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MEASUREMENT AND IRREVERSIBILITY IN INFINITE SYSTEMS[1]

PAVEL BÓNA*, Bratislava

The problem of measurement in the quantum theory is an old problem. There have been various attempts to solve it. None of the proposed solutions was generally accepted. During the last few years there have been attempts to use a mathematical approach suitable for the description of infinite systems. It seems that the approach may give a formal solution to the measurement problem.

In the following talk I would like to show why it is necessary to turn to infinite systems if one wants to find a solution to the measurement problem. The talk is just a noncomprehensive survey with no pretence to originality. It is divided into three parts. The first part contains the schematic formulation of the problem. In the second part we shall exclude some approaches which, from our point of view, are unsuccessful. The main part of the talk is the third one. We shall dicuss there some new possibilities to solve the problem of measurement via the introduction of systems with infinitely many degrees of freedom.

I. SCHEMATIC FORMULATION OF THE PROBLEM

The problem is essentially the one of the "reduction of the wave packet". In the von Neumann formulation of quantum mechanics there are two basically different processes of the time evolution of the physical system. The "process of the second kind" is described by the one parametric group of time translation corresponding to the Schrödinger equation. The "process of the first kind" is a rapid and irreversible change of the density matrix ϱ. The change occurs when some quantity Q is measured. The matrix ϱ may describe either the microscopic system or the compound system micro plus apparatus. In the same way Q represents respectively a quantity related to the microsystem or to the compound. If not stated otherwise in the following the "system"

* Katedra teoretickej fyziky Prírodovedeckej fakulty UK, 816 31 BRATISLAVA, Mlynská dolina.

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means the compound. Let ϱ_0 be the density matrix before the interaction of the micro and the macro systems (i. e. the apparatus). After the ineraction of the closed system (i. e. the process of the 2nd kind only) we obtain the density matrix ϱ. According to von Neumann the observer then enters the game and ϱ changes to ϱ' by the process of the 1st kind:

$$\varrho_0 \underset{2^{nd}}{\to} \varrho \underset{1^{st}}{\to} \varrho' \equiv \sum_i P_i \varrho P_i \ ,$$

where P_i are projections on eigensubspaces of Q. The quantity Q represents here a "pointer position" of the system. According to our point of view the observer plays no role. Consequently ϱ and ϱ' should be physically equivalent. The expectation values of all observables should then be the same in the states ϱ and ϱ'. This immediately leads to the condition $[A, P_i] = 0$ for all P_i and all observables A. This condition is physically equivalent to the existence of a superselection rule, which makes the vector-states from different subspaces P_i mutually incoherent.

The problem we want to discuss here is the question of whether it is possible to describe the irreversible process of measurement without postulating any special "processes of the 1st kind". We shall call an "ideal solution" of our problem the strict equivalence of the processes of the first and the second kind in the sense specified above. Then the first problem for the ideal solution is to construct a mathematical theory of big systems, where some superselection rule really works. The second problem is then the question of the time evolution in such systems. The last problem is the consstruction of realistic models.

The question of the description of measurement-like processes is connected with the question of general irreversible processes. The complex system object plus apparatus is only an example of big systems with an irreversible behaviour. We consider here measurement as an objective physical process independent on an observer — contrary to the von Neumann approach. For a given big system we may then expect the occurrence of some objective measurement-like processes regardless of their usefulness for any measurements. For example, in some big system we expect such a behaviour as if each elementary subsystem were measured by the "rest" of the big system (see Fig. 1).

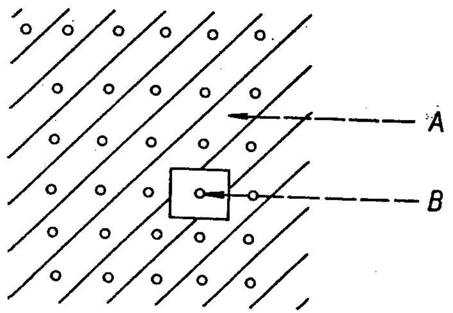

Fig. 1. A — macroscopic "rest'; B — arbitrarily chosen "elementary part".

II. EXCLUSION OF UNSUCCESSFUL SOLUTIONS

Here we want to determine in a few words our way of seeking an ideal solution.

We can get the wanted "reduction" or "irreversibility" in open systems by introducing suitable statistical fluctuations to the interaction or to the boundary conditions [8]. Nevertheless, the restriction to open systems is in some way similar to the von Neumann influence of an observer: this is not the case we are here interested in. Other approaches are based on radical changes of quantum mechanics, e. g. the introduction of some nonlinear processes [9]. What we want to discuss here are the approaches, which keep the standard formalism of quantum mechanics undisturbed as far as possible. In the following we only exclude the "reduction postulate" from the standard theory.

