acta phys. slov. 23 (1973), No. 3

MESON-MESON INTERACTIONS[1]

JENS LYNG PETERSEN,* Geneva**

The history and the present status of our theoretical and phenomenological understanding of the meson-meson scattering is briefly reviewed with emphasis on the processes $\pi\pi \to \pi\pi$, $K\pi \to K\pi$, and $\pi\pi \to K\bar{K}$.

I. INTRODUCTION

The unique theoretical singnificance of meson-meson interactions (in this talk I shall confine myself to the processes $\pi\pi \to \pi\pi$, $\pi K \to \pi K$, $\pi\pi \to K\bar{K}$) has been realized for more than a decade. Is arises from the following characteristics:

(i) absence of spin implies a great simplification in understanding the dynamics and in interpreting phenomenological amplitudes;

(ii) equality (or near equality) of masses and their small values (relative to typical resonance energies) implies that the system has a fundamentally relativistic nature.

On the other hand, the lack of direct experimental information about these processes means that for the $\pi\pi$ scattering the invariant amplitudes are known even less than for the πN scattering. Nevertheless, pion production experiments analyzed according to the Chew-Low-Goebel idea [1] have in recent years made the field a phenomenologically respectable one. Also, progress in the understanding of the role of one-pion exchange in these processes (for summaries with several references see for example Refs. [2]) together with promises given by recently published experiments or by the ones in a late stage of analysis, has made several poeple feel that this field is before a major break-through that may, in a few years time, let meson-meson amplitudes rival the pion-nucleon ones as detailed examples from which theoreticians can get inspirations.

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[1] Talk given at Elementary Particle Physics Seminar at Pezinská Baba, September 22—25, 1971.

* CERN — Div. TH, 1211 GENEVA 23, Switzerland.

** On leave of absence from the Niels Bohr Institute, Copenhagen.

In Section II I shall, by means of key-words, indicate what have been, in my mind, the high-lights of the past decade of development. In the rest of the talk I shall be concerned with the events of the last year or so. In Section III I shall describe the fairly unambiguous picture that has emerged for low energy $\pi\pi$ scattering. In Section IV, I shall talk about elastic and inelastic $\pi\pi$ interactions in the above ϱ region which promises to be in the focus of interest in the near future. In Section V I shall give the latest results on $K\pi$ interaction, related by crossing to the $\pi\pi \to K\bar{K}$ one emphasized in Section IV. Finally, conclusions will be attempted in Section VI.

II. BRIEF HISTORICAL OUTLINE

1. Early 60's

Phenomenologically the ϱ meson is well known from the pion production. From the analysis of the low energy pion-nucleon interaction, $I = J = 0$ $\pi\pi$ scattering is known to be attractive and of an appreciable magnitude [3]. On the theoretical side, the basic formalism is developed and the most important first principle consequences of the analyticity crossing and unitarity (ACU) are worked out. The Chew-Mandelstam theory (near constant amplitudes) is unable to generate a $\pi\pi$ scattering that fits experimental information [4]. Chew's bootstrap program is formulated [5] but despite an enormous effort no very satisfactory results were obtained (for a recent examination see Basdevant [6]).

2. Up to 1968

Experiments disclose that throughout the field of low and intermediate energy hadron physics, *resonances* play a predominant role. The natural spin parity ones group nicely into two SU(3) nonets:

$$J^P = 1^-: \{\varrho(765),\ \varphi(1019),\ \omega(784),\ K^*(890)\} \tag{1}$$

$$J^P = 2^+: \{A_2(1300),\ f'(1514),\ f(1270),\ K^{**}(1420)\}$$

with a near ideal (quark model [7]) mixing among the central members:

$$\begin{matrix} m_\omega \approx m_\varrho \\ m_{A_2} \approx m_f \end{matrix} \quad \text{implies } \mathrm{tg}^2 Q \approx \tfrac{1}{2}\,. \tag{2}$$

No "exotic" states (i.e., those pertaining to SU(3) representations other than {1}, {8} for mesons and {1}, {8}, {10} for baryons) are found. Regge trajectories seem to exist and all the known ones (except for the Pomeranchukon) are roughly linear parallel and exchange denerate (i.e., the signature is rendered almost irrelevant by nature).

