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DISCONTINUITY OF THE REGGE TRAJECTORY IN THE DUAL RESONANCE MODEL[1]

Planar case

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The discontinuity of the second-order correction to the Regge trajectory is calculated in the dual resonance model. The evaluation is based on planar loop diagrams. In contrast to the behaviour of the self-energy whose discontinuity is not positive throughout the correction to the Regge trajectory remains positive in the region considered.

I. INTRODUCTION

Recently, Neveu and Scherk [1] have investigated the planar box diagram in the dual resonance model and derived an expression

$$\alpha_{new} = a + \tfrac{1}{2}\, t + g^2\, \Sigma(t) + \ldots \qquad (1)$$

for the leading output trajectory up to the second order[2]. In order to interpret their result based on a study of the asymptotic behaviour of the box diagram they had to assume the reggeization of the model in higher orders.

This result should allow a test of the applicability of the perturbative approach to unitarity in the following sense: If a finite number of terms in expression (1) are expected to lead to a reasonable description, then they must be small in comparison with the input term $\alpha_t = a + \frac{1}{2}\, t$, especially the output trajectory must be asymptotically linear. Furthermore, already the second order term of the discontinuity should satisfy the positivity condition necessary

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[2] The unit μ in wich masses and fourmomenta are measured is fixed by the condition that the slope α' of the trajectory in these units takes the value $\frac{1}{2}$, that means $\mu = (2\alpha')^{-1/2}$. As usual the external (spinless) particles are taken on the same trajectory, i. e. they have the mass $m_0 = (-a/\alpha')^{1/2} = (-2a)^{1/2}\mu$.

for a resonance interpretation. In the case of the scalar self-energy in the dual resonance model we have convinced ourselves that the discontinuity attains negative values in certain intervals. On the other hand there is some resemblance between the self-energy and the correction to the trajectory such that doubts with regard to the positivity of the latter arise. In the following section we derive an expression for the disc $\Sigma(t)$ and evaluate it explicitely up to $t = 100$ for various intercepts, with the result that the discontinuity is positive for the t values considered. We are aware of the renormalization ambiguities [2] which, however, will not affect the discontinuity (to order g^2).

To complete the investigation in the second order, one should also calculate the contributions from nonplanar diagrams to the trajectory. These will be given in a further paper.

II. DISCONTINUITY OF THE TRAJECTORY

Using a particular kind of renormalization [1], the second order correction to the trajectory is given by

$$\sum(t) = 4\,\pi^2 \int_0^1 \mathrm{d}x\,\mathrm{d}y\,\frac{(xy)^{-a-1}}{\log^2(xy)}\,[(1-x)(1-y)]^{a-1}(1-xy)\,[\mathrm{P}(xy)]^{-4}e^{ah}\times$$

$$\times[e^{\alpha_t H}-1], \tag{1}$$

where

$$\mathrm{P}(xy) = \prod_1^\infty [1-(xy)^n] \tag{2}$$

$$h(x,y) = \frac{\log x \log y}{\log(xy)} + 2\sum_1^\infty \frac{x^n+y^n-2x^ny^n}{n(1-x^ny^n)}$$

$$H(x,y) = -h(x,y) + \log\left\{\sum_1^\infty \frac{n(x^n+y^n)}{1-x^ny^n} - \frac{1}{\log(xy)}\right\}.$$

To calculate disc $\sum(t)$ it is sufficient to integrate in the vicinity of $x = y = 0$. Then the counterterm is no longer necessary and can be dropped. Therefore

$$\mathrm{disc}\sum(t) = 4\pi^2\,\mathrm{disc}\int_0^\varepsilon \mathrm{d}x\mathrm{d}y\,\frac{(xy)^{-a-1}}{\log^2(xy)}\,[(1-x)(1-y)](1-xy)\times$$

$$\times[P(xy)]^{-4}\left\{\sum_1^\infty\frac{n(x^n+y^n)}{1-x^ny^n}-\frac{1}{\log(xy)}\right\}^{\alpha_t}\times$$

$$\times\exp\left\{t\left[\sum_1^\infty\frac{2x^ny^n-x^n-y^n}{n(1-x^ny^n)}-\frac{\log x\log y}{2\log(xy)}\right]\right\}. \tag{3}$$