The ideal solution is impossible, however, in closed systems with a finite number of degrees of freedom. In such systems all bounded hermitean operators in the respective Hilbert space correspons to observables, i. e. there is no superselection rule. We can illustrate this by the fact that the C^*-algebra of the system of a finite number of harmonic oscillators has only one irreducible representation (up to the unitary equivalence).

Summarizing we can say that the only way to approach an ideal solution is to deal with systems with infinitely many degrees of freedom. This assertion seems physically acceptable, because, as far as I know, irreversibility is specific for big systems.

III. INFINITE SYSTEMS

To make things clear I shall start with a simle example and later on I shall pass to more general considerations.

1. An example

Let us consider an infinite chain of spins $\frac{1}{2}$ with two-dimensional Hilbert spaces C_n^2, $n = 0, 1, 2, \ldots$. Then the W^*-algebra of extended operators on the Complete Direct Product Space (C.D.P.S.) $\overset{\otimes}{_n} C_n^2 \equiv \mathscr{H}_{\otimes}$,

$$\bar{A}_n \equiv 1_0 \otimes 1_1 \otimes \ldots 1_{n-1} \otimes A_n \otimes 1_{n+1} \otimes \ldots,$$
$$A_n \in B(C_n^2),$$

where A_n are operators in C_n^2 (Pauli matrices), forms the algebra of all observables in an infinite chain. Denote this algebra as $B_{\#}$. Let $|\vec{e}_n\rangle$ be the state of the n-th spin oriented in the direction $\vec{e}_n$. $|\{e\}\rangle \equiv \bigotimes_{n=0}^{\infty} |\vec{e}_n\rangle$ is the vector product state in the C.D.P.S. $\mathscr{H}_{\otimes}$. (For details regarding the theory of C.D.P.S. see [1] — general case, and [2] — a short survey for a spin chain). Let us sketch here the structure of C.D.P.S. and the algebra $B_{\#}$. The scalar product of the two vectors $|\{e\}\rangle$ and $|\{e'\}\rangle$ is defined by the relation

$$\langle\{e'\}|\ \{e\}\rangle \equiv \prod_n \langle \vec{e}_n' \,|\, \vec{e}_n\rangle = \prod_n e^{i\varphi_n} \,|\, \langle \vec{e}_n' \,|\, \vec{e}_n\rangle \,|.$$

If the phases $\arg \langle \vec{e}_n' \,|\, \vec{e}_n\rangle \equiv \varphi_n$ fulfil the condition

$$\sum_n |\varphi_n| = \infty,$$

we define the scalar product as zero.

To each sequence $\{\vec{e}_n\} \equiv \{e\}$ we define the equivalence class $C\{e\}$ and also the week equivalence class $C_w\{e\}$ of the product vectors $|\{e'\}\rangle$ by the relations:

$$C\{e\} : |\{e'\}\rangle \approx |\{e\}\rangle \Leftrightarrow \sum_n |\langle \vec{e}_n \,|\, \vec{e}_n'\rangle - 1 \,| < \infty,$$

$$C_w\{e\} : |\{e'\}\rangle \overset{w}{\approx} |\{e\}\rangle \Leftrightarrow \sum_n ||\langle \vec{e}_n \,|\, \vec{e}_n'\rangle \,| - 1 \,| < \infty.$$

By completions of these classes we obtain respectively a separable Hilbert space $\mathscr{H}_{C\{e\}}$ — the co called I.D.P.S. (the incomplete direct product space), and a nonseparable Hilbert space $\mathscr{H}_{C_w\{e\}}$. Any $\mathscr{H}_{C_w}$ is a direct sum of inenumerably many $\mathscr{H}_C$'s differing from each other by an infinite phase factor. From one $\mathscr{H}_C$ to another $\mathscr{H}_{C'}$ contained in the same $\mathscr{H}_{C_w}$ we can pass by a unitary operator $U(\{z\})$, where

$$U(\{z\}) \,|\, \{e\}\rangle \equiv \bigotimes_{n=0}^{\infty} (z_n \,|\, \vec{e}_n\rangle), \quad |z_n| = 1, \ \sum_n |\arg z_n| = \infty.$$

