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THE VALIDITY OF THE POMERANCHUK THEOREM AND THE LOW ENERGY DATA FOR THE FORWARD πN SCATTERING AMPLITUDES[1]

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A short survey of the present numerical analysis of the validity of the Pomeranchuk theorem is given. A sum rule for the inverse amplitude is derived; it holds *only* if the Pomeranchuk theorem is *violated*. For the case of the πN scattering we present a fairly extensive numerical analysis of this relation using the dipole model input.

I. INTRODUCTORY REMARKS, NOTATION, FITS

In this talk we shall deal mainly with some problems related to the validity of the Pomeranchuk theorem. To be more precise we would like to discuss here more quantitatively some of the high energy models which have been recently suggested [1] in order to explain the new data for $\sigma_{tot}^{\pm}$ (cf. Ref. [2]). Certain general questions connected with the violation of the Pomeranchuk theorem have been discussed in Ref. [3] (and references quoted therein). It would be, however, desirable to have also a better *numerical* analysis of this problem. As we shall see later it is a slightly difficult task and the conclusions obtained till now are rather vague. We therefore derive at the end of this talk a dispersion formula for the inverse scattering amplitude under the assumption that the Pomeranchuk theorem is violated and explain why it has a good chance to be very sensitive to our asymptotical (model dependent) evaluations.

Let us start with the notation. Since for the case of $\pi^{\pm}p$ scattering

i. we do not have in the dispersion relations any additional terms related to the non-physical contributions

ii. the experimental information about $\sigma_{tot}^{\pm}(k)$ and $\alpha^{\pm}(k)$ (the ratio of the

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real to the imaginary part) is better known at high energies than for other reactions
we would like to restrict ourselves only to the πN scattering.

The elastic forward scattering for the processes $\pi^\pm + p \to \pi^\pm + p$ is described by the amplitudes $T^\pm(\omega)$, where $\omega = (k^2 + \mu^2)^{1/2}$ is the energy of the incoming pion in the lab. system and μ stands for the pion mass. We prefer, however, to work with the even and odd combinations of $T^\pm(\omega)$

$$T_1(\omega) = \tfrac{1}{2}[T^-(\omega) + T^+(\omega)],$$
$$T_2(\omega) = \tfrac{1}{2}[T^-(\omega) - T^+(\omega)].$$

In order to make our subsequent considerations more clear we concentrate on three specific asymptotic models: on the Regge pole model, on the model including poles and cuts, and on the dipole model. All these models lead to the same general asymptotic form for $T_{1,2}(\omega)$

$$T_1(\omega) \simeq i\gamma_p k - \gamma_{p'} e^{-\frac{1}{2}i\pi\alpha_{p'}} k^{\alpha_{p'}} + i\gamma_C k\left(\ln k - A - i\frac{\pi}{2}\right)^\lambda + B, \qquad (1a)$$

$$T_2(\omega) \simeq -\frac{2}{\pi}\varepsilon k\left(\ln k - C - i\frac{\pi}{2}\right) + i\gamma_\varrho e^{-\frac{1}{2}i\pi\alpha_\varrho} k^{\alpha_\varrho}, \qquad (1b)$$

differing by fixed values of λ, ε and γ_C. We use here the system of units for which $\hbar = c = \text{GeV} = 1$ and the amplitudes $T^\pm$ are normalized by the optical theorem $\text{Im}\, T^\pm(\omega) = k\sigma^\pm_{tot}(k)$. Let us fix the values of $\alpha_{p'}$ and α_ϱ putting $\alpha_\varrho = 0.5$ and $\alpha_{p'} = 0.4$. What gives us the best χ^2-value fit for these three models? Fitting 102 experimental points of $\sigma^\pm_{tot}$ and $\alpha^\pm$ published in Refs. [2, 4, 5, 6], we have obtained the following most likely sets of values for the free parameters (Table 1).

We see that the dipole model, which embodies in a certain way the violation of the Pomeranchuk theorem, is strongly competitive. The value of ε we get from this fit leads to $\sigma^-_{tot}(\infty) - \sigma^+_{tot}(\infty) \simeq 0.75$ mb. Now consider only the difference $\sigma^-_{tot}(k) - \sigma^+_{tot}(k)$. With 18 experimental points [2, 5] and the parametrization

$$\Delta\sigma(k) = \sigma^-_{tot}(k) - \sigma^\pm_{tot}(k) = \eta + A/p^\delta \qquad (2)$$

we obtain the following values for the free parameters (Table 2).

