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STATISTICAL ISOSPIN MODEL AND CROSS SECTIONS OF THE REACTIONS $\pi^-p \to N + (2, \ldots, 6)\pi$ WITHIN 1—16 GeV ENERGY RANGE

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Using a statistical isospin model as a basis, cross section ratios were calculated for different final charge states of $\pi^-p \to N + (2, \ldots, 6)\pi$ reactions. The comparison of the model predictions with the experimental data for these reactions was performed at various energies of incident π^--mesons. It was shown that the statistical isospin model predicts ratios between reaction cross sections $\pi^-p \to N + (2, \ldots, 6)\pi$ within the 1—16 GeV energy range with an accuracy of $\approx 10\,\%$.

I. INTRODUCTION

The present paper presents the results of the statistical isospin model [1] application for describing cross section ratios for the reactions $\pi^-p \to N + (2, \ldots, 6)\pi$ within the 1—16 GeV energy range.

The statistical isospin model is based on the assumption that for the reaction with a given number of secondary particles of a certain type (N, π, K, etc.) possible (i. e. those conserving isospin and charge) isospin end states are equally probable. This assumption permits to obtain cross section ratios for different charge states of the given reaction.

In the last years this model known already for more than 15 years, attracted attention again [2—5], first, in connection with a series of new models (for example, a multiperipheral Regge model [6], an additive quark one [7] etc.)[1]; secondly, experimental data were accumulated on the cross sections of various reactions, which made it possible to check the model predictions more correctly; thirdly, it was necessary to estimate the cross sections for reactions with many secondary neutral particles, which are difficult to measure.

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[1] For comparison of some predictions of these models with the experiment one has to know ratios between different charge configurations of the given final states.

Refs. [3, 4] presented the results for cases where the statistical isospin models were employed for the cross sections of $\pi^- p$ interactions with multiparticle production and it was concluded that the model agreed satisfactorily with the experiment. However, this conclusion was drawn from rather limited data[2].

In the present work based on a far greater number of data ($\sim$ 120 experimental points)[3] an attempt was made to determine the quantitative limits for the model application and the accuracy of its predictions. Besides, the question of the accuracy of model predictions with the account of the resonance production in the investigated reactions was studied.

II. METHOD OF CALCULATING CROSS SECTION RATIOS FOR THE REACTIONS $\pi^- p \to N + (2, \ldots, 6)\pi$

Let us denote by n_+, n_-, n_0 the number of π^+, π^- and π°-mesons in the final state, $\sigma(n_+, n_-, n_0)$ is the cross section corresponding to the given charge state, σ_k is the total cross section of the π-meson production irrespective of their charges, $(k = n_+ + n_- + n_0)$, $w(n_+, n_-, n_0) = \sigma(n_+, n_-, n_0)/\sigma_k$ the relative probability of the given charge state.

To compute cross section ratios for the reactions $\pi^- p \to N + (2, \ldots, 6)\pi$, the following formulae were used:

$$w(n_+, n_-, n_0) = p(n_+, n_-, n_0)/\sum_{n_+, n_-, n_0} p(n_+, n_-, n_0)$$

$$p(n_+, n_-, n_0) = \frac{(n_+ + n_- + n_0)!}{n_+!\, n_-!\, n_0!}\left[\tfrac{1}{3}C(\tfrac{3}{2}, -\tfrac{1}{2}|n_+, n_-, n_0) + \tfrac{2}{3}C(\tfrac{1}{2}, -\tfrac{1}{2}|n_+, n_-, n_0)\right].$$

Here $C(T, T_3|n_+, n_-, n_0)$ are isospin coefficients ($T = 1/2$, $3/2$ and $T_3 = 1/2$ is an isospin of the initial state and its third projection) computed by the formulae [8]:

$$C(\tfrac{3}{2}, -\tfrac{1}{2}|n_+, n_-, n_0) = 2^{-(n_+ + n_- + 1)} \int_{-1}^{+1} (1 + x)^{n_+ + n_- + 1} x^{n_0} (3x - 1)\mathrm{d}x$$

$$C(\tfrac{1}{2}, -\tfrac{1}{2}|n_+, n_-, n_0) = 2^{-(n_+ + n_- + 1)} \int_{-1}^{+1} (1 + x)^{n_+ + n_- + 1} x^{n_0}\mathrm{d}x.$$

[2] Previously published papers [3, 4] had a comparison of cross sections for 5–6 reactions, i. e. only with charged particles in the final state at 3–4 energy values of the incident π^--meson.

[3] We used the data from refs. [12–39].

Table 1

Relative probabilities of charge configurations for the reactions $\pi^- p \to N + (2, \ldots, 6)\pi$

Final state	Probability	Final state	Probability
$K = 2$		$K = 5$	
$n\pi^+\pi^-$	0.467	$n2\pi^+2\pi^-\pi^\circ$	0.338
$n\pi^\circ\pi^\circ$	0.155	$n\pi^+\pi^-3\pi^\circ$	0.181
$p\pi^-\pi^\circ$	0.378	$n5\pi^\circ$	0.008
$K = 3$		$p2\pi^+3\pi^-$	0.130
$n\pi^+\pi^-\pi^\circ$	0.462	$p\pi^+2\pi^-2\pi^\circ$	0.301
$n3\pi^\circ$	0.061	$p\pi^-4\pi^\circ$	0.042
$p\pi^+2\pi^-$	0.277	$K = 6$	
$p\pi^-2\pi^\circ$	0.200	$n3\pi^+3\pi^-$	0.096
$K = 4$		$n2\pi^+2\pi^-2\pi^\circ$	0.336
$n2\pi^+2\pi^-$	0.210	$n\pi^+\pi^-3\pi^\circ$	0.095
$n\pi^+\pi^-2\pi^\circ$	0.316	$n6\pi^\circ$	0.003
$n4\pi^\circ$	0.021	$p2\pi^+3\pi^-\pi^\circ$	0.248
$p\pi^+2\pi^-\pi^\circ$	0.358	$p\pi^+2\pi^-3\pi^\circ$	0.204
$p\pi^-3\pi^\circ$	0.095	$p\pi^-5\pi^\circ$	0.017

