\infty} a_n z^n .$$

After the expansion of the pole terms

$$f^{II}(z) = \sum_{-\infty}^{-1} \{\alpha \lambda^{-n-1} + \alpha^* \lambda^{*-n-1}\{ z^n + \sum_{0}^{\infty} a_n z^n .$$

Let us define

$$Q_n = \frac{1}{2\pi i} \oint f^{II}(z)\, z^{n-1} \mathrm{d}z . \tag{3}$$

If we konw the function $f^{II}(z)$ exactly, then $Q_n = 0$, for the case of no singularities and $Q_n = \alpha\lambda^{-(n+1)} + \alpha^*\lambda^{*-(n+1)}$, if there is a pair of complex conjugated poles. Since we know the values of $f^{II}(z)$ only with a given accuracy, we can say according to the statistical approach that Q_n are random distributed variables with mean values equal to zero (if there are no singularities) or $\alpha\lambda^{-(n+1)} + \alpha^*\lambda^{*-(n+1)}$ (the pair of poles). We construct the χ^2 function

$$\chi^2_N = \sum_{1}^{N} |Q_n - \alpha\lambda^{-(n+1)} - \alpha^*\lambda^{*-(n+1)}|^2 \tag{4}$$

and by minimizing it we find the parameters α, λ. λ is a position of a pole in the z-plane. Doing the inverse mapping we find its position in the s-plane and from that it is easy to determine the resonance parameters.

We did the minimization of χ^2_N with $N = 8$. For the calculation of Q_n we have used instead of (3) a slightly modified rormula

$$Q_n = \frac{1}{2\pi i} \oint f^{II}(z) w(z) z^{n-1} \mathrm{d}z , \tag{5}$$

where $w(z)$ is a weight function constructed from errors. It would suppress the contribution from those parts of the boundry where the function is not known (left-hand cuts). The values of Q_n were of the order 10^2, which means that the function is not without singularities. The data indicate clearly the presence of a pair of compex conjugated poles. The argument in more detail proceeds as follows. We first calculate the value of

$$\chi^2 = \textstyle\sum_1^N |Q_n|^2 .$$

For $N \sim 10$ we obtain χ^2 of the order 10^5. This means that the function is not analytic, since if it were, χ^2 would be about 10. Then we assume that the function has a pair of complex conjugated poles. We construct χ^2, which tests this hypothesis, in our case it is just χ^2 given by eq. (4). This is minimized with respect to α and λ. At the minimum we find χ^2 of the order 10—50, which indicates that the introduction of a pair of complex conjugated poles diminishes the χ^2 from 10^5 down to $10-10^2$. Within the assumption made it is thus reasonably safe to assume that the pair of complex conjugated poles is the only statistically significant singularity on the second sheet.

So far we have only preliminary results as regards the pole position. There appear some complications at the technical level which should first be studied on simplified model examples. It is, for instance, not clear beforehand what Q_n, s are to be chosen most advantageously for the determination of resonance parameters. The criterion for such a choice is probably the lowest sensitivity towards the assumed behaviour of the amplitude on the (second sheet) left-hand cut.

Our preliminary results indicate that the mass of the $N33$ (1236) resonance is somewhat lower than it is generally believed. The determination based on methods described in Sect. II. gives something between 1230 and 1236 MeV, Ball et al. [1] found 1230 MeV and our calculations give so far 1204 $\pm$ 10 MeV.

The width is more sensitive fo fine technnical details of the calculation and I shall rather not give too preliminary and probably misleading numbers.

We intend now to check the method on some artificial example where the pole position is known beforehand and the phase shift is then calculated and then the data produced this way are used as an input for the method described above. The pole which is found is then compared with the true position.

Apart from that we have to use the data of Carter et. al., which, as regards the P_{33} partial wave are more accurate then the CERN 67 analysis which we have been using so far.

And so, it is quite possible that our preliminary results will be radically changed. Let me stress again that it was not the purpose of my talk to give the final results but that I wanted to describe instead the method and to show that it has a real chance to work.

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