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THE EFFECTIVE RHO-OMEGA INTERFERENCE AND PION FORMFACTOR

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The most recent Orsay data on the elmg. pion formfactor are explained by the interference of the rho and omega, both objects considered with an effective mass and width. Leaving the rho mass and width as free parameters, the best fit procedure gives $m_\varrho = 775.6$ MeV and $\Gamma_\varrho = 126.8$ MeV.

I. INTRODUCTION

It has been shown lately [1] that the recent data on the isovector elmg pion formfactor are satisfactorily explained by the $\varrho - \omega$ interference if ϱ is described by the Gounaris-Sakurai form, [2], which involves a quadratic term in energy squared in its denominator.

There is still an open question how to relate the experimental mass and width of a resonance, directly observed, with the location of a simple pole in the complex energy plane. However, expanding the denominator of a resonant formula at the resonance position and neglecting the quadratic and higher terms, it is possible to introduce the effective mass and the effective width of a resonance. It is shown in the present contribution that the formulae derived in that way lead also to a satisfactory explanation of the aforementioned recent data. Moreover, the introduction of the effective parameters of the resonance might explain the shifts in masses and widths observed in different collisions. Furthermore, by means of those parameters it is possible to understand also the difficulties encountered when a sharper value is to be ascribed to the mass and width of the low-energy two-pion system with $I^G J^P = 0^+0^+$, [3].

II. EFFECTIVE PARAMETERS

Let us describe a resonant object with the angular momentum l by a Breit-Wigner amplitude of the form

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$$m_0^2/[m_0^2 - s - im_0\Gamma(s)], \tag{1}$$

where

$$\Gamma(s) = \Gamma_0 \left[\frac{q_{1s}q_{2s}}{(q_{1s}q_{2s})_0}\right]^{(2l+1)} \frac{\varrho(s)}{\varrho(m_0^2)}; \tag{2}$$

m_0 and Γ_0 are the (constant, "unperturbed") mass and width of the resonance. The expression (1) is chosen quite conventionally in the s-channel, $m_1 + m_2 \to m_3 + m_4$. The c. m. s. momenta q_{1s} and q_{2s} give $q_{1s}q_{2s} = h(s)/4s$, where

$$h(s) = [(s^2 - s\Sigma + \varkappa)^2 - 4s(\lambda s + \nu)]^{1/2}$$

and

$$\Sigma = m_1^2 + m_2^2 + m_3^2 + m_4^2, \quad \varkappa = (m_1^2 - m_2^2)(m_3^2 - m_4^2),$$
$$\lambda = (m_1^2 - m_3^2)(m_2^2 - m_4^2), \quad \nu = (m_1^2 m_4^2 - m_2^2 m_3^2)(m_1^2 - m_2^2 - m_3^2 - m_4^2).$$

In rel. (2), $\varrho(s)$ is usually adjusted to give the best description at the resonance (compare, e. g. ref. [4]).

On the other hand, a resonance is often described by a simple pole

$$f(m_0^2) = m_0^2/const.\ [m_{ef}^2 - s - im_{ef}\Gamma_{ef}]. \tag{3}$$

To be able to relate both denominators in rels. (1) and (3), the energy dependent width (2) is expanded in series at the resonance position, $s = m_0^2$. Neglecting the terms $(s - m_0^2)^N$ with $N \geqslant 2$, exactly the form (3) is obtained, where

$$m_{ef}^2 - im_{ef}\Gamma_{ef} = m_0^2 - im_0\Gamma_0/(1 + im_0\Gamma_0 V) \tag{4}$$

and the factor "*const*" in (3) is uniquely determined. In rel. (4), $V = \frac{1}{2}(2l + 1)(A - m_0^{-2}) + B$ and $A = h'(m_0^2)/h(m_0^2)$, $B = \varrho'(m_0^2)/\varrho(m_0^2)$: the prime denotes the first derivative. If the external particles are stable, the following expressions result

$$m_{ef} = m_0[1 + V\Gamma_0^2/(1 + m_0^2\Gamma_0^2V^2)]^{1/2}, \tag{5a}$$

$$\Gamma_{ef} = \Gamma_0[1/(1 + m_0^2\Gamma_0^2V^2)](m_0/m_{ef}). \tag{5b}$$

The parameter B is influenced only by the $\varrho(s)$-function. From a great variety of different forms let us choose the following one (it is a slight adaptation of that given by eq. (7) in ref. [5]),

