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DETERMINATION OF CDD-POLE PARAMETERS IN THE πN $I = J = \frac{1}{2}$ STATE FROM THE REGGE ASYMPTOTIC BEHAVIOUR OF PARTIAL WAVE AMPLITUDES[1]

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As recently shown, the attempt to combine the Regge asymptotic behaviour with partial wave dispersion relations in a consistent manner leads to an infinite set of finite energy sum rules for each partial wave.

These sum rules are used to determine CDD-pole parameters in the pion nucleon $I = J = 1/2$ state. To avoid model dependent ambiguities we take in a modified N/D approach an input calculated from empirical phase shifts.

I. INTRODUCTION

In the present paper we study the question of how far the high energy behaviour of a single partial wave amplitude, following from the Regge pole dominance of the whole amplitude, determines CDD-pole parameters [1].

To approach the problem, we use finite energy sum rules for partial waves, which have been derived for the scattering of scalar particles under the following assumptions [2—4].

i. The whole amplitude in the s-channel is asymptotically determined by the Regge poles in the t- and u-channel. (The generalization to the Regge cuts is straightforward.)

ii. Only the sharp forward and backward peaks contribute to the partial wave projection. In this region the trajectories are assumed to be linearly rising.

iii. The partial wave amplitudes obey dispersion relations. In these relations a cut off S_c is introduced, only to indicate the region, where the Regge asymptotic behaviour is valid.

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Then the sum rules follow from the fact that no pure S^{-n} powers occur in the asymptotic expansion of the partial wave amplitude. It is possible to formulate such sum rules for the realistic case of the πN scattering too [5]. We write them down in the w plane. Therefore in all calculations the S_{11} and the P_{11} waves are joined together by the Max Dowell symmetry [6]. In a modified N/D formalism [7] we compute phase shifts with an empirical input potential. These phase shifts generally are functions of unknown CDD pole parameters. To determine the parameters we substitute these functions into the sum rules. If we satisfy the first two sum rules by one CDD pole in addition to the direct channel nulceon pole, then the corresponding output phase shifts are in good agreement with the experimental ones.

The sum rules are written down in Section II. In Section III we formulate the N/D equations and their input potential. The results are given in Section IV.

II. FINITE ENERGY SUM RULES FOR PARTIAL WAVE AMPLITUDES

We consider the sum rules for the πN scattering in the $I = J = \frac{1}{2}$ state. We have [5]

$$\frac{1}{\pi}\int_{u}^{iW_c} dw' \operatorname{disc} h(w') w'^{r-1} + \frac{1}{\pi}\int_{M+1}^{W_c} dw' \{\operatorname{Im} h(w') + (-1)^r \operatorname{Im} h(-w')\} w'^{r-1} +$$

$$+ \frac{3}{8\pi} g^2_{\bar{N}N\pi} (-M)^{r-1} = \gamma_r^{(r)} \qquad r = 1, 2, \ldots \tag{1}$$

where $M(= 6.75)$ is the nucleon mass ($m_\pi = 1$) and $g_{\bar{N}N\pi}(= 13.5)$ is the pion-nucleon coupling constant. Further we have assumed for $W \geqslant M + 1$

$$h(w) = \frac{\eta(w) e^{2i\delta(w)} - 1}{2i\varrho(w)} \equiv h_{0+}(w) = \frac{\eta_{0+}(w) e^{2i\delta_{0+}(w)} - 1}{2i\varrho_{0+}(w)} \tag{2}$$

$$-h(-w) = -\frac{\eta(-w) e^{2i\delta(-w)} - 1}{2i\varrho(w)} \equiv h_{1-}(w) = \frac{\eta_{1-}(w) e^{2i\delta_{1-}(w)} - 1}{2i\varrho_{1-}(w)}$$

therefore

$$\operatorname{Im} h(w) = \frac{1 - \eta(w)}{2\varrho(w)} + \frac{\eta(w)\sin^2\delta(w)}{\varrho(w)}. \tag{3}$$

The function $\varrho(w)$ is asumed to produce the correct threshold behaviour. The first term in (1) represents the sum of all "lefthand" cut contributions up to the cut off on the imaginary axis. The constants $\gamma^{(r)}$ on the right-hand side of