We would not like to put too much emphasise on the χ^2-test. The results depend in this case very much on the choice of experimental data and change significantly with higher experimental accuracy. Actually the models we started the fitting procedure with may also be not quite satisfactory from some points of view. With the same asymptotic behaviour we may, e.g., not

Table 1

The values of parameters obtained from the χ^2-fit to $\sigma^\pm$ and $\alpha^\pm$

	Regge pole model	Poles and cuts $\lambda = -1.1$	Dipole model $\lambda = 1.0$
γ_C	—	−185.61	2.71
A	—	− 2.48	4.50*
ε	—	—	1.02
C	—	—	2.32
γ_p	56.90	75.85	56.49
$\gamma_{p'}$	62.73	134.54	106.68
γ_ϱ	11.81	11.78	6.27
B	13.88	50.31	57.45
χ^2	222.93	178.23	111.31

* Fixed

take properly into account the non-leading terms. Nevertheless we see that we cannot flatly reject the possibility of the violation of the Pomeranchuk theorem. The values of $\Delta\sigma(\infty)$ make us to think about. And once the audience is convinced it is worth to go deeper into this bussines.

II. DISPERSION RELATIONS, SUM RULES

Since we are considering the case where ε is not put equal to zero a priory, we have also introduced a "dipole" term into $T_1(\omega)$ in order to get finite values of $\alpha^\pm$ at infinity. Our main interest, however, is still concentrated on the asymptotic behaviour of $T_2(\omega)$ and in what follows we discuss this point more extensively.

The once subtracted dispersion formula for $T_2(\omega)$ reads (we change now the normalization to $\text{Im}\, T_2(\omega) = (k/4\pi)\,\sigma_2(k)$)

$$\left[\frac{\text{Re}T_2(\omega)}{2\omega} - \frac{k^2}{4\pi^2}\int_0^\infty \frac{dk'}{\omega'}\frac{\sigma_2(\omega')}{k'^2 - k^2}\right](\omega^2 - \omega_B^2) =$$

$$= f^2 + (\omega^2 - \omega_B^2)\left[\frac{\text{Re}T_2(\mu)}{2\mu} - \frac{f^2}{\mu^2} + \frac{k^2}{4\pi^2}\int_{k_0}^\infty \frac{dk'}{\omega'}\frac{\Delta\sigma_2(k')}{k'^2 - k^2}\right], \qquad (3)$$

where $\omega_B = \mu^2/2M$ (M = the proton mass), f^2 is the coupling constant. The integral on the rihgt-hand side is introduced for possible model dependent

Table 2

Results of the best fits to the differences $\sigma_{tot}^{-}(k) - \sigma_{tot}^{+}(k)$

	Power law	Dipole model
η	0.0*	0.57 mb
A	3.95 mb	5.85 mb
δ	0.31	0.5*
χ^2	6.05	6.71

* Fixed

Table 3

Qualitative results of the analysis done by Ferrari and Violini

Δ_π	β	Agreement
0	$\neq 0$	yes
$\neq 0$	0	no
$\neq 0$	$\neq 0$	yes

corrections [7]. The left-hand side can be compared with the relevant experimental estimates if we assume something about the asymptotic behaviour of $\sigma_2(\omega)$ (starting from the value $k = k_0$). If we have good reasons to prefer a different ansatz we correct our previous values of Re $T_2(\omega)$, integrating over $\Delta\sigma_2(\omega) = \sigma_2^{model\ 1}(\omega) - \sigma_2^{model\ 2}(\omega)$. Taking $k = 20$ GeV/c we calculate the corrections due to different choices of our asymptonic models. It appears that from the threshold up to 15 GeV/c they are completely *negligible*. Moreover, these corrections are so small (usually two orders of magnitude smaller than the experimental errors) that they even do not show up as a summary effect after some integration (cf. the last section). Therefore we conclude that the usual dispersion relation is not a very useful tool in analyzing the high energy models (cf. Refs. [8, 9, 10]).

What about the sum rules? Ferrari and Violini [11] analyzed the family of sum rules

$$I_m = \frac{1}{N^{m+1}} \int_0^N \omega^m \operatorname{Im} T_2(\omega)\, d\omega, \quad m = 0, 2, \ldots, 20$$

parametrizing $\sigma_2(\omega)$ in the form

$$\delta_2(\omega) = \Delta_\pi + \beta/\omega^{\alpha-1}.$$

The conclusions are shown in Table 3. These results have been confirmed in an independent way. By chance the authors of Refs. [12] and [13] have analyzed the same sum rule

$$\tfrac{1}{3}\left(1 + \frac{\mu}{M}\right)(a_1 - a_3) = \frac{2f^2}{1 - (\mu/M)^2} + \frac{\mu^2}{2\pi^2}\int_0^N dk \frac{\sigma_2(k)}{(k^2+\mu^2)^{1/2}} -$$

$$-\frac{\sigma_2(\infty)}{2\pi^2}\mu^2 \ln\left\{\frac{N}{\mu} + [(N/\mu)^2 + 1]^{1/2}\right\} + \varrho\text{-pole term.} \tag{4}$$

The values of $2\sigma_2(\infty)$ are

0.4 mb if the ϱ-pole is included,
0.07 mb without the ϱ-pole term.