$$\varrho(s) = s^{-1/2}[\sum_{j=0}^{n}(q_{1s}q_{2s}R^2)^j]^{-1}, \tag{6}$$

where R determines the range of the interaction (in our computations we have fixed R at about 2 *fm* for ϱ as well as for ω). In this case

$$B = -[1 + 2lm_0^2 W]/2m_0^2,$$

where

$$W = C(m_0^2 A - 1)(\sum_{j=0}^{n} jC^{j-1})/\sum_{j=0}^{n} C^j$$

and

$$C = R^2 h(m_0^2)/4m_0^2 = (q_{1s}q_{2s})_0 R^2.$$

III. RESULTS AND CONCLUSIONS

The amplitude for the pion formfactor is chosen in the form

$$F_\pi = f(m_\varrho^2) + ce^{i\varphi} f(m_\omega^2), \tag{7}$$

where $f(m_0^2)$ is given by rel. (3). In rel. (7), $c = 822\ B_\pi B_e (\Gamma_\omega/m_\omega)/\beta_\pi^{3/2}$ and $\beta_\pi = (1 - 4m_\pi^2/s)^{1/2}$, [1]. In rel. (6) we set $n = 1$. As input we fixed m_ω at 783 MeV, Γ_ω at 11.9 MeV and $B_\pi = 0.2$, $B_e = 9 \times 10^{-3}$; the numerical values of the branching ratios $B_\pi \equiv B^{1/2}_{\omega\to 2\pi}$ and $B_e \equiv B^{1/2}_{\omega\to e^+e^-}$ are taken from ref. [1]. Minimizing with respect to the first ten experimental points[1], the best fit [6] gives for our free ("unperturbed") parameters[2],

$$m_\varrho = 775.6 \text{ MeV}, \quad \Gamma_\varrho = 126.8 \text{ MeV and } \varphi \approx 119^0. \tag{8}$$

Those values are influenced only very slowly by the variation of the ω input parameters in the range of several MeV. The effective parameters of m_ω, Γ_ω and Γ_ϱ are nearly the same as those „unperturbed" values (for instance, $(m_\omega)_{ef} = 783.978$ MeV), however, $(m_\varrho)_{ef} = 768$ MeV. Our best fit is seen in Figs. 1a and 1b, with a confidence level of about 78 % ($\chi^2 \approx 3.1$).

Nearly the same fit is obtained with the dependence $\varrho(s)$ as given by rel. (6), using rels. (1) and (2) directly: now the confidence level is about 81 % ($\chi^2 \approx$ ≈ 2.96) and the output values for $m_\varrho, \Gamma_\varrho$ and φ are nearly the same as in rel. (8).

Because the χ^2-criterion cannot significantly distinguish between those two approaches we have used also the "τ-test", ref. [7]: here a criterion is suggested which allows to choose one of the two alternative hypotheses and also a method of linearization of the non-linear problems is described. However, in our case, the τ-test gave no sign in favour of any of the two methods mentioned above.

The method of the effective parameters is convenient also in the dispersion relation techniques: in the effective pole term also the shape of the resonance as the singularity on the unphysical sheet can be taken into account; without that notion the evaluation of the crossed channel contributions encounters many complications, namely when the inelastic processes are considered.

