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INFINITE HELICITY SUMS AND GENERALIZED O(1.2) EXPANSIONS[1]

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We give a brief survey of the O(1.2) expansion formalism for power bounded multiparticle amplitudes and show that the occurrence of infinite helicity sums in the formalism does not present any serious convergence problem. The role of the so-called nonsense terms is clarified, and it is shown that these necessarily cancel against identical terms in the O(1.2) integrals. A modified formalism in which the necessary cancellations are built in explicitly is suggested.

I. INTRODUCTION

In this paper we want to discuss certain problems which are connected with the occurrence of infinite "helicity" sums in a generalized complex angular momentum "partial-wave" analysis of multi-particle amplitudes.

That there may be convergence problems connected with infinite helicity sums has been noted already in 1964 in a peper by Omnès and Alessandrini [1], which, among other things, deals with the extension of the Froissart-Gribov formalism to three-particle amplitudes. These questions were taken up again in a more recent paper by Dash [2] in which is was suggested that (divergent) helicity sums should be regularised by introducing convergence parameters, which, hopefully, could be made to disappear in the end of some dynamical calculation scheme involving complex J unitarity equations.

Here we want to show within the context of a generalised O(1.2) expansion formalism of the kind developed by Klink and the present author [3] (this paper will be referrend to as I in what follows), that the infinite helicity sums need not give rise to serious convergence problems. Furthermore we clarify the role of the discrete terms (so-called non-sense terms) in the generalized O(1.2) expansion formalism, and indicate how these terms can be included

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explicitly in the integral representation of the amplitude which group-theoretically corresponds to the principal series of representations of O(1.2). This last step is made possible by slight modification of the formalism of I, and will be given in Sec. IV below.

The rest of this paper is organised as follows. In Sec. II we give a brief survery of the group theoretical variables for a $2 \to n$ multiparticle amplitude and write down the classical O(1.2) expansion formula for an amplitude which is square integrable on the group manifold, i.e. $O(s^{-1/2})$ for large s, where s is the c.m. energy squared in the channel of the incoming particles. Sec. III contains a brief summary of the generalised O(1.2) formalism valid for general power-bounded amplitudes, and a discussion of the convergence of the helicity sums. In the next Section we then indicate how the formalism can be modified so that the sum of nonsense terms in the expansion gets included in the integral representation which corresponds to the principal series of O(1.2) representations. In this way we arrive at a simple and compact expansion of the amplitude, which should be of value in practical applications of the formalism. The final Sec. V constains a brief summary and a discussion of our results.

We would like to stress that the discussion given in Sec. IV is entirely non-rigorous and heuristic, since we prefer, for reasons of clarity, to defer complicated proofs to later publications [4].

II. O(1.2) VARIABLES AND THE CLASSICAL EXPANSION FORMULAE

The choice of group theoretical variables for a multiparticle amplitude given below has been put forward in an excellent paper by Bali, Chew and Pignotti [5] to which we refer for details. This paper again is a development of an earlier work by Toller, and for details of this and other developments in "generalized partial-wave analysis" we refer to Toller's 1969 review paper [6].

Decomposing the amplitude for the N-particle production process

$$A + B \to 1 + 2 + \ldots + N$$

into two clusters with M and $N - M$ outgoing particles, respectively, linked by a fourmomentum transfer Q_{AB}, as indicated on Fig. 1, we can consider the amplitude as a function of the invariant momentum transfer $t_{AB} = Q_{AB}^2$, $3M - 4$ internal cluster variables V_A for cluster A and $3(N - M) - 4$ similar variables V_B for cluster B, *and* three "Euler"-angles α, β, γ which specify a Lorentz-transformation of the momenta of cluster B relative to the cluster A which leaves Q_{AB} unchanged,

$$\langle 1, \ldots, N|T|AB\rangle = F(t_{AB}; \alpha, \beta, \gamma; V_A, V_B). \qquad (2)$$

For a space-like momentum transfer, which we consider here and in what follows, the Lorentz-transformations which do not change Q_{AB} are O(1.2) transformations which can be decomposed into a z-axis rotation given by γ, and x-axis boost specified by the rapidity variable β followed by a z-axis rotation given by α,

$$0 \leq \alpha < 2\pi, \; 0 \leq \beta < \infty, \; 0 \leq \gamma < 2\pi. \qquad (3)$$

Explicit expressions for these "Euler" angles in terms of scalar invariants have been given in the paper by Bali et al. referred to above, and we note here only that the hyperbolic cosine of the rapidity variable β is linearly related to the variable $s = (p_A + p_B)^2$.