Note that the integrand in Eq. (3) differs from that of the scalar self-energy by the factor

$$\left\{\sum_1^\infty\frac{n(x^n+y^n)}{1-x^ny^n}-\frac{1}{\log(xy)}\right\}^{\alpha_t}$$

only, which is singular at $x = y = 0$. Nevertheless it is possible to apply the Cutkosky technique to Eq. (3). Using

$$\int \mathrm{d}^4Q\, x^{-\alpha(Q_1^2)-1}\,y^{-\alpha(Q_2^2)-1} = \frac{4\,\pi^2}{\log^2(xy)}(xy)^{-a-1}\exp\left[-\frac{\log x\log y}{2\log(xy)}\,t\right] \tag{4}$$

$$(Q_1+Q_2)^2 = t$$

we write Eq. (3)

$$\mathrm{disc}\,\Sigma(t) = \mathrm{disc}\int_0^\varepsilon \mathrm{d}x\,\mathrm{d}y\int\mathrm{d}^4Q\,x^{-\alpha_1-1}\,y^{-\alpha_2-1}[(1-x)(1-y)]^{a-1}\times$$

$$\times(1-xy)\,[P(xy)]^{-4}\left\{\sum_1^\infty\frac{n(x^n+y^n)}{1-x^ny^n}-\frac{1}{\log(xy)}\right\}^{\alpha_t}\exp\left[t\sum_1^\infty\frac{2x^ny^n-x^n-y^n}{n(1-x^ny^n)}\right]. \tag{5}$$

To get an expression explicitely given by Feynman propagators we expand the integrand in Eq. (5)

$$\mathrm{disc}\sum(t) = \mathrm{disc}\int_0^\varepsilon \mathrm{d}x\mathrm{d}y\int\mathrm{d}^4Q\sum_{k,l,r} C_{klr}(t)\,x^{-\alpha_1-1+k}y^{-\alpha_2-1+l}\,[-\log(xy)]^{-\alpha_t+r}. \tag{6}$$

For any fixed value of t only a finite number of terms does contribute to disc $\sum(t)$. Again we use the independence of disc $\sum(t)$ of the upper integration boundary and calculate

$$\int_0^1 \mathrm{d}x\,\mathrm{d}y\,x^{-\alpha_1-1+k}\,y^{-\alpha_2-1+l}\,[-\log(xy)]^{-\alpha_t+r} =$$

$$= \frac{1}{\Gamma(\alpha_t - r)} \int_0^\infty dz \frac{z^{-1+\alpha_t-r}}{(\alpha_1 - k - z)(\alpha_2 - l - z)} . \quad (7)$$

We substitute Eq. (7) into Eq. (6) and apply Cutkosky rules after undoing the Wick rotation in a finite number of terms

$$\mathrm{disc} \sum (t) = \sum_{k,l,r} C_{klr} \frac{1}{\Gamma(\alpha_t - r)} \mathrm{disc} \int_0^\infty dz\, z^{-1+\alpha_t-r} (-i) \int d^4 Q \times$$

$$\times \frac{1}{(\alpha_1 - k - z)(\alpha_2 - l - z)} = \quad (8)$$

$$= - i(2\pi i)^2 \sum_{k,l,r} C_{klr} \frac{1}{\Gamma(\alpha_t - r)} \int_0^\infty dz\, z^{-1+\alpha_t-r} \times$$

$$\times \int d^4 Q\, \delta_+ (\alpha_1 - k - z)\, \delta_+ (\alpha_2 - l - z) =$$

$$= i\pi^3\, 16 \times 2^{1/2}\, t^{-1/2} \sum_{k,l,r} \frac{C_{klr}}{\Gamma(\alpha_t - r)} \int_0^\infty dz\, z^{-1+\alpha_t-r} \times$$