(1) depend on the Regge pole parameters. The dominant contributions are due to meson trajectories. If w_c is sufficiently high, in the first two sum rules a trajectorie with the signature $-1(\varrho)$ in the second two a trajectorie with the gisnature $+1$ (Pomeron P) dominate. We have [5]

$$\gamma^{(1)} = -\frac{1}{2\pi^2}\frac{C^\varrho_+(0)}{(1-\alpha_\varrho(0))\alpha'_\varrho(0)}\left(\frac{2M}{w_c^2}\right)^{1-\alpha_\varrho(0)}\frac{1}{\ln w_c} \tag{4}$$

$$\gamma^{(2)} = -2M\gamma^{(1)},$$

$$\gamma^{(3)} = -\frac{1}{4\pi^2}\frac{C^P_+(0)}{\alpha'_P(0)}\frac{w_c^2}{\ln w_c}$$

$$\gamma^{(4)} = -2M\gamma^{(3)}.$$

Here the Pomeron intercept is put $\alpha_P(0) = 1$. The $\alpha'_i(0)$ represent the slopes and $C^i_+(0)$ the residues of the Regge ansatz in the invariant A' amplitude at $t = 0$ [8]. For the numerical calculations we take the parameter values of a fit of Chiu et al. [9]. In their notation and parametrization we have:

$$C^\varrho_+(0) = \left(\frac{1}{E_0}\right)^{\alpha_\varrho(0)} C^\varrho_0(\alpha(0) + 1) \tag{5}$$

$$C^P_+(0) = -2\left(\frac{1}{E_0}\right)^{\alpha_P(0)} C^P_0,$$

E_0 is a scale constant (~ 1 GeV). The parameter values are given in Tab. 1.

There are reasons to take this fit with big slopes of P and P'. We have calculated the asymptotic behaviour of $\delta(w)$ and $\eta(w)$ depending upon results of several fits [9]. We found the smaller the slopes of P and P', the higher the point W'_c where δ and η start to grow monotonously to asymptotic limits

$$\left(\eta(w) \xrightarrow[w\to\pm\infty]{} 1, \quad \delta(w) \xrightarrow[w\to\pm\infty]{} \frac{n}{2}\pi, n \text{ integer}\right).$$

When we took such fits with $\alpha'_P(0) \sim 0.10$, we obtained $w'_c \sim 10^3$. On the other

Table 1

Regge — pole parameters [9] taken in the numerical calculations

	$\alpha(0)$	$\alpha'(0)$ [GeV^{-2}]	C_0 [mb GeV]
ϱ	0.58	1.02	1.49
P	1.00	0.34	7.43
P'	0.72	0.34	16.60

hand with the values in (6) we got $w_c' \sim 10^2$. Of course we should chose $w_c \simeq \simeq w_c'$. Therefore the compromise between good accuracy of the numerical solution of the N/D equations and the use of the best fit lead us to (6) and $w_c = 100$.

III. THE POTENTIAL OF THE N/D EQUATIONS

To calculate the phase shifts of the P_{11} and S_{11} waves we use the N/D equations in a formalism similar to Frye and Warnock [7]. In these equations the above mentioned cut off w_c occurs. Therefore from the mathematical point of view we have to solve the same type of equations as in the strip approximation [10]. For practical purposes it is sufficient to use the method of the matrix inversion [11].

For $M + 1 \leq |w| \leq w_c$ we have in the presence of n_c CDD poles (L_i, C_i)

$$\frac{2\eta(w)\mathrm{Re}N(w)}{1+\eta(w)} = \mathrm{Re}B(w) + \sum_{i=1}^{n_c} C_i \frac{w\mathrm{Re}B(w) - L_i\mathrm{Re}B(L_i)}{w - L_i} + $$
$$+ \frac{1}{\pi}\left(P\int_{-w_c}^{-M-1} + P\int_{M+1}^{w_c}\right)\mathrm{d}w' \frac{2\varrho(w')\mathrm{Re}N(w')}{(1+\eta(w'))w'(w'-w)} \times$$
$$\times \{w'\mathrm{Re}B(w') - w\mathrm{Re}B(w)\} \tag{7}$$

$$\mathrm{Re}D(w) = 1 + w\left[\sum_{i=1}^{n_c} \frac{C_i}{w - L_i} - \frac{1}{\pi}\left(P\int_{-w_c}^{-M-1} + P\int_{M+1}^{w_c}\right) \times \right.$$
$$\left. \times\ \mathrm{d}w' \frac{2\varrho(w')\mathrm{Re}N(w')}{(1+\eta(w'))w'(w'-w)}\right]. \tag{8}$$

With

$$\mathrm{Im}D(w) = -\frac{2\varrho(w)}{1+\eta(w)}\mathrm{Re}N(w)$$

we have for the output phase shift

$$\mathrm{tg}\,\delta_{out}(w) = -\frac{\mathrm{Im}D(w)}{\mathrm{Re}D(w)}. \tag{9}$$