Having displayd the dependence of the amplitude on the O(1.2) variables α, β, and γ it is natural to perform an O(1.2) harmonic analysis of the amplitude, i.e. write down its O(1.2) Fourier transform. For amplitudes which are square integrable on the group-manifold, i.e. such that

$$\int_0^{2\pi} d\alpha \int_0^{2\pi} d\gamma \int_1^{\infty} d\,\mathrm{ch}\beta \, |F(\alpha, \beta, \gamma)|^2 < \infty \qquad (4)$$

the expansion reads as follows

$$F(\alpha, \beta, \gamma) = \sum_{\mu,\nu=-\infty}^{\infty} f_{\mu\nu}(\mathrm{ch}\beta)\, e^{-i\mu\alpha - i\nu\gamma}, \qquad (5)$$

with

$$f_{\mu\nu}(\mathrm{ch}\beta) = \frac{1}{2i} \int_{\frac{1}{2}+i0}^{-\frac{1}{2}+i\infty} \frac{dl(2l+1)}{\tan \pi(l+\varepsilon)} a_{\mu\nu}(l)\, d^l_{\mu\nu}(\mathrm{ch}\beta) + \sum_{l=\varepsilon}^{M-1} \left(l + \frac{1}{2}\right) a_{\mu\nu}(l)\, d^l_{\mu\nu}(\mathrm{ch}\beta) \qquad (6)$$

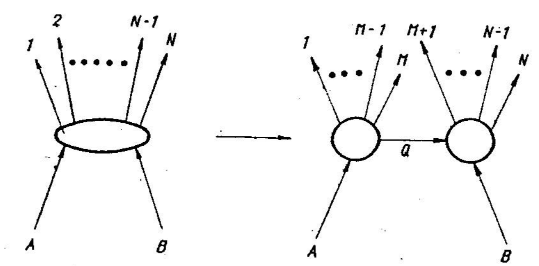

Fig. 1. Cluster decomposition of a general $(2 \to N)$ multiparticle amplitude.

and

$$M = \begin{cases} \min(|\mu|, |\nu|) & \text{for } \mu\nu \geqslant 0 \\ 0 & \text{for } \mu\nu < 0 \end{cases} \tag{7}$$

with the convention that $\sum_{l=\varepsilon}^{M-1} = 0$ if $M = 0$ or $\frac{1}{2}$. The number ε which occurs in the formulae above is either zero or one half, depending on whether μ and ν are integers or half odd integers, the rule being that if the total spin projection (defined according to some appropriate convention) of the particles in the cluster A (and B) is a half-odd integer then so are μ and ν. Otherwise they are integers. The coefficients $f_{\mu\nu}(\mathrm{ch}\beta)$ and $a_{\mu\nu}(l)$, respectively, in Eqs. (5) and (6) are given as follows,

$$f_{\mu\nu}(\mathrm{ch}\beta) = \frac{1}{4\pi^2}\int_0^{2\pi} \mathrm{d}\alpha \int_0^{2\pi} \mathrm{d}\gamma F(\alpha, \beta, \gamma) e^{i\mu\alpha + i\nu\gamma} \tag{8}$$

and

$$a_{\mu\nu}(l) = \int_1^{\infty} \mathrm{d}\,\mathrm{ch}\beta\, f_{\mu\nu}(\mathrm{ch}\beta)\, \mathrm{d}_{\mu\nu}^{-l-1}(\mathrm{ch}\beta). \tag{9}$$