Weinberg writes down his soft meson theory of low energy parameters [8]. Thus, not only do resonances reflect the symmetry properties of hadron physics via their multiplet structure, but so do low energy parameters via low energy theorems provided by current algebras at off-shell points and interpreted on the shell by PCAC. A mysterious link between these two structures (low energy parameters and resonances) is discovered (KSFR relation) [9]:

$$\frac{1}{F_\pi^2} \approx \frac{96\pi\Gamma_\varrho}{m_\varrho^3}\,((2.5 \pm 0.3)m_\pi^{-2} \approx (1.6 \pm 0.3)m_\pi^{-2})\,. \tag{3}$$

Here F_π is the pion decay constant in terms of which all Weinberg's low energy parameters are measured.

3. 1968—1970

3.1. Duality

The following propositions about nature
(i) exchange degenerate linear trajectories;
(ii) exact crossing symmetry;
(iii) resonance dominance;
(iv) quark model quantum number structures
are discovered not to be independent. The link between them, first provided by finite energy sum rules [10] is called duality [11]. The above statements are explicitly satisfied by the Veneziano-Lovelace models [12] for meson-meson scattering:

$$\text{for } \pi\pi \to \pi\pi\,,\ A(s,t,u) = f^2(V_{\varrho\varrho}(s,t,) + V_{\varrho\varrho}(s,u) - V_{\varrho\varrho}(t,u))\,; \tag{4}$$

$$\text{for } K\pi \to K\pi\,,\ A^{(\pm)}(s,t,u) = \tfrac{1}{2}f^2\,(V_{K^*\varrho}(s,t) \pm V_{K^*\varrho}(u,t))\,;$$

$$\text{for } KK \to KK\,,\ A^{(0)}(s,t,u) = f^2\,(V_{\varphi\varrho}(t,u) - V_{\varphi\varrho}(u,t))\,;$$

$$A^{(1)}(s,t,u) = f^2\,(V_{\varphi\varrho}(t,u) + V_{\varphi\varrho}(u,t))\,,$$

where

$$V_{xy}(s,t) = -\frac{\Gamma(1-\alpha_x(s))\Gamma(1-\alpha_y(t))}{\Gamma(1-\alpha_x(s)-\alpha_y(t))}$$

$$= -(1-\alpha_x(s)-\alpha_y(t))\,B(1-\alpha_x(s),\,1-\alpha_y(t))\,.$$

In addition they generate the Weinberg low energy amplitudes approximately provided the off-shell forms satisfy the Adler conditions, which implies the following quantization conditions on the trajectories:

$$\alpha_\varrho(\mu^2) = \tfrac{1}{2},\ \alpha_{K^*}(s) \equiv \alpha_\varrho(s) - \Delta,\ \alpha_\varphi(s) \equiv \alpha_{K^*}(s) - \Delta$$

(μ is pion mass)

$$\Delta \equiv \frac{m_K^2 - \mu^2}{2(m_\varrho^2 - \mu^2)} \approx \frac{1}{4}. \tag{5}$$

Apart from satisfying all known SU(3) nonet results, these amplitudes provide an infinity of new predictions: the free parameter in the Gell-Mann-Okubo mass formula [13] is fixed; new "symmetries" are generated:

$$m_f^2 - \mu^2 = 3(m_\varrho^2 - \mu^2) \tag{6}$$

$$m_g^2 - \mu^2 = 5(m_\varrho^2 - \mu^2) \text{ etc. } \ldots;$$

an infinity of daughter states are predicted. Of special interest for us are

(i) the $\varepsilon : m_\varepsilon = m_\varrho,\ I^G(J^P)C = 0^+(0^+)+,\ \Gamma_\varepsilon/\Gamma_\varrho = \frac{9}{2}$

(ii) the $S^* : m_{S^*} = m_\varphi,\ I^G(J^P)C = 0^+(0)+.$

In Eqs. (4) S^* couples only to $K\overline{K}$ and not to $\pi\pi$, however, due to unitarity its effect on the $\pi\pi$ scattering could be noticeable (see Section IV). The amplitudes of Eqs. (4) have the following well-known drawbacks:

(i) unrealistic absorptive parts ($\operatorname{Im} A \alpha \sum_i \delta(s - m_i^2)$);
(ii) no diffractive component is present;
(iii) lack of general factorization.

All of these reflect violations of unitarity, nevertheless, the model continues to be of great inspiration to phenomenologists in the field.