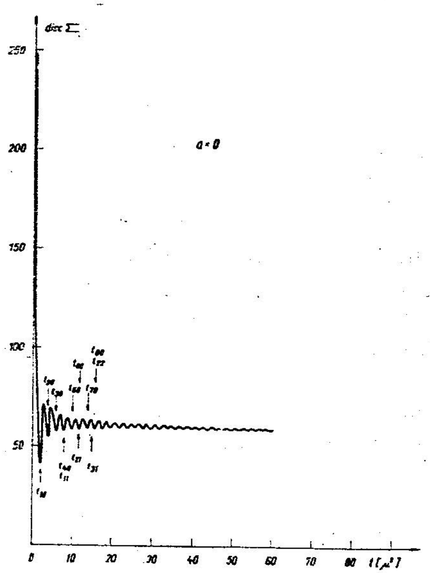

Fig. 1.

$$\times \Theta[t^2 - 4(k + l - 2a)t + 4(k - l)^2] \{[t^2 - 4(k + l - 2a)t + 4(k - l)^2] \times$$

$$\times (8t)^{-1} - z\}^{1/2} .$$

The z integration can be performed analytically and we arrive finally at

$$\mathrm{disc} \sum(t) = i\pi^{7/2}\, 128^{1/2}\, t^{-1/2} \sum_{k,l,r} \frac{C_{klr} \Theta[t^2 - 4(k + l - 2a)t + 4(k - l)^2]}{\Gamma(3/2 + \alpha_t - r)} \times$$

$$\times \left\{ \frac{t^2 - 4(k + l - 2a)t + 4(k - l)^2}{8\, t} \right\}^{1/2+\alpha_t-r} . \quad (9)$$

Eq. (9) is in accordance with the general result on the threshold behaviour of Regge trajectories [3]

$$\mathrm{Im}\, \alpha \sim (t - t_s)^{1/2+\alpha(t_s)} \quad (10)$$

restricted to the second order. (One has to take into account the restriction $0 \leq r < \alpha_t$ implied by the Θ function in Eq. (9).)

Strictly speaking, Eq. (7) would hold for $\alpha_t < 2$ only, but we could introduce a regularization, e. g. by a factor $(1 - x^n y^n)^N$. This would warrant convergence of the integration on both sides of Eq. (7) without altering the discontinuity below $\alpha_t = 8(n - a)$.

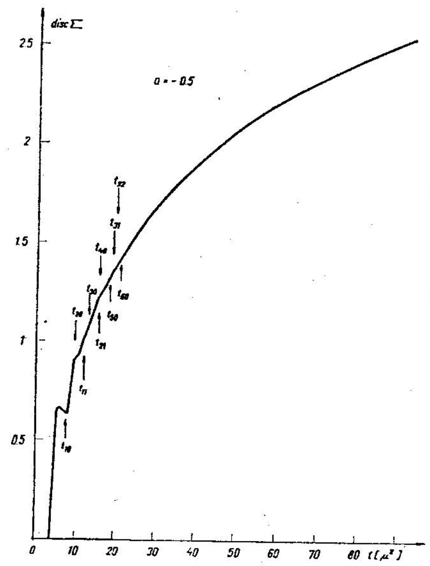

Fig. 2

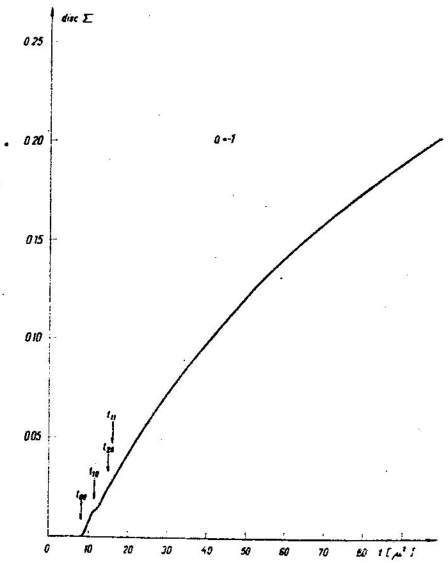

Fig. 3.

Figs 1 to 3 show the function disc $\sum(t)$ for various values of the intercept a. The marks t_{kl} denote the positions of the thresholds $t_{kl} = (M_k + M_l)^2$.

The numerical results suggest an asymptonic behaviour like $\sum(t) \sim (\log t)^{-3a}$. We do not see as present a way to prove this hypothesis.

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