For a discussion of the formulae above and a definition of the O(1.2) d-functions which occur in these formulae we refer to Vilenkin's monograph on group theory and special functions [7]. Here we may recall that the functions $\mathrm{d}_{\mu\nu}^{l}(\mathrm{ch}\beta)$ behave essentially like Legendre functions of the first kind, in fact $\mathrm{d}_{00}^{l}(\mathrm{ch}\beta) = P_l(\mathrm{ch}\beta)$. Before proceeding further, let us note that the occurrence of the infinite sum over μ and ν in Eq. (5) is due to the fact that we are dealing with a multiparticle amplitude; for a $2 \to 2$ amplitude the sum in Eq. (5) would degenerate into one term only, with μ and ν equal to the differences of she crossed channel helicities of particles A and 1 and B and 2, respectively.

Let us then discuss the convergence of the (μ, ν) sums in the representation (5). From the mathematical point of view the convergence of the double Fourier sum in Eq. (5) presents no problem as all, since according to the classical O(1.2) expansion theory we know that the representation given by Eqs. (5) and (6) for square integrable functions holds in the sense of the L^2-norm on O(1.2). This means that we have the Parseval-Plancherel formulae,

$$4\pi^2\int\int\int \mathrm{d}\alpha\, \mathrm{d}\gamma\, \mathrm{d}\beta\, \mathrm{sh}\beta |F(\alpha, \beta, \gamma)|^2 = \sum_{\mu,\nu} \mathrm{d}\beta\, \mathrm{sh}\beta |f_{\mu\nu}(\mathrm{ch}\beta)|^2 \tag{10}$$

and

$$4\pi^2 \int_0^{\infty} \mathrm{d}\beta\, \mathrm{sh}\beta |f_{\mu\nu}(\mathrm{ch}\beta|^2 = \frac{1}{2i}\int_{-\frac{1}{2}+i0}^{-\frac{1}{2}+i\infty} \frac{\mathrm{d}l(2l+1)}{\tan \pi(l+\varepsilon)} |a_{\mu\nu}(l)|^2 + D.T., \tag{11}$$

where

$$D.T. = \sum_{l=\varepsilon}^{M-1} (l + \tfrac{1}{2}) \frac{\Gamma(l+\nu+1)\,\Gamma(\nu - l)}{\Gamma(l+\mu+1)\,\Gamma(\mu - l)} |a_{\mu\nu}(l)|^2. \tag{12}$$

The equations above show in what sense the representation given by Eqs. (5) and (6) holds for L^2-functions. For physical applications it is reasonable to assume much more than just L^2-summability of the amplitude $F(\alpha, \beta, \gamma)$ with respect to α and γ; in fact one may assume that $F(\alpha, \beta, \gamma)$ is of bounded variation and continuous in α and γ, in which case the (μ, ν) sum in Eq. (5) converges uniformly within intervals of continuity of the amplitude $F(\alpha, \beta, \gamma)$. Therefore under such reasonable conditions on the amplitude the convergence of the (μ, ν) sum is not in doubt.

We can equally well interchange the (μ, ν) sums with the integral and discrete sum over l in Eq. (6) with the result

$$\begin{aligned} F(\alpha, \beta, \gamma) = {} & \frac{1}{2i}\int_{-\frac{1}{2}+i0}^{-\frac{1}{2}+i\infty} \frac{\mathrm{d}l(2l+1)}{\tan \pi(l+\varepsilon)} \sum_{\mu,\nu} a_{\mu\nu}(l)\, D_{\mu\nu}^{l}(\alpha, \beta, \gamma) + \\ & + \sum_{l=\varepsilon}^{\infty} (l + \tfrac{1}{2}) \sum_{\mu,\nu=-\infty}^{-l-1} a_{\mu\nu}(l)\, D_{\mu\nu}^{l}(\alpha, \beta, \gamma) + \\ & + \sum_{l=\varepsilon}^{\infty} (l + \tfrac{1}{2} \sum_{\mu,\nu=l+1}^{\infty} a_{\mu\nu}(l)\, D_{\mu\nu}^{l}(\alpha, \beta, \gamma), \end{aligned} \tag{13}$$

where

$$D_{\mu\nu}^{l}(\alpha, \beta, \gamma) = e^{-i\mu\alpha}\, \mathrm{d}_{\mu\nu}^{l}\, (\mathrm{ch}\beta)\, e^{-i\nu\gamma}. \tag{14}$$

