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THE BOLTZMANN EQUATION FOR THE RIGID SPHERE SUSPENSION

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Let us consider a fluid in which solid particles much greater than the fluid molecules are suspended. Let us suppose that the particles and the fluid molecules have the form of rigid spheres. We can use the Enskog modification of the Boltzmann equation for dense gases and liquids [1]. Let $f(\mathbf{x}, \mathbf{v}, t)$, $h(\mathbf{x}, \mathbf{u}, t)$ be the distribution functions for the fluid molecules and the solid particles, respectively. The number of fluid molecules, which move in the time $(t, \mathrm{d}t)$ in the space interval $(\mathbf{x}, \mathrm{d}\mathbf{x})$ with the velocity $(\mathbf{v}, \mathrm{d}\mathbf{v})$, does not change only by collisions with other molecules, but also with the solid particles. (We suppose the suspension to be so diluted that the mutual collisions among the suspended particles can be neglected). The former change is the same as that in pure liquids. To express the latter change, with Θ^+, Θ^- for the number of molecules which enter and leave the considered interval by means of collisions with the suspended particles, respectively, we can wrute:

$$f\left(\mathbf{v} + \frac{\mathrm{F}}{m}\mathrm{d}t, \mathbf{x} + \mathbf{v}\mathrm{d}t, t + \mathrm{d}t\right) - f(\mathbf{v}, \mathbf{x}, t) = \Theta^+ - \Theta^-. \tag{1}$$

Let us consider a part dS of the parcicle surface. The probable number of molecules with the veolocity $(\mathbf{v}, \mathrm{d}\mathbf{v})$, which collide with this area in the time $(t, \mathrm{d}t)$ is

$$f(\mathbf{x}, \mathbf{v}, t)(\boldsymbol{gn})\mathrm{d}S\mathrm{d}\mathbf{v}\mathrm{d}t, \tag{2}$$

where $\boldsymbol{g} = \mathbf{v} - \boldsymbol{u}$ and $\boldsymbol{n}$ is the unit vector of dS.

The whole number of molecules colliding with the particle is obtained by integration over the part of its surface, where $(\boldsymbol{gn}) \leq 0$, that is

$$\int_{(\boldsymbol{gn})\leq 0} f(\mathbf{x}, \mathbf{v}, t)(\boldsymbol{gn})\mathrm{d}S\mathrm{d}\mathbf{v}\mathrm{d}t. \tag{3}$$

By multiplying this expression with the number of particles at the point $(\boldsymbol{x} - \frac{1}{2}(a + \sigma)\boldsymbol{n}, \mathrm{d}\boldsymbol{x})$, ($a$ and σ are the diameters of the particle and the molecule, respectively) and the collision parameter $Y(\boldsymbol{x} - \frac{1}{2}\sigma\boldsymbol{n})$, and by integration over all $\boldsymbol{u}$ we obtain:

$$\Theta^- = \iint_{(\boldsymbol{gn})\leq 0} f(\mathbf{x}, \mathbf{v}, t)h\left(\boldsymbol{x} - \frac{a+\sigma}{2}\boldsymbol{n}, \boldsymbol{u}, t\right) Y\left(\boldsymbol{x} - \frac{\sigma}{2}\boldsymbol{n}\right)(\boldsymbol{gn})\mathrm{d}S\mathrm{d}\boldsymbol{u}. \tag{4}$$

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For the computation of Θ^+ we must express the inverse collisions. By the collisions of the molecules with the part dS of the particle surface their velocities $\mathbf{v}$ change to $\mathbf{v}'$ and $\mathbf{u}$ to $\mathbf{u}'$. For the inverse collision we must consider the collision of the molecules with the velocity $\mathbf{v}'$ on the part $\mathrm{d}S'(\mathbf{n}' = -\mathbf{n})$ of the surface of the particle with the velocity $\mathbf{u}'$. Let $\mathbf{g}' = \mathbf{v}' - \mathbf{u}'$. Then we must integrate over the part of the particle surface, where $(\mathbf{g}'\mathbf{n}') \leq 0$. Thus we obtain:

$$\Theta^+ \mathrm{d}\mathbf{x}\mathrm{d}\mathbf{v}\mathrm{d}t = \iint\limits_{(\mathbf{g}'\mathbf{n}')\leq 0} f(\mathbf{x}, \mathbf{v}', t) h\left(\mathbf{x} - \frac{a+\sigma}{2}\mathbf{n}', \mathbf{u}', t\right) Y\left(\mathbf{x} - \frac{\sigma}{2}\mathbf{n}'\right) \cdot$$
$$\cdot (\mathbf{g}'\mathbf{n}')\mathrm{d}S'\mathrm{d}\mathbf{u}'\mathrm{d}\mathbf{v}'\mathrm{d}t\mathrm{d}\mathbf{x}. \tag{5}$$

From the law of the momentum conservation it follows that $(\mathbf{g}'\mathbf{n}') = (\mathbf{g}\mathbf{n})$ and because we deal with the contact transformation, we have $\mathrm{d}\mathbf{u}'\mathrm{d}\mathbf{v}' = \mathrm{d}\mathbf{u}\mathrm{d}\mathbf{v}$. Then

$$\Theta^+ = \iint\limits_{(\mathbf{g}\mathbf{n})\leq 0} f(\mathbf{x}, \mathbf{v}', t) h\left(\mathbf{x} + \frac{a+\sigma}{2}\mathbf{n}, \mathbf{u}', t\right) Y\left(\mathbf{x} + \frac{\sigma}{2}\mathbf{n}\right)(\mathbf{g}\mathbf{n})\mathrm{d}\mathbf{u}\mathrm{d}S. \tag{6}$$

The expression for the change caused by collisions with the suspended particles is

$$\Theta^+ - \Theta^- = \iint\limits_{(\mathbf{g}\mathbf{n})\leq 0} \left\{ Y\left(\mathbf{x} + \frac{\sigma}{2}\mathbf{n}\right) f(\mathbf{x}, \mathbf{v}', t)\, h\left(\mathbf{x} + \frac{a+\sigma}{2}\mathbf{n}, \mathbf{u}', t\right) - \right.$$
$$\left. - Y\left(\mathbf{x} - \frac{\sigma}{2}\mathbf{n}\right) f(\mathbf{x}, \mathbf{v}, t)\, h\left(\mathbf{x} - \frac{a+\sigma}{2}\mathbf{n}, \mathbf{u}, t\right)\right\} (\mathbf{g}\mathbf{n})\mathrm{d}S\mathrm{d}\mathbf{u}. \tag{7}$$

If we set $a = \sigma$, we shall obtain the right-hand side of the Boltzmann equation for the pure rigid-sphere fluid. [2].

REFERENCES

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