Another sort of information is obtained from Dumbrais' calculations [14]. He evaluated the right-hand side of the aquation

$$\frac{1}{2\pi}\int_\mu^W d\omega k \sigma_2(\omega) - 0.017\, g_{\pi NN}^2 = R(W) \tag{5}$$

for different models and different values of W and compared the results with the left-hand side input. Table 4 shows an almost obvious "draw".

Another work would like to mention here is that by Ellis and Weisz [15]. They obtained, among other results, that the value of $|g_A/g_V|$ ($\simeq 1.17$) calculated from the sum rule

$$\frac{1}{f^2}[1 - (g_A/g_V)^2] = \frac{4}{\pi}\int_\mu^N k \frac{d\omega}{\omega^2}\sigma_2(\omega) + \frac{4}{i}\int_C \frac{T_2(\omega)}{\omega^2} d\omega$$

did not depend on any reasonable high energy input.

Resuming the situation we see that, surprisingly enough, we do not have at the moment any sufficiently precise numerical tool to distinguish among the models which have already been proposed for the high energy scattering

Table 4

Dumbrais' evaluation of the left-hand side and right-hand side of Eq. (5)

Parameters taken from the papers by	$\Delta\sigma(\infty)$	W			
		10 GeV	20 GeV	30 GeV	60 GeV
Phillips and Rarita	0.0	25	75	137	451
Barger and Phillips	0.0	29	79	137	404
Arnowitt and Rotelli	0.8	23 ± 8	74 ± 24	144 ± 47	553 ± 183
Horn	1.3 ± 0.3	—	—	120 ± 28	561 ± 130
LHS		23.8 ± 0.1	75 ± 2	145 ± 13	580 ± 46

amplitudes. This stimulates us to look for further constraints on $T_2(\omega)$ and one of them will be discussed in the next section.

III. INVERSE AMPLITUDE DISPERSION RELATION

In this section we present first a short derivation of a sum rule which holds only if the Pomeranchuk theorem is *violated* and then we give a more detailed numerical analysis of it. We start with the two experimental observations.

i. the difference $\sigma_{tot}^{-}(k) - \sigma_{tot}^{+}(k)$ is non-negative for $1.65 \text{ GeV/c} \leq k \leq$ ≤ 65 GeV/c

ii. none of the fits presented above leads to a cross-over effect for this difference at momenta greater than 65 GeV/c.

Therefore we take for granted

$$\sigma_2(k) \geqslant 0, \qquad k \geqslant k_N = 1.65 \text{ GeV/c}.$$

Now, if $\sigma_2(\infty) = \text{const.} \neq 0$, then

$$\lim_{k\to\infty} \left| \frac{k}{T_2(k)} \right| \simeq \frac{const.}{\ln\omega} = 0 \tag{6}$$

and

$$\lim_{k\to\infty} \operatorname{Im} \frac{k}{T_2(k)} \simeq \lim_{k\to\infty} \frac{const.}{(\ln k)^2} = 0. \tag{7}$$

Suppose for a moment that $T_2(\omega)/\omega$ does not have any zero in the complex $z = k^2$ plane. Then the asymptotic behaviour of $T_2(\omega)$ given by (6) and (7) would allow us to write a dispersion formula for $\omega/T_2(\omega)$ without any subtraction. Note that this statement should put rather severe constraints on the values of the parameters describing, e.g., the dipole model. Indeed, the integral

$$\int^{\infty} \frac{dx}{x} \frac{1}{(\ln x)^2}$$

converges but the integral

$$\int^{\infty} \frac{dx}{x} \frac{1}{\ln x}$$

is already divergent. Therefore we can expect that the asymptotic contributions from our new dispersion integral will be

i. sensitive to the structure of the high energy models taken as the input

ii. essential for the low energy evaluations of $\omega/T_2(\omega)$. These two qualitative predictions are confirmed very well by our subsequent numerical analysis.