All vectors $U(\{z\}) \,|\, \{e\}\rangle$ belong to the same $\mathscr{H}_{C_w\{e\}}$. The vectors $|\{e\}\rangle$ and $U(\{z\}) \,|\, \{e\}\rangle$ belong to the same $\mathscr{H}_C$ if and only if

$$\sum_n |\arg z_n| < \infty, \quad -\pi < \arg z_n \leq \pi.$$

A basis in $\mathscr{H}_{C\{e\}}$ may be constructed as follows: Reverse a finite number of

spins in the state $|\{e\}\rangle$, all states obtained in such a way from the same state form a wanted basis. The whole C.D.P.S. $\mathscr{H}_{\otimes}$ is a direct sum (or integral) of $\mathscr{H}_{C_w}$:

$$\mathscr{H}_{\otimes} = \bigotimes_{C_w} \mathscr{H}_{C_w}.$$

Denote the projector onto $\mathscr{H}_C$ as P_C and let $U(\{z\})$ be the unitary operator described above. The structure of the algebra of observables $B_{\#}$ in our spin chain is described then by the von Neumann theorem [1]: The bounded linear operator A in the C.D.P.S. belongs to $B_{\#}$ if and only if it fulfil the relations

$$[A, P_C] = 0, \quad [A, U(\{z\})] = 0,$$

for all P_C and all $U(\{z\})$.

This theorem is true also in the general case of the C.D.P.S. In our terms the theorem implies the existence of a wanted superselection rule in the algebra of observables. The superselection sectors are the subspaces of $\mathscr{H}_{C_w}$. Representations of the algebra of the observables $B_{\#}$ in all the subspaces I.D.P.S. $\mathscr{H}_C$ of an $\mathscr{H}_{C_w}$ are unitarily equivalent (on the question of equivalence and irreducibility of representations of the algebra $B_{\#}$ in various $\mathscr{H}_C$ see, e. g. [3]). The form of a general element of algebra of observables in the C.D.P.S. is illustrated by our Figure 2.

Possible operators of the macroscopic observables are here the projectors P_{C_w} and their real linear combinations. The classification and interpretation

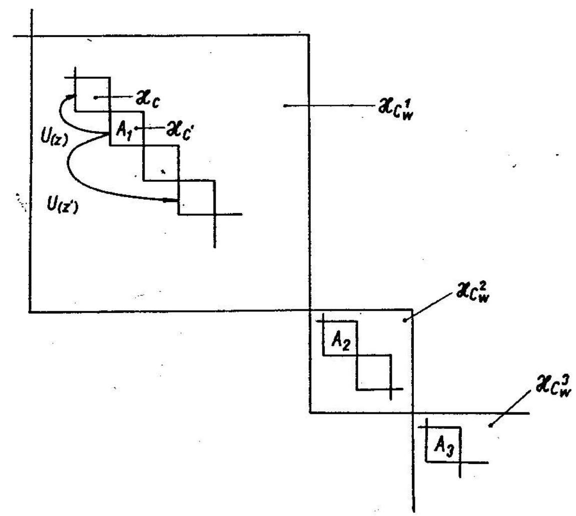

Fig. 2.

of such operators is, however, non-trivial. In our example, e. g. the operator of the average spin $\vec{s}_\infty$ does not exist in arbitraty $\mathscr{H}_{C_\omega}$, [2]. In a given $\mathscr{H}_{C_\omega}$ determined by a sequence $\{\vec{e}_n\}$ it is:

$$2\vec{s}_\infty \equiv \lim_{N\to\infty} 2\vec{s}_N \equiv \lim_{N\to\infty} \frac{1}{N}\sum_{n=0}^{N} \vec{\sigma}_n = \lim_{N\to\infty} \frac{1}{N}\sum_{n=0}^{N} \vec{e}_n$$

and the existence of $\vec{s}_\infty$ depends on the convergence of the righthand side. We can expect a similar situation for other intensive quantities. For extensive quantities the situation is even worse [2].

What we want to discuss next is the question of the time evolution. As an example of measurement-like evolution we define [4] the unitary group $W(t)$ on the D.P.S. $\mathscr{H}_\otimes$ by the following relation

$$W(t) \equiv P_0^+ + P_0^- U(t)\ ,\quad P_0^\pm \equiv \frac{1 \pm \sigma_3^{(0)}}{2}$$

$$U(t) \mid \{e\}\rangle \equiv |\vec{e}_0\rangle \otimes \prod_{n=1}^{\infty} \otimes \exp(i\sigma_1^{(n)} t) \mid \vec{e}_n\rangle.$$