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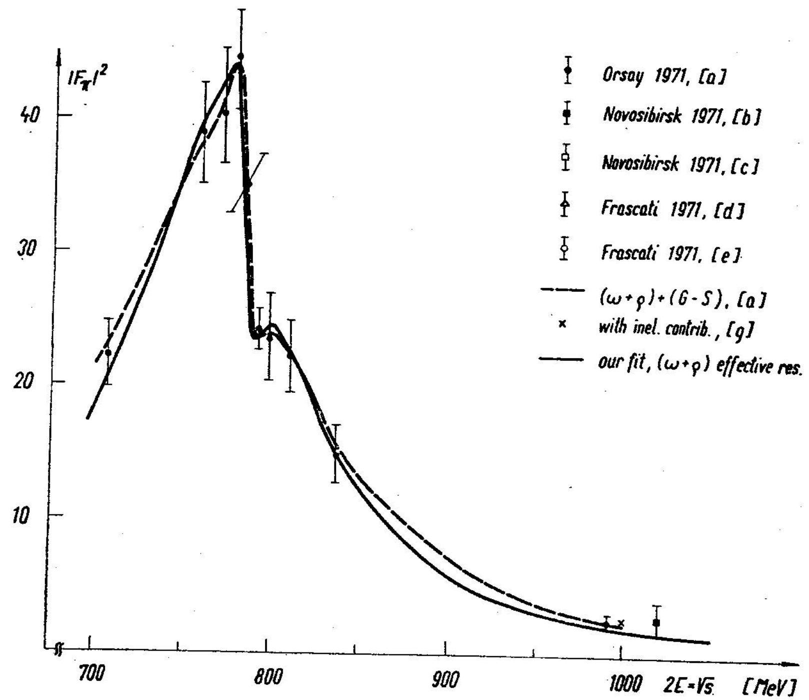

Fig. 1a. The pion formfactor vs energy, until $\sqrt{s} \sim 1$ GeV. [a] Lefrançois J., in Ref. [f], p. 52. [b] Balakin V. E., et al., Phys. Letts. *34 B* (1971), 328. [c] Sidorov V. A., in Ref. [f], p. 66. [d] Borelli V. A., et al., (BCF group); quoted by Bernardini C., in Ref. [f], p. 38. [e] Barbiellini G. et al., (muon-pion group); quoted by Bernardini C., in Ref. [f], p. 38. [f] Proc. 1971 Internat. Symp. on Electron and Photon Interactions at High Energies, Cornell Univ., Ithaca, N. Y. 1971 (Ed. by Mistry N. B). [g] Baier V. N., Fadin V. S., Pisma v JETP (Letts.) *15* (1972), 219.

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[1] The minimization with respect to the first nine experimental points from Fig. 1a brings essentially no difference.

[2] The external error correlation matrix in the subroutine MIGRAD using parabolic approximation gives the following values for errors: $\Delta m_\varrho = 3.8$ MeV, $\Delta\Gamma_\varrho = 2.3$ MeV and $\Delta\varphi \sim 0.1°$.

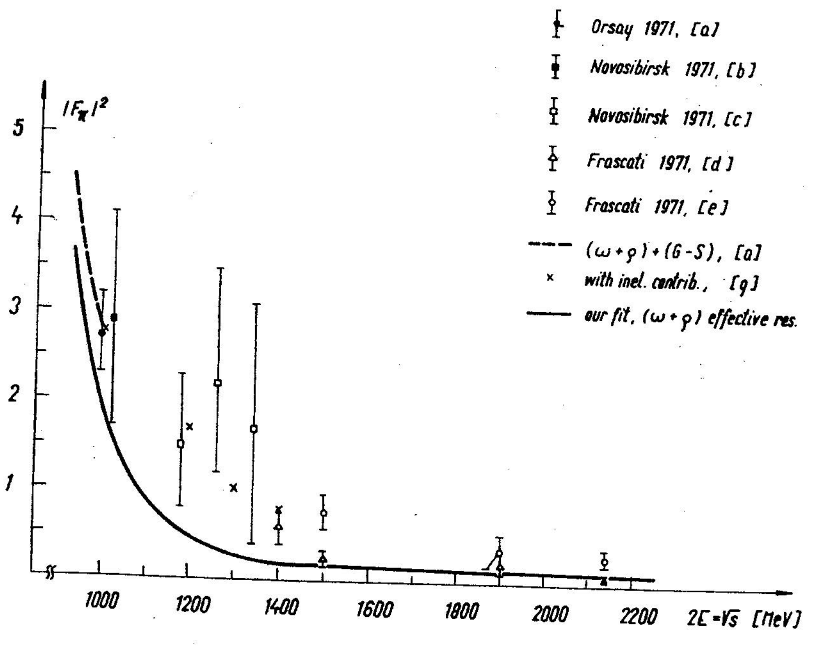

Fig. 1b. The pion formfactor vs energy, for $\sqrt{s} \geqslant 1$ GeV (the notation is the same as in Fig. 1a).

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