3.2. The emergence of phase shift language for the $\pi\pi$

Despite important fundamental difficulties (see for example, Refs. [2]) the Chew-Low-Goebel analysis of pion production data reaches such a level of credibility that $\pi\pi$ phase shifts between 600 and 900 MeV can be suggested:

(i) $I = J = 1$ or δ_1^1 shows an elastic resonance (ϱ) with mass and width being determinable to $\approx \pm 20$ MeV [14];

(ii) $I = 2$, $J = 0$ or δ_0^2 is slowly varying and negative. At $m_{\pi\pi} = m_\varrho, \delta_0^2 \approx \approx -11° \pm 10°$;

(iii) $I = 0$, $J = 0$ or δ_0^0 is near 90° at $m_{\pi\pi} = m_\varrho$. Away from that energy measurement of s-p interference only, implies the existence of the well-known up-down ambiguity [15];

(iv) a conspicuous $K\overline{K}$ production just above threshold may or may not indicate the existence of an S^* resonance [16];

(v) a large and slowly varying s-p interference in the final $K\pi N$ states in KN interaction is interpreted as evidence for an $I = 1/2$, $J = 0$ $\varkappa$ resonance at $\approx$ 110 MeV.

3.3. First principle constraints on partial-wave amplitudes

Martin [17] shows that the $\pi\pi$ partial wave amplitudes in the triangle $s \geqslant 0,\ t \geqslant 0,\ u \geqslant 0$ are severely constrained by crossing and positivity ($\operatorname{Im} f_l^I(s) \geqslant 0$ for $s \geqslant 4\mu^2$). Particularly useful are the complete set of crossing constraints derived by Roskies and others [18]. These immediately kill a number of $\pi\pi$ models. Their limitation lies partly in the two following facts:

(i) they leave the transformations

$$A(s, t, u) \to A'(s, t, u) = a + bs + cA(s, t, u),\ c > 0$$

invariant;

(ii) they apply only to a non-physical region.

III. LOW ENERGY $\pi\pi$ INTERCATION

1. The Weinberg predictions

For $\pi\pi$ scattering in a state of isospin I and orbital angular momentum l we define the scattering lengths a_l^I and the effective ranges r_l^I from the phase shift δ_l^I and the c.m. momentum q by

$$q^{21+1} \cot \delta_l^I = (a_l^I)^{-1} + 1/2\, r_l^I q^2 + \ldots. \tag{7}$$

Then Weinberg predicts

$$L \equiv \tfrac{1}{6}\,(2a_0^0 - 5a_0^2) = 3\mu^2 a_1^1 = 0.10\mu^{-1} \tag{8}$$

$a_1^1 = 0.033\mu^{-3},\ a_0^0 = 0.17\mu^{-1},\ r_0^0 = -7.3\mu^{-1},\ a_0^2 = -0.050\mu^{-1},\ r_0^2 = 6.0\mu^{-1},$

$a_0^0/a_0^2 = -\frac{7}{2}$ (the non-exoticity assumption) compared to the Chew-Mandelstam theory result, $a_0^0/a_0^2 \approx +\frac{5}{2}$. The above numbers have an uncertainty of $\pm$ 10 % due to ambiguities in the determination of the pion decay constant (PCAC constant).

2. Low energy $\pi N \to 2\pi N$

In a recent final state interaction analysis of data (including important new $\pi^\circ\pi^\circ$ ones by Maung et al. [19]) below the πN^* threshold, Botke [19] gives

$$a_0^0 = \left(0.2 \begin{matrix} +0.08 \\ -0.10 \end{matrix}\right)\mu^{-1} \tag{9}$$

$$a_0^0 = 0.2 \quad \text{implies} \quad r_0^0 = \left(-5.2 \begin{matrix} +0.2 \\ -0.5 \end{matrix}\right)\mu^{-1}$$

$$a_0^0 = 0.15 \text{ implies } r_0^0 = (-6.5 \pm 0.5)\mu^{-1}$$

in good agreement with (8).