The representation (13) expresses more clearly than Eqs. (5) and (6) the group-theoretical content of the O(1.2) expansion; the integral in Eq. (13) corresponds to the principal series of representations of O(1.2), whereas the discrete sums in Eq. (13) correspond to the discrete series (ascending and descending) of representations of O(1.2).

An expansion of the form given by Eq. (13) formed the starting point in the discussion of the (supposedly divergent) helicity sums in the paper by Dash referred to previously. However, it should be observed that Dash replaced the genuine O(1.2) expansion coefficients $a_{\mu\nu}(l)$ defined by Eq. (9) in the integral representation (13) by the corresponding Froissart-Gribov amplitudes (assuming of course dispersion relations in chβ). This can be done formally, but involves a series of manipulations, some of which must be illegal, if the convergence difficulties observed by Dash are real, since we have already seen that the genuine O(1.2) expansion does not involve any convergence difficulties.

The genuine O(1.2) expansion coefficients $a_{\mu\nu}(l)$ are frequently in the literature ascribed properties which one may expect Froissart-Gribov amplitudes to possess, but these two objects in fact have little in common.

The characteristic property of a Froissart-Gribov amplitude is that it should be analytic in its l-plane for sufficiently large values of Re l. However, it can immediately be seen from Eq. (9) which defines the O(1.2) expansion coefficient $a_{\mu\nu}(l)$ that this object cannot have such properties (except possibly if $f_{\mu\nu}(\text{ch}\beta)$ decreases faster than any inverse power as ch$\beta \to \infty$). Consider for simplicity $a_{00}(l)$. The asymptotic behaviour of $\text{d}_{00}(x)$ is

$$|\text{d}^l_{00}(x)| \equiv |P_l(x)| = x^{|\text{Re}\, l + 1/2| - 1/2}\left(1 + 0\frac{1}{x}\right). \tag{15}$$

From Eq. (15) it follows that the coefficient $a_{00}(l)$ can at most be analytic in some finite strip centred around Re $l = -\frac{1}{2}$ if the function $F_{00}(\text{ch}\beta)$ decreases sufficiently fast as ch$\beta \to \infty$. The circumstances indicated above should be enough to cast some doubt on the legitimacy of replacing the O(1.2) expansion coefficient in Eq. (13) by a corresponding Froissart-Gribov amplitude.

Luckily enough this question becomes largely irrelevant in dealing with realistic amplitudes (which are not square integrable in chβ) since for such amplitudes one has to generalize the O(1.2) formalism anyway, and it turns out that this generalization can be made to include the best features of both the classical O(1.2) formalism and the Froissart-Gribov formalism. In the next section we give a brief summary of the generalized O(1.2) expansion formalism developed in I, to which we refer for technical details, analysis of spin complications, and references to previous work on the subject.

III. THE GENERALIZED O(1.2) EXPANSION

Let us now consider a realistic multiparticle amplitude which again may be considered as a function of the variables α, β and γ of the previous section, suppressing the dependence on the momentum transfer and the internal

cluster variables. We denote this amplitude by $F(\alpha, \beta, \gamma)$ and project out its double Fourier components just as in Sec. II,

$$f_{\mu\nu}(\text{ch}\beta) = \frac{1}{4\pi^2}\int_0^{2\pi} \text{d}\alpha \int_0^{2\pi} \text{d}\gamma\, F(\alpha, \beta, \gamma)\, e^{i\mu\alpha + i\nu\gamma}. \tag{16}$$

A conservative estimate (the Froissart bound) of the asymptotic behaviour of $f_{\mu\nu}(\text{ch}\beta)$ is $f_{\mu\nu}(\text{ch}\beta) = O(\beta^2\, \text{ch}\beta)$ as $\beta \to \infty$.