The spin with $n = 0$ plays here the role of the measured microobject and the rest of the spin chain $n = 1, 2, \ldots$ is a measuring device. If we choose the initial state of the device as ψ_+ with all spins pointing up, then the two states of the compound Φ_+ and Φ_- are weakly inequivalent (and hence macroscopically different) in arbitrarily short times $t \neq 0$: we have

$$\Phi_\pm \equiv W(t) \mid \sigma_3^{(0)} = \pm 1 \rangle \otimes \psi_t.$$

Any observable $A \in B_\#$ of the compound system at such times has vanishing matrix elements between Φ_+ and Φ_-. As an initial state of the micro-object we take an arbitrary $|\vec{e}_0\rangle = \alpha_+ \mid \sigma_3^{(0)} = +1\rangle + \alpha_- \mid \sigma_3^{(0)} = -1\rangle$ and the initial state of the compound system is $\Phi(0) = \alpha_+\Phi_+(0) + \alpha_-\Phi_-(0)$. Then for an arbitrary short $t \neq 0$ and all observables A we have

$$\langle \Phi(t) \mid A\Phi(t)\rangle = |\alpha_+|^2 \langle \Phi_+(t) \mid A\Phi_+(t) \rangle + |\alpha_-|^2 \langle \Phi_-(t) \mid A\Phi_-(t)\rangle.$$

We may interprete the last formula as the wanted "reduction" obtained without any "reduction postulate". The process is periodical in t — there is no true irreversibility. The time evolution in this case is weakly discontinuous. The continuous evolutions may lead to the reduction in the limit $t \to \infty$ only [4].

Let us now condider a big but finite system. For such a system there is no superselection rule in our sense. Let N be the number of spins in the chain chain and let the finite-vector-product-states $|\{e\}_N\rangle$ and $|\{e'\}_N\rangle$ be in the

limit $N \to \infty$ weakly inequivalent. Let A_M be an operator acting only on the first M spins, $||A_M|| \leqslant 1$. Then for $N \to \infty$ we have [4]

$$|\langle\{e'\}_N \mid A_M \mid \{e\}_N\rangle \mid \leq 0[\exp(-\alpha N^\varepsilon)],\ \ \alpha > 0,\ \varepsilon > 0,$$

uniformly for all $||A_M|| \leq 1$ (M fixed) and some α, ε.

The last relation shows now the "overlap" for macroscopically different states decreases with an increasing number of degrees of freedom. Hence we can hope that an ideal solution with infinite systems will be a good approximation for real systems with a large but finite number of degrees of freedom.

2. General considerations

In the general case of infinitely many systems with Hilbert spaces $\mathscr{H}_\alpha$, $\alpha \in I$ (I-infinite set of indices) the general structure of the C.D.P.S. and of the algebra $B_\#$ of operators in it is the same as in the example of the spin chain (Fig. 2). In all such cases we obtain a superselection rule automatically. There is, however, an ambuguity in it: The associative law in the D.P.S. is not fulfilled without restrictions [1]. If we divide the manifold I into infinitely many nonintersecting manifolds I_γ with more than one element: $I = \bigcup_{\gamma\in\tilde{I}} I_\gamma$, $\tilde{I}$ infinite, we obtain a new C.D.P.S. $\tilde{\mathscr{H}}_\otimes$ by the relation: $\tilde{\mathscr{H}}_\otimes \equiv \bigotimes_{\gamma\in\tilde{I}} (\bigotimes_{\alpha\in\tilde{I}} \mathscr{H}_\alpha) \equiv \bigotimes_{\gamma\in\tilde{I}} \tilde{\mathscr{H}}_\gamma$. The structure of the respective algebras $B_\#$ in $\mathscr{H}_\otimes$ and $\tilde{\mathscr{H}}_\otimes$ in general differs, $B_\# \neq \tilde{B}_\#$. This fact may play a nontrivial role in systems with interactions. In finite decompositions ($\tilde{I}$ finite) the associative law is fulfilled. Problems with the above mentioned ambiguity will not be considered here.

In a general W^*-algebra $\mathfrak{A}$ of observables the classical observables are defined as elements belonging to the centrum Z of $\mathfrak{A} : Z \equiv \mathfrak{A} \cap \mathfrak{A}'$, where $\mathfrak{A}'$ is the commutant of $\mathfrak{A}$. Operators in the centrum commute with all the observables of the system. The centrum of $B_\#$ in the C.D.P.S. is generated by all projectors P_{C_ω}.

