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DISPERSION THEORY OF LOW ENERGY PARTICLE-RESONANCE SCATTERING[1]

DALIBOR KRÚPA,* Bratislava

The N/D dispersion theoretical model for the low energy particle-resonance scattering amplitude with the dynamical input of the one particle exchange mechanism have been solved. It was found that depending on the two particle resonance parameters the particle-resonance amplitude can show a resonance behaviour, i.e. a three-particle-resonance can be generated.

I. INTRODUCTION

In my short talk at this colloquium I should like to speak about some aspects of the effective two-body approximation to the three-body problem and to inform about the main results concerning the importance of the Peierls exchange mechanism in production of higher resonances. The more complete treatment of the problem can be found in [1].

In the scattering processes where the scattering channel consists of three or more particles we cannot use the usual two-body scattering theory and it is necessary to employ rather complicated theories as, for instance, that of Faddeev or others. When, however, two particles in a three-particle state interact strongly via a resonance, a certain reasonable approximation can be made, i.e. the three-particle state may be replaced by a quasi two-body one consisting of an unstable particle or "isobar" corresponding to the resonating pair and the third particle. The three particle → three particle scattering amplitude can then be approximated in a certain energy region by an "isobar" amplitude describing the quasi two-body process of the third particle + resonance → the third particle + resonance.

In the case where two of the particles from the three-body state form a bound state rather than a resonance state, the quasi two-body approach can also be used. It has been applied by Barton and Phillips [2] to the neutron-deuteron scattering. They applied the on-shell N/D method to elastic s-wave

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* Fyzikálny ústav SAV, 899 30 BRATISLAVA, Dúbravská cesta.

scattering where as the input they took the single proton exchange mechanism. This input contains the deuteron binding energy as a parameter. Barton and Phillips found that in the case of spinless "nucleons" the one particle exchange force is strong enough to produce a three-body "n-d" bound state. The fact that the same result has been derived from the three-body Faddeev equations for the spinless "nucleons" interacting through Yukawa or exponential potentials [3] gives us a confidence that by using the simple dispersion relation quasi two-body approach reasonably good quantitative results can be achieved.

We can ask ourselves the following question: Would the one particle exchange mechanism similar to that of the Barton and Phillips, where the two-particle bound states are replaced by the two-particle resonances, generate three-body resonances in analogy to the Barton-Phillips calculation of the three-body bound state? There is a possibility that it will be so, and it has been suggested by R. F. Peierls [4] to explain higher resonances in the π-N scattering. Since Peierls proposed his exchange mechanism (Fig. 1) as one being responsible for higher scattering resonances there has been a great deal of controversy about this question, but most treatments have centred on triagle graphs incorporating the Peierls mechanism [5]. However, such graphs are best regarded as terms arising in an iteration solution of the full N/D equations. For this reason and also because of the similarity of the particle-resonance scattering amplitude with the particle-bound state one and encouraging results of Barton an Phillips we solved the N/D equations for the particle-resonance amplitude taking the Peierls exchange mechanism as the dynamical input. We found that the proposed dynamical mechanism does lead to a resonance enhancement if the parameters of the two-body resonance conspire favourably.

II. N/D EQUATIONS FOR THE PARTICLE-RESONANCE AMPLITUDE

In the following we restrict ourselves to non-relativistic kinematics. We suppose that a resonance a^* of mass m_r is formed of two particles b and c of different masses m and μ, where $m_r = m + \mu + \varepsilon_r$ and $\varepsilon_r = \varepsilon - i\delta$, ε being the resonance excitation energy and δ its half width. The particle-resonance amplitude describes the process

$$a^* + c \to a^* + c,$$

where the stable particle c is identical with the particle of mass μ in a^*. For the sake of simplicity we consider a^*, b, c as being spinless.

In setting up dynamical equations for the particle-resonance amplitude $M(k^2)$ in the nonrelativistic limit we assume that the amplitude has only the right-hand cut along the real positive k^2 axis, where k is the particle-resonance centre of mass momentum and the left-hand cut, which depends on the effective exchange mechanism representing the dynamical input in the N/D calculation. The exchange mechanism which is expected to be a dominant one is just the Peierls mechanism, Fig. 1, since it contains the left-hand singularities nearest to the physical region.

The discontinuity of the partial wave particle-resonance amplitude across the right-hand cut is given by

$$\text{disc. } M^l(k^2) = 2\, ikM^l(k_+^2)\, M^l(k_-^2) \tag{1}$$

and can be derived from the unitarity relation of the three-body amplitude [1, 6].

Specializing now to s-wave scattering $l = 0$, we calculate the s-wave projection of Fig. 1 in order to determine the left-hand cut and the related discontinuity. We find the following expression:

$$B^{l=0}(k^2) = \frac{\Gamma^2}{16\pi(m+2\mu)2k^2} \log \frac{-2m\varepsilon_r + \dfrac{(m+2\mu)^2}{\mu(m+\mu)}k^2}{-2m\varepsilon_r + \dfrac{m^2}{\mu(m+\mu)}k^2}. \tag{2}$$

It is convenient to introduce a new dimensionless variable [2]

$$z = \frac{(m+2\mu)k^2}{2\mu(m+\mu)\varepsilon}, \tag{3}$$

which measures the kinetic energy in units of the excitation energy, and also two dimensionless parameters

$$g = \frac{\delta}{\varepsilon}, \qquad f = \frac{\mu}{m} \tag{4}$$

of the two-body resonance.