In order to deal with asymptotically growing functions Eq. (9) of the previous section can simply be generalized by replacing the function $d^{-l-1}_{\mu\nu}(\text{ch}\beta)$ by a second kind function $\text{e}^l_{\mu\nu}(\text{ch}\beta)$, which is related to $\text{d}^l_{\mu\nu}(\text{ch}\beta)$ in the same way as the second kind Legendre function $Q_l(x)$ is related to $P_l(x)$; in fact $\text{e}^l_{00}(\text{ch}\beta) = Q_l(\text{ch}\beta)$. We thus define the expansion coefficient $b_{\mu\nu}(l)$

$$b_{\mu\nu}(l) = \int_1^\infty \text{d}\, \text{ch}\beta\, f_{\mu\nu}(\text{ch}\beta)\, \text{e}^l_{\mu\nu}(\text{ch}\beta). \tag{17}$$

It follows from the properties of $\text{e}^l_{\mu\nu}(\text{ch}\beta)$ (as shown in I) that $f_{\mu\nu}(\text{ch}\beta)$ must have kinematic zeroes of the order $\frac{1}{2}(\mu - \nu)$ at ch$\beta = 1$ in order that Eq. (17) be meaningful, or strictly speaking one must require that

where
$$f_{\mu\nu}(x) = \left(\frac{x-1}{x+1}\right)^{\frac{1}{2}|\mu-\nu|} \overline{\Psi}_{\mu\nu}(x), \tag{18}$$

$$\int_1^\infty \text{d}x(x^2 - 1)^{-1/4}\, x^{-p-1/2}\, |\overline{\Psi}_{\mu}(_\nu x)| < \infty. \tag{19}$$

The number $p > -\frac{1}{2}$ in Eq. (19) indicates a power bound on $f_{\mu\nu}(x)$ for $x \to \infty$, i.e. roughly speaking $f_{\mu\nu}(x) < x^p$ as $x \to \infty$. If the function $f_{\mu\nu}(x)$ satisfies the conditions (18), (19), then the function $b_{\mu\nu}(l)$ defined by Eq. (17) is analytic for Re $l > p$ (save for fixed poles due to the $\text{e}^l_{\mu\nu}(x)$ function at $l = \pm\mu - 1$, $\pm\mu - 2, \ldots$) and we have the following representation

$$f_{\mu\nu}(\text{ch}\beta) = \frac{1}{2\pi i}\int_{\text{Re}\, l = p_{\mu\nu}} \text{d}l(2l+1)\, b_{\mu\nu}(l)\, \text{d}^l_{\mu\nu}(\text{ch}\beta), \tag{20}$$

where $p_{\mu\nu} > \max(p, \pm\mu - 1, \pm\nu - 1)$. Eqs. (17) and (20) constitute a unique generalization of Eqs. (6) and (9) for the case of power-bounded functions $f_{\mu\nu}(\text{ch}\beta)$.

The total amplitude is obtained as

$$F(\alpha, \beta, \gamma) = \sum_{\mu,\nu=-\infty}^{\infty} f_{\mu\nu}(\text{ch}\beta)\, e^{-i\mu\alpha - i\nu\gamma}. \tag{21}$$

Since the coefficients $f_{\mu\nu}(\text{ch}\beta)$ are the double Fourier coefficienss of a Fourier series which by hypothesis is convergent, it is clear that one may insert the representation (20) in Eq. (21) without destroying the convergence of the double Fourier series, since she right-hand side of Eq. (20) merely reproduces the given coefficient $f_{\mu\nu}(\text{ch}\beta)$.