In order to derive the pronounced sum rule we rewrite the standard dispersion formula (3) with one subtraction as (we drop also the subscript "2")

$$\frac{T(\omega)}{\omega} + \frac{2f^2}{\mu^2} \frac{\omega^2 - \mu^2}{\omega^2 - \omega_B^2} - \frac{T(\mu)}{\mu} - \frac{k^2}{\pi} \int_0^{k_N^2} \frac{dk'^2}{k'^2} \frac{\operatorname{Im}T(k')}{\omega'(k'^2 - k^2)} =$$

$$= \frac{k^2}{\pi} \int_{k_N^2}^{\infty} \frac{dk'^2}{k'^2} \frac{\operatorname{Im}T(k')}{\omega'(k'^2 - k^2)}. \tag{8}$$

Consider an auxiliary function

$$F(z) = -\alpha^2 + \frac{z}{\pi} \int_{k_N^2}^{\infty} \frac{dk'^2 \operatorname{Im}T(\omega')}{k'^2\, \omega'(k'^2 - z)}, \quad \alpha = \alpha^*,\ z = k^2 \tag{9}$$

which has the same asymptotic behaviour as $T(z)$ and at the "threshold" $F(k_N^2) < 0$. This is sufficient to show that $F(z)$ does not have any zeros in the complex z-plane ($\sigma(k)$ is positive for $k \geqslant k_N$) and to write the following Cauchy formula for $1/F(z)$

$$-\frac{1}{F(z)} = \frac{1}{\pi} \int_{k_N^2}^{\infty} \frac{dk'^2 \operatorname{Im}T(\omega')}{\omega' |F(k')|^2 (k'^2 - z)}. \tag{10}$$

In particular, if we put $z = 0$ we get from (9) and (10)

$$\frac{1}{\alpha^2} = \frac{2}{\pi} \int_{k_N}^{\infty} \frac{dk' \operatorname{Im}T(k')}{k'\omega'|F(k')|^2}.$$

We have evaluated this integral in the natural system of units for three different values of α^2 and three different sets of values for ε, C and γ_ϱ taken from Refs. [1], [16] and Tab. 1. The medium energy input (from 1.65 GeV/c to 10 GeV/c) has been taken from the last Karlsruhe tables [17]. Table 5 gives the contributions to the integral in the right-hand side of Eq. (11) from different energy regions.

What kind of conclusions can be drawn from Table 5?

1. If we like to emphasize the medium energy input we should take a rather small value of α^2. Note, however, that α^2 cannot be arbitralily small since $F(k_N^2)$ must be negative according to our assumptions.

2. For large values of α^2 (e.g. for $\alpha^2 = 0.5$) the asymptotic contribution to the integral (11) is extremely important. If for example we neglect the contribution from the 8th column when $\alpha^2 = 0.5$, we get for the ratio of the right-hand side to the left-hand side the value $\simeq 0.58$ whereas it should be $\simeq 1$.

Table 5

Results of the numerical analysis of Eq. (11)

ε	C	γ_e	LHS ($= \alpha^{-2}$)	1.65–10 GeV/c	10–100 GeV/c	100–10^{325} GeV/c	10^{325}–∞ GeV/c	RHS
0.83	4.3	7.3	60.70 10.00 2.0	18.43 0.66 0.028	6.36 0.35 0.0159	37.12 7.76 1.072	1.56 1.38 0,89	63.47 10.15 2.005
1.02	5.0	6.0	60.70 10.0 2.0	18.43 0.66 0.028	6.77 0.35 0.016	38.08 8.02 1.13	1.28 1.16 0.79	64.48 10.20 2.06
1.02	2.32	6.27	60.70 10.0 2.0	18.33 0.66 0.028	5.57 0.34 0.0162	34.35 7.81 1.16	1.28 1.16 0.79	59.53 9.97 2.00

The value of $C = 2.32$ is twice smaller than those coming from other fits. In fact such a value of C does not give a smooth transition to the medium energy values of $\mathrm{Re}\, T_2(\omega)$ (and it is obvious why: we have not fitted the values of $\mathrm{Re}\, T_2(\omega)$, which can be read from the experiment, but the values of $\alpha^{\pm}$). Therefore if we agree that C should take a value around 4.5 and the value of α^2 will get smaller (probably) with some more accurate data from higher energies, we may suggest that the violation of the Pomeranchuk theorem would contradict (within the model which has been analyzed) Eq. (11). The values of the right-hand side obtained for different values of α^2 and different sest of parameters have a clear tendency to be greater than the left-hand side.

Our analysis was in fact only a "one point evaluation". Obviously we can also look upon the relation (10) in the same manner as we do with the usual dispersion relations and calculate the left-hand side and the right-hand side for different values of z. This is, however, a more complicated task and certainly beyond the scope of the present talk.

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