We also define

$$M(z) = \left[\frac{2\mu(m+\mu)}{m+2\mu}\right]^{1/2} M(k^2) \tag{5}$$

and similarly for $B(z)$, so that

$$B(z) = \frac{g}{4z}\cdot\frac{(1+f)^2}{f(1+2f)^{1/2}} \log \frac{1(+2f)z - 1 + ig}{z(1+2f)^{-1} - 1 + ig}, \tag{6}$$

where we have dropped the superscript $l = 0$ on M and B.

It is worth-while to notice that the logarithmic left-hand cut is located in the lower half complex z plane and runs from

$$t_1 = \frac{1-ig}{1+2f} \text{ to } t_2 = (1+2f)(1-ig). \tag{7}$$

The slope of this cut is controlled by g and the length of this cut is determined by the value of f. The relative positions of both cuts of the particle-resonance amplitude are shown in Fig. 2.

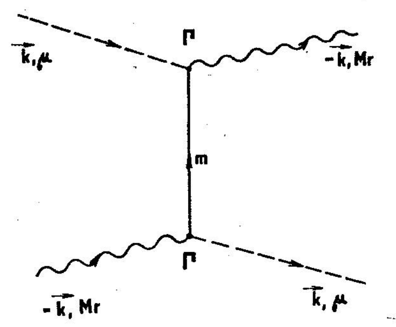

Fig. 1. The Peierls exchange mechanism for the particle-resonance scattering.

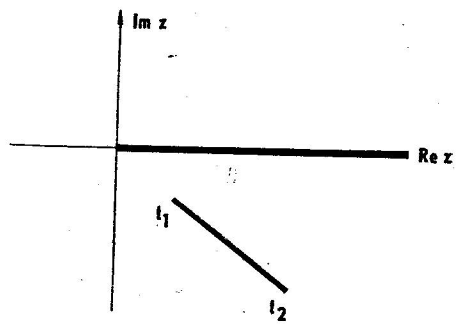

Fig. 2. The positions of the right-hand and left-hand cuts of the particle-resonance amplitude.

From the definition of $M(z)$ and Eq. (1) it follows that the discontinuity of $M(z)$ across the kinematical cut is

$$\text{disc. } M(z) = (z)^{1/2}\, M(z_+)\, M(z_-), \tag{8}$$

which implies that assuming convergence and using the standard N/D method we have to solve the following dispersion relations:

$$D(z) = 1 - \frac{z}{\pi}\int_t^\infty \frac{z\,N(t)}{t(t-z)}\,dt \tag{9}$$

$$N(z) = \frac{1}{\pi}\int_{t_1}^{t_2} \frac{D(t)\,\text{disc. } B(t)}{t-z}\,dt.$$

We solve these coupled integral equations by the method of Pagels [7] generalizing his calculation for the case where the left-hand cut is off the real axis [6]. The advantage of the method is that the solution of Eqs. (9) can be expressed in closed form in terms of $B(z)$.

The functions $N(z)$ and $D(z)$ for z real and positive have been evaluated numerically for various values of the two parameters f and g. In all cases we find that the particle-resonance amplitude has a peak structure. In some cases the peak structure is very sharp (the zero of $D(z)$ is close to the real z axis) in other cases the peak is rather shallow. Numerical investigation of the solutions for different values of f and g shows that for each value of f in (0.1, 1.0) there is a value of g for which the zero of $D(z)$ is close to the real z axis, $z \approx x_0$. The zero of $D(z)$ is a pole of the amplitude. If this pole is close to the real axis we shall interpret it as a resonance in the particle-resonance system. The corresponding kinetic energy of the particle-resonance system at the resonance pole is $E_k = x_0 \varepsilon$, which implies that the corresponding three-body resonance energy is

$$E = (m + 2\mu + \varepsilon) + x_0 \varepsilon.$$

The mutual dependence of f, g and x_0, which was numerically investigated, predicts therefore a specific correlation between the position of a three-particle-resonance and the two-particle subsystem resonance parameters g and f.

When the present model was further generalized to include the two-particle-resonances with spin, the particle-resonance amplitude still showed the resonance enhancement but not so marked as in the spinless case.

III. CONCLUSION

The solutions we have obtained indicate that for certain values of the two-body resonance parameters f and g the Peierls exchange mechanism is strong enough to cause the resonance enhancement in the particle-resonance scattering amplitude, i.e. the resonance formed of three particles.

Allthough our model is quite simple, for instance we do not consider more than one two-body resonance in the three particle states and we made other simplifying assumptions, the model considered above permits us to investigate quantitatively the possible generation of three particle resonances via this mechanism. We may also expect the resonance pole of the particle-resonance amplitude to cause an effect in all channels coupled to the particle-resonance one, therefore the investigation of the Peierls mechanism should be extended in the future to include the complete coupled channels solution.

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