3. Dispersion relation phenomenology

I restrict myself to the work of Morgan and Shaw [20]. They use forward dispersion relations to provide for most of the important crossing constraints. Phase shifts are fitted to various experimental information on δ_0^0, δ_0^2, δ_1^1 in the region 500–900 MeV. The low energy region is obtained using the dispersion relations to extrapolate from the intermediate energy region in a way consistent with ACU. The obtained low energy parameters are functions of the assumed experimental forms in 500–900 MeV and of the high energy contributions needed to get consistency with unitarity. Despite the fact that a considerable range of solutions for (a_0^0, a_0^2) is obtained, they are discovered to lie on a "universal curve" (interpreted by Olsson [21]), so that an accurate estimate is obtained for

$$\tfrac{1}{6}(2a_0^0 - 5a_0^2) = (0.095 \pm 0.015)\mu^{-1} \tag{10}$$

in close agreement with Eq. (8). Further, the high energy contributions are difficult to interpret in terms of known mechanisms except if a_0^0, a_0^2 and a_1^1 separately are close to Eqs. (8).

4. Theoretical models

A. The Padé calculations [22] in field theory models (the σ model and the Yang-Mills fields model) demonstrate convincingly that Eqs. (8) are consistent with unitarity, which was far from obvious in their derivation. In order for this to happen, ϱ and ε appear to dynamically require each other.

B. Many models have been constructed based on [23]

(i) the elastic unitarity below 1 GeV;

(ii) the saturation by s and p waves below 1 GeV;

(iii) the existence of the ϱ meson;

(iv) the imposition of crossing in the triangle $s, t, u \geqslant 0$ by the Martin inequalities and the Roskies relations;

(v) the use of analyticity to extrapolate to the physical region.

Results seem to require Eqs. (8) and the existence of a broad ε resonance. Although as a dynamical calculation this is quite modest compared to the (unsuccessful) bootstrap ambitions, it is highly non-trivial that all long range forces *and* the short range force (measured by a_0^0/a_0^2, for a discussion see for example Serebryakov and Shirkov [24]) are predicted from one component of the long range forces (the ϱ).

5. Integral equations for partial wave amplitudes

Recently an important new tool has become available which achieves the long sought goal of imposing rigorous crossing constraints on physical region partial wave amplitudes. For an amplitude that satisfies a fixed t unsubtracted dispersion relation, they are immediately obtained by projecting out partial wave amplitudes giving relations like [25]

$$f_l^I(s) = \sum_{I'=0}^{2} \sum_{l'=0}^{\infty} (2l'+1) \int_{4\mu^2}^{\infty} \mathrm{d}s'\, G_{l,I}^{l',I'}(s',s)\, \mathrm{Im}\, f_{l'}^{I'}(s'). \tag{11}$$

When subtractions are present, additional terms occur. Roy and Basdevant et al. [26], however, showed that these are fixed by crossing in terms of physical quantities (for $\pi\pi$ scattering). Preliminary studies indicate that they provide a highly sensitive tool.

6. The K_{e4} decay

The decay $K^+ \to \pi^+\pi^-e^+\nu_e$ has been known for many years to provide a means of measuring the on-shell $\pi\pi$ scattering below ≈ 400 MeV. Recently, the experimental troubles associated with the very small branching ratio were overcome [27] and by providing 1600 events the Geneva-Saclay group succeded for the first time in analyzing their data according to the completely model independent Pais-Treiman scheme [28]. Results for a_0^0 are in qualitative agreement with Eqs. (8).

7. Summary

For the below ϱ region there is thus good evidence in favour of Weinberg's low energy parameters. The up-down ambiguity is resolved in favour of "between" in agreement with the two latest π production experiments [29] carried out at very different beam momenta.

IV. THE ELASTIC AND INELASTIC $\pi\pi$ INTERACTION ABOVE THE ϱ

I shall deal only with the $I = J = 0$ wave which is the most interesting one at present.

1. The $\pi^\circ\pi^\circ$ production

The existence of the up-down ambiguity reflects the experimental inability to separate the isotropic part of the $\pi\pi$ distribution due to the s wave scattering from that, due to the decay of depolarized ϱ's. To do so one must have a handle on the production mechanism other than OPE. The difficulty is circumvented in $\pi^0\pi^0$ production, but so far different experimentators have been unable to reach mutual agreement [30].

2. The f^0 (1270) region

A large s-d interference seems to have been observed in that region [31]. Consequently it has been suggested that δ_0^0 stays clost to 90° from 700 MeV to 1300 MeV. As we shall see this appears to be quite impossible.