Once we have defined the algebra $\mathfrak{A}$ of observables, we can forget the specific representation in the C.D.P.S. A state of the physical system can be described then by a normed positive linear functional $\omega(A)$, $A \in \mathfrak{A}$. This shift from a specific faithful representation to an abstract C^*-algebra is sometimes necessary (a short survey of C^*-algebras and the application to QFT see in [7]: for a good survey of algebraic methods in contemporary physics see [10]). If e. g. the time evolution is defined by such an automorphism τ_t of $\mathfrak{A}$, $A_t = \tau_t A$, which cannot be described by a unitary transformation in the Hilbert space H_π of a representation $\pi(A)$ of $\mathfrak{A}$,

$$\pi(A_t) \neq U(t)\pi(A)U^{+}(t),$$

the state shifted in time cannot be described by a vector ψ_t from the same Hilbert space $\mathscr{H}_\pi$ as was the initial-state-vector ψ_0. As seen above, the existence of a superselection rule is fully given by commutation relations, i. e. as a property of an algebra of observables independent from any specific (faithful) representation.

In the example above the time evolution was defined as a unitary transformation $W(t)$ of the vectors in the C.D.P.S. The transformation is weakly discontinuous and the Stone theorem deosen't apply. In our case the system has in fact no Hamiltonian [4]. The operators $W(t)$ don't belong to the algebra $B_{\#}$ and the corresponding unitary transformation leads from $B_{\#}$: $W(t)AW^{+}(t) \equiv A_t (\notin B_{\#}$ in general). The isomrophism of $B_{\#}$ defined by $W(t)$ is not an automorphism. However, an automorphic time evolution occurs in the usual treatments of the general theory of big systems (see, e. g. [5], [6] and [7]) — at least as the automorphism of the weak closure of the respective representation. For automorphic time evolutions there appear two points:

a. the impossibility of "reduction" in finite times,

b. the non-implementability of a general automorphism by a unitary transformation in a given representation.

The general theorem asserts [4] that an automorphic time evolution cannot lead to reduction:

$$\alpha \in \operatorname{aut} \mathfrak{A}, \quad \omega_1 \between \omega_2, \quad \omega_i(\alpha A) \equiv \tilde{\omega}_i(A) \Rightarrow \tilde{\omega}_1 \between \tilde{\omega}_2.$$

The relation $\between$ ("disjointnes") may be interpreted as "incoherence in any representation in which ω_1 and ω_2 are vector states", i. e. if $\psi_i \in \mathscr{H}_\pi$, $\omega_i(A) = (\psi_i, \pi(A)\psi_i)$, $i = 1, 2$ and $\omega_1 \between \omega_2$, then $(\psi_1, \pi(A)\,\psi_2) = 0$ for all $A \in \mathfrak{A}$. Nevertheless, it is possible to construct such automorphic time evolutions that the wanted reduction is obtained in the limit $t \to \infty$. Examples of such models are in paper [4]. Let us consider such an automorphism τ_t. Let ψ_1 and ψ_2 be coherent vector states in some representation π and let states $\tilde{\omega}_i(A) \equiv \lim_{t\to\infty} (\psi_i, \pi(\tau_t A)\psi_i)$ be disjoint: $\tilde{\omega}_1 \between \tilde{\omega}_2$. Then one can prove that for coherent superposition $\psi = \alpha_1\psi_1 + \alpha_2\psi_2$ we get in $t \to \infty$ a mixture $\tilde{\omega}$:

$$\tilde{\omega}(A) \equiv \lim_{t\to\infty} (\psi, \pi(\tau_t A)\psi) = |\alpha_1|^2\tilde{\omega}_1(A) + |\alpha_2|^2\tilde{\omega}_2(A), \quad A \in \mathfrak{A}.$$

In this way we can get the wanted reduction in $t \to \infty$. This is a "true irreversibility" because of the infinite time interval.

By an automorphic evolution it may happen that the automorphism τ_t is not unitarily implementable in some representation π. Then for the initial state $\psi \in \mathscr{H}_\pi$ and $A \in \mathfrak{A}$ we have

$$\omega_t(A) \equiv (\psi, \pi(\tau_t A)\psi) \neq (\psi_t, \pi(A)\psi_t) \text{ for any } \psi_t \in \mathscr{H}_\pi.$$

The time developed state is now the linear form ω_t, which is not expressible as a vector state in $\mathscr{H}_\pi$.

Summarizing, we can say that for the description of infinite systems it is convenient to use the abstract algebraic formulation. The realistic models in such a formulation, as far as I know, are still unknown. The general features of such theories indicate, as I have tried to show above that the abstract C^*-algebraic formalism might be an appropriate formalism for the solution of the problem of irreversible processes, the special case of which is the problem of measurement in the the quantum theory.

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