Admittedly the representation (20) is nos very useful as such, since the line of integration depends on the summation variables μ and ν. Taking into account the fixed poles of the integrand in Eq. (20) which are due to the functions $e^l_{\mu\nu}(x)$ and $d^l_{\mu\nu}(x')$, and shifting the line of integration to $\text{Re}\, l = p$ in Eq. (20) we obtain

$$f_{\mu\nu}(\text{ch}\beta) = \frac{1}{2\pi i}\int_{\text{Re}\, l=p} dl(2l+1)\, b_{\mu\nu}(l)\, d^l_{\mu\nu}(\text{ch}\beta) + \sum_{l=p_\varepsilon}^{M-1} (l+\tfrac{1}{2})\, a_{\mu\nu}(l)\, d^l_{\mu\nu}(\text{ch}\beta), \tag{22}$$

where

$$a_{\mu\nu}(l) = \int_1^\infty d\, \text{ch}\beta f_{\mu\nu}(\text{ch}\beta)\, d^{-l-1}_{\mu\nu}(\text{ch}\beta) \tag{23}$$

and

$$M = \begin{cases} \min(|\mu|, |\nu|) & \text{if} \quad \mu\nu \geqslant 0 \\ 0 & \text{if} \quad \mu\nu < 0 \end{cases} \tag{24}$$

with

$$p_\varepsilon = [p+\varepsilon] + 1 - \varepsilon \tag{25}$$

and

$$\sum_{l=p_\varepsilon}^{M-1} = 0 \quad \text{if} \quad M - 1 < p_\varepsilon.$$

Eqs. (22—25) indicate clearly the similarity between the generalized O(1.2) representation and the classical one given by Eq. (6).

Now let us substitute the representation (22) into Eq. (21) and formally interchange the orders of summation and integration. We obtain

$$F(\alpha, \beta, \gamma) = \frac{1}{2\pi i}\int_{\text{Re}\, l=p} dl(2l+1) \sum_{\mu,\nu} b_{\mu\nu}(l)\, D^l_{\mu\nu}(\alpha,\beta,\gamma) + \tag{26}$$

$$+ \sum_{l=p_\varepsilon}^{\infty} (l+\tfrac{1}{2}) \sum_{\mu,\nu=-\infty}^{-l-1} a_{\mu\nu}(l)\, D^l_{\mu\nu}(\alpha,\beta,\gamma) + \sum_{l=p_\varepsilon}^{\infty} (l+\tfrac{1}{2}) \sum_{\mu,\nu=l+1}^{\infty} a_{\mu\nu}(l)\, D^l_{\mu\nu}(\alpha,\beta,\gamma).$$

It should be noted that now we cannot rely on any group theoretical arguments to show that the right-hand side in Eq. (26) is in fact convergent. However, if the (μ, ν) sum under the integral sign in Eq. (26) is in fact divergent, as indicated by the discussion given by Dash, then one may conclude that the discrete sums are also divergent but that these divergences cancel when the discrete terms are combined with the integral, since we know that the (μ, ν) sum is convergent if it is performed after the integration and summation over l.

The problems indicated above would of course become much simple if one could combine the discrete terms with the integral in Eq. (22) into a compact expression. That this is in fact possible will be shown in the next section.

IV. THE DISCRETE TERMS AND THEIR REMOVAL

Let us now return to Eq. (22). The discrete contribution to Eq. (22) consists of non-sense terms ($l < \min |\mu|, |\nu|$). For non-sense values of l the $d^l_{\mu\nu}(x)$--functions have the following asymptotic behaviour

$$d^l_{\mu\nu}(x) = O(x^{-l-1}). \tag{27}$$

l = non-sense

Thus for $p \geqslant -\frac{1}{2}$ the integral in Eq. (22) behaves in general like $O((\text{ch}\beta)^p)$ as $\text{ch}\beta \to \infty$, whereas the discrete terms behave like $O((\text{ch}\beta)^{-p_\varepsilon-1})$.