3. Inelasticities

Baton et al. get a better fit to the angular distribution by allowing for a large 4π inelasticity below 1 GeV (the threshold = 560 MeV), but new measurements show that essentially no 4π events are present and the amplitude is by now considered generally elastic below 1 GeV [32]. Above 1 GeV 4π the inelasticity rapidly grows and at 1100 MeV may account for 20—30% of the $K\bar{K}$ inelasticity. The last one is very strong (the S^* effect) resulting in the following estimates

$$\langle\eta_0^0\rangle_{1000}^{1100} \approx 0.45 \begin{matrix}+0.13\\-0.43\end{matrix} \quad \text{(Hayms et al. [32])} \tag{12}$$

$$\eta_0^0(1020) \approx 0.60 \pm 0.07 \quad \text{(Alston-Garnjost et al. [32]).}$$

4. Implications

By unitarity a large s wave $K\bar{K}$ production implies a square-root drop of η_0^0 at $4m_K^2 \approx 1$ GeV:

$$(\eta_0^0)^2 = 1 - \frac{8kq}{s}|A_0^0(\pi\pi \to K\bar{K})|^2, \tag{13}$$

where q and k are the pion and kaon c.m. momenta. By *analyticity* this requires δ_0^0 to have an infinite positive slope just below 1 GeV (the Ball-Frazer effect, see also Morgan [33]). In fact, from Eq. (12) we expect $\Delta\delta_0^0$ over the last 100 MeV before 1 GeV to be $\approx 50° \pm 20°$. For this reason the suggestion in subsection 2 must be discarded.

5. The experimental consequence

In a recently published experiment Alston-Garnjost et al. [32] found beautiful evidence for this, in a violent discontinuity, particularly in the s-p interference and in the $\pi\pi$ mass spectrum and the effect is shown to be due to the s wave. Since the $\pi\pi$ mass spectrum is found to drop by about the s wave unitarity limit, evidences are that δ_0^0 is large just below 900 MeV and the down solution is highly favoured up to that point. Between 900 MeV and 1 GeV δ_0^0 must change from down to up which is entirely possible when due allowances for uncertainties are made [29].

V. THE $K\pi$ INTERACTION

To provide a complete treatment of the $K\bar{K}$-S^* effect a coupled $\pi\pi - K\bar{K}$ channel analysis has to be carried out near the $K\bar{K}$ threshold. The "forces" of the S^* are then the $\pi\pi$ scattering and the $K\pi$ scattering. A study of this last process is therefore necessary for a crossing symmetric treatment to be given. Also, it is of great interest in itself to obtain low energy $K\pi$ parameters for comparison with the soft meson prediction [8] and to sort out the possible daughter role of the $\varkappa$ as well as to get good $K\pi$ phase shifts in general.

The Chew-Low-Goebel extrapolation studies of pion production in the KN interaction is a much less matured field than that for the πN interaction and several experimental difficulties exist compared to those of the latter [39]. The latest result of the International K^+ collaboration [35] suggests in addition to the well-known K^* (890)

(i) weak interaction in the exotic $I = \frac{3}{2}$ channel: an almost vanishing p wave and an s wave of negative magnitude giving

$$\sigma_{K\pi}(\tfrac{3}{2}) \approx (2 \pm 1)\,\mathrm{mb} \tag{14}$$

up to 1200 MeV;

(ii) the interesting s wave $\delta_0^{1/2}$ exhibits an up-down ambiguity much as in the $\pi\pi$ scattering. The down branch appears to stay below 80° all the way up to 1100 MeV (which is not evidence against a pole on the second sheet corresponding to a very wide object), whereas the up-branch follows the p wave near $m_{K^*} \approx 890$ MeV and continues parallel to the down branch, however, displaced upwards by 180°.

VI. CONCLUSIONS

The past year has seen an impressive *qualitative* progress on several points:

a. an increased accuracy and credibility of experimental information;
b. increased regions in which it is available;
c. improved phenomenological tools;
d. an improved understanding of the underlying dynamics.

Progress on points a. and b. is almost certain to continue for at least a couple of years to come and when taken together with c. there is every hope that a phenomenological effort will lead to considerable quantitative progress so that in a few years time maps of the invariant amplitude up to ≈ 2 GeV are likely to exist. There is no reason that they should have a lesser impact on our field than the ones for pion-nucleon scattering have had during the last five years.

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Received September 22nd, 1971