The input function $\mathrm{Re}B(w)$ has to fulfil two conditions:

i. Along the finite physical cuts $\mathrm{Re}B(w)$ must be identical with [12]

$$\mathrm{Re}B_{emp.}(w) \equiv \frac{\eta(w)\sin 2\,\delta(w)}{2\varrho(w)} - \frac{1}{\pi}\left(P\int_{-w_c}^{-M-1} + P\int_{M+1}^{w_c}\right)\mathrm{d}w' \frac{\eta(w')\sin^2\delta(w')}{\varrho(w')(w'-w)}, \tag{10}$$

where the right-hand side is calculated with empirical phase shifts.

ii. The discontinuity of the finite nonphysical cuts disc $h(w)$ must be consistent with the finite energy sum rules (Eq. (1)) calculated with empirical phase shifts too.

Only in this way one can hope to get results comparable with the experimental data. Such a potential could be constructed for instance by a pole approximation of long range forces (considering crossing relations) and of suitably adjusted short range forces.

However, for our calculations it is not necessary to know completely the input, i. e. to have an explicit knowledge of the disc $h(w)$. Therefore we only assume that a function $\mathrm{Re}B(w)$ should exist satisfying the two above mentioned conditions. Then we solve the N/D equations with $\mathrm{Re}B_{emp.}(w)$ and calculate the integrals

$$\frac{1}{\pi}\int_{u}^{iw_c} \mathrm{d}w' \mathrm{disc}h(w')w'^{r-1}$$

using the sum rules (1) with empirical phase shifts [13]. Thus we can replace an approximative pole input by a model independent one. (Of course our input contains assumptions about the smoothed behaviour of the phase shifts in the energy region where the phase shift analysis fails.)

We remark that $\mathrm{Re}B_{emp.}(w)$ contains the direct channel nucleon pole. To examine the case without this pole, we have to subtract its contribution

$$-\frac{3}{8\pi}\frac{g^2_{NN\pi}}{w+M}$$

from $\mathrm{Re}B_{emp.}(w)$.

IV. RESULTS

We study the N/D equations for three cases:

(a) neither the direct channel nucleon pole nor a CDD pole,

(b) with the nucleon pole but without a CDD pole,

(c) both with the nucleon pole and with one variable CDD pole (C_1, L_1).

In all cases the first four sum rules are tested by substituting the output phases δ_{out} into (1). The numerical calculations give the following results.

For the cases (a) and (b) the sum rules cannot be fulfilled. The disagreement between the left- and the right-hand sides of (1) is extreme for the case (a), but becomes smaller for the case (b). In the case (c) we sought the points where the individual sum rules are satisfied by variation of C_1 and L_1 (Fig. 1). The shown region is the only one in a wide range where the sum rules I, II, III and IV are all fulfilled together within a small distance from each other.

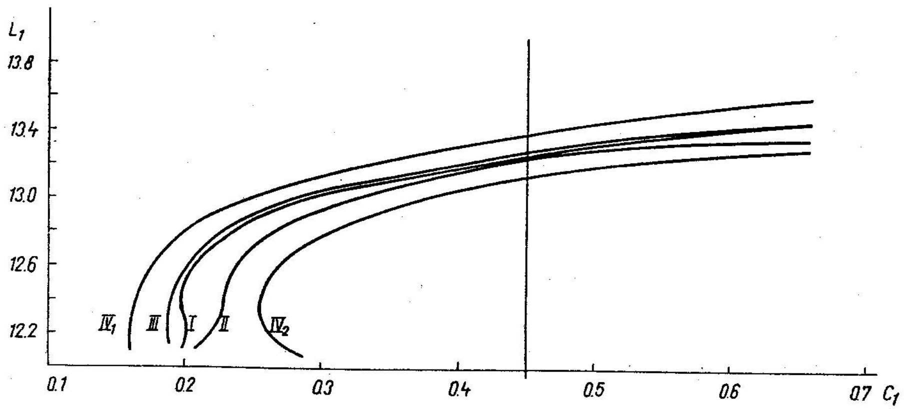

Fig. 1. Geometric locus where the sum rules I, II, III and IV are satisfied. Optimal CDD parameters: $L_1 = -13.25$, $C_1 = 0.45$.