A question which may arise is whether the discrete terms "really are there", i.e. whether they — because of some dynamical reason (such as unitarity) — are in fact zero. That this is not the case can be shown heuristically as follows. Consider a function $f^N_{\mu\nu}(\text{ch}\beta)$ with the following asymptotic behaviour.

$$f^N_{\mu\nu}(\text{ch}\beta) = (\text{ch}\beta)^{p-\delta}\,(1 + O((\text{ch}\beta)^{-N})), \tag{28}$$

$$\delta > 0$$

where N is a large positive number. An application of Eq. (22) would then result in a representation for $f^N_{\mu\nu}(\text{ch}\beta)$ in which discrete terms with asymptotic behaviour $O((\text{ch}\beta)^{-p_\varepsilon-1})$ occur. However, choosing $f^N_{\mu\nu}(\text{ch}\beta)$ positive and remembering that $d^{-l-1}_{\mu\nu}(\text{ch}\beta)$ has no zeroes for $\text{ch}\beta > 1$ one cannot obtain the result $a_{\mu\nu}(l) = 0$ which could explain the paradox above. Therefore one must conclude that the discrete terms are *exactly cancelled* by similar terms which are built in into the integral. It is not difficult to convince oneself that this is also true in more general cases. One may then ask whether it is possible to achieve this cancellation explicitly by an appropriate modification of the integrand in Eq. (22). In order to see how this can be done let us consider the following integral representation for the function $d^l_{\mu\nu}(\mathrm{ch}\beta)$,

$$d^l_{\mu\nu}(\mathrm{ch}\beta) = \int_{-\beta}^{\beta} \frac{d\tau e^{(l-\nu+\frac{1}{2})\tau}}{2(\mathrm{ch}\beta - \mathrm{ch})^{1/2}} z_+^{\mu-\nu}\left(\mathrm{ch}\frac{\beta}{2} + z_+ \mathrm{sh}\frac{\beta}{2}\right)^{2\nu} + \qquad (29)$$

$$+ z_-^{\mu-\nu}\left(\mathrm{ch}\frac{\beta}{2} + z_- \mathrm{sh}\frac{\beta}{2}\right)^{2\nu}$$

$$z_\pm = (e^\tau - \mathrm{ch}\beta \pm ie^{\tau/2}[2(\mathrm{ch}\beta - \mathrm{ch}\tau)]^{1/2})/\mathrm{sh}\beta.$$

From Eq. (29) it follows that $d^l_{\mu\nu}(\mathrm{ch}\beta)$ is essentially symmetric under $l \to -l - 1$ and that its asymptotic expansion contains terms which are $O(e^{\pm(l+1/2)\beta})$ for large values of β. Intuitively it is perhaps clear that this last mentioned fact is responsible for the occurrence of the discrete terms which nevertheless are cancelled by identical terms in the integral. In fact, preliminary calculations [4] indicate that the discrete terms in Eq. (22) can simply be dropped provided one replaces the function $d^l_{\mu\nu}(\mathrm{ch}\beta)$ in the integral in Eq. (22) by essentially the function which is obtained from Eq. (29) by integrating on τ from 0 to β instead of from $-\beta$ to β. The convergence of the (μ, ν) sum in the new representation is thus simple to investigate and in fact it is clear that the (μ, ν) sum certainly converges if it is taken outside the integral in the modified representation. However, more important than the question of interchange-ability of the l-integration and (μ, ν) summation in the modified representation is the fact that this representation already as a built-in feature contains the cancellations which are implicit in Eq. (22) and therefore presumably gives a better starting point for approximations of the kind which are invitable in practical applications than the representation given in Eq. (22).

V. SUMMARY AND DISCUSSION

We have given a brief review of the ordinary and generalized O(1.2) expansion formalism for multiparticle amplitudes and shown that the occurrence of infinite helicity sums does not pose any serious convergence problem, in contrast to the case of multi-Froissart-Gribov expansions.

A further modification of the generalized O(1.2) expansion formalism was suggested, which enables one to include the so-called non-sense terms explicitly in the generalized O(1.2) integral representation. Since the modified expansion formula contains as a built-in feature the necessary cancellations between the discrete terms and the integral representation in the previously formulated expansion theory, it is hoped that the modified expansion formula will be a good starting point for dynamical calculations which inevitably involve approximations.

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