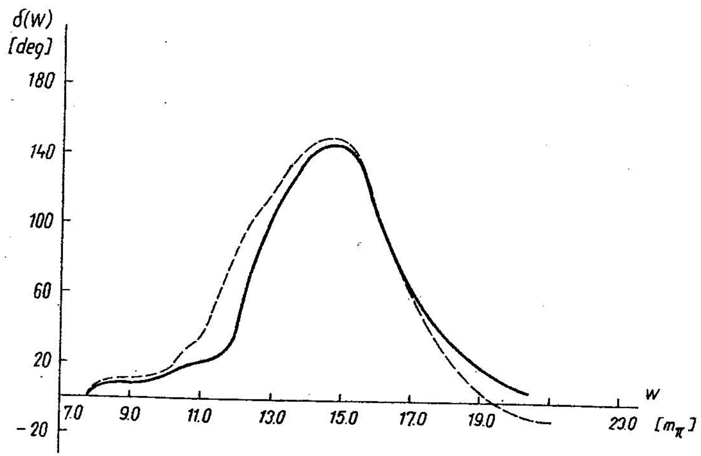

Fig. 2. S_{11} phase shift. — — — empirical; ——— theoretical.

We have not drown other curves in other regions where for instance only one sum rule is satisfied. Thus we see that there is one point where the first two sum rules are fulfilled simultaneously ($L_1 = -13.25$ and $C_1 = 0.45$). The

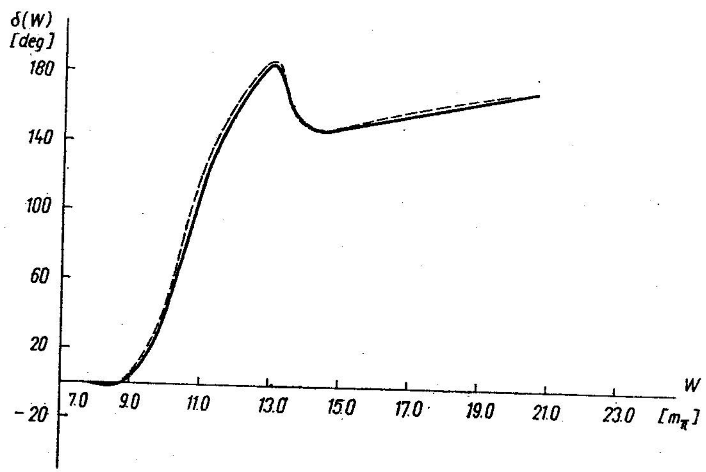

Fig. 3. P_{11} phase shift. — — — empirical; ——— theoretical.

Table 2

Values of the individual parts of the first four sum rules (UPC, PC, NP or Σ, resp. denote symbolically the contributions of the nonphysical cut, the physical cut, the nulceon pole or the whole left-hand side of the r^{th} sum rule (1), respectively.)

case	r	$\gamma^{(r)}$	Σ	PC	NP	UPC
(a)	1	−0.02	−11.36	−15.02	21.93	−18.27
(b)		−0.02	0.82	−2.84	21.93	−18.27
(c)		−0.02	−0.02	−3.68	21.93	−18.27
(a)	2	0.3	117.9	248.0	−147.7	17.6
(b)		0.3	−1.1	129.0	−147.7	17.6
(c)		0.3	0.3	130.4	−147.7	17.6
(a)	3	9.65×10^3	8.79×10^3	-1.93×10^3	1.00×10^3	9.72×10^3
(b)		9.65×10^3	9.91×10^3	-0.81×10^3	1.00×10^3	9.72×10^3
(c)		9.65×10^3	9.58×10^3	-1.14×10^3	1.00×10^3	9.72×10^3
(a)	4	-1.30×10^5	-0.71×10^5	4.13×10^5	-0.07×10^5	-4.77×10^5
(b)		-1.30×10^5	-1.12×10^5	3.72×10^5	0.07×10^5	-4.77×10^5
(c)		-1.30×10^5	-1.35×10^5	3.49×10^5	-0.07×10^5	-4.77×10^5

third and the fourth sum rule are not satisfied at this point, but the "tube" treated by the two solutions of the fourth sum rule has the smallest "radius" there. Thus the determined CDD parameters are the optimum for the higher sum rules too. To get a better agreement of all sum rules, we should introduce more CDD poles. Furthermore we see that the determined CDD pole position L_1 agrees with that point, where the empirical P_{11} phase shift crosses π for the second time. The determined parameters correspond to the output phase shifts in a very good agreement with the experimental ones (Fig. 2, 3).

Table 2 shows the behaviour of the individual parts of the sum rules (1) in the considered three cases. For the third case we give the results for the above mentioned CDD parameter values.

Finally we conclude that the finite energy sum rules derived from the Regge asymptotics of the partial wave amplitudes are able to determine CDD pole parameters if a correct input potential is used.

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