The computation results of relative probabilities are listed in Table 1.[4]

III. COMPARISON OF MODEL PREDICTIONS WITH EXPERIMENT

Figs. 1 and 2 present reaction cross section ratios obtained from the experiments [12–39]

$$\sigma(\pi^- p \to n\pi^+\pi^-)/\sigma(\pi^- p \to p\pi^-\pi^\circ) \tag{1}$$

$$\sigma(\pi^- p \to n2\pi^\circ)/\sigma(\pi^- p \to p\pi^-\pi^\circ) \tag{2}$$

$$\sigma(\pi^- p \to n\pi^+\pi^-\pi^\circ)/\sigma(\pi^- p \to p\pi^+2\pi^-) \tag{3}$$

$$\sigma(\pi^- p \to n3\pi^\circ)/\sigma(\pi^- p \to p\pi^+2\pi^-) \tag{4}$$

$$\sigma(\pi^- p \to n2\pi^+2\pi^-)/\sigma(\pi^- p \to p\pi^+2\pi^-\pi^\circ) \tag{5}$$

$$\sigma(\pi^- p \to n\pi^+\pi^-2\pi^\circ)/\sigma(\pi^- p \to p\pi^+2\pi^-\pi^\circ) \tag{6}$$

$$\sigma(\pi^- p \to n4\pi^\circ)/\sigma(\pi^- p \to p\pi^+2\pi^-\pi^\circ) \tag{7}$$

$$\sigma(\pi^- p \to n3\pi^+3\pi^-)/\sigma(\pi^- p \to p2\pi^+3\pi^-\pi^\circ) \tag{8}$$

as a function of the incident π^--meson momentum.

[4] Our calculations agree with the values $w(n_+, n_-, n_0)$ given in refs. [3, 9, 10]. Moreover, these were checked by NISCO programme [11] calculations.

The estimate of the accuracy of model predictions was achieved using χ^2--analysis. For each cross section ratio (1—8) and different Δy_T the value χ^2 was calculated:

$$\chi^2 = \sum_i (x_i - y_T)^2/[(\Delta x_i)^2 + (\Delta y_T)^2],$$

where x_i is the experimental cross section ratio; Δx_i is the corresponding error; y_T is the predicted cross section ratio; Δy_T is the prediction accuracy (parameter varied).

χ^2-analysis revealed that the model describes all the considered experimental cross section ratios ($p(\chi^2) > 1$ %) with the accuracy of about 10 % ($\Delta y_T/y_T = 0.1$), except in the case of the cross section ratio (2) $[\sigma(\pi^- p \to n 2\pi^0)/\sigma(\pi^- p \to p\pi^-\pi^0)]$.[5)]

Besides, weighted average cross section ratios were calculated

$$y_e \pm \Delta y_e = (\sum_{i=1}^{n} \omega_i x_i / \sum_{i=1}^{n} \omega_i) \pm [1/(\sum_{i=1}^{n} \omega_i)^{1/2}],$$

where

$$\omega_i = 1/(\Delta x_i)^2,$$

with $\chi_e^2 : \chi_e^2 = \sum_{i=1}^{n} \omega_i (x_i - y_e)^2$. If $\chi_e^2 > n - 1$, the error Δy_e was increased by the factor $k = [\chi_e^2/(n-1)]^{1/2}$. The calculation results for y_e and χ_e^2 for cross section ratios (1—8) are shown in Table 2.

Further, it was considered for what reactions and at which energies of incident π^--mesons there is an agreement with accurate model predictions ($\Delta y_T = 0$). It was found that:

1. The ratios of cross sections (1), (3), (5) and (6) agree with the accurate model predictions ($\Delta y_T = 0$) starting at the ≈ 5 GeV energy of primary π^---mesons and the ratios of cross sections (4), (7) and (8) agree along the overall energy range in question.

However, this observation contains no information concerning the improvement of model predictions at an increasing energy of incident π^--mesons or of the secondary particle number, since both the ratios (4), (7) and (8) and those for other reactions at energy ranges of π^--mesons of more than 5 GeV are known only with a 10—50 % accuracy.

2. Systematic displacement of the cross section ratio (1) from $y_T = 1.23$ to $y_e = 1.49$ and of that (5) from $y_T = 0.59$ to $y_e = 0.47$ (Table 2) is observed. If in the χ^2 analysis y_e values are used instead of y_T, then $p(\chi_e^2) > 1$ %.

We tried to explain this displacement by the absence of the resonance consideration in the used model. Table 3 shows the probabilities of different resonance productions in reactions (1), (5) and (8) obtained as a result of averaging experimental data of refs. [13—39].

As it is seen from the table in reactions (1), (5) and (8) resonances ϱ, f, ω and Δ are formed for the most part.

The resonance production was considered in a similar way that in ref. [2].

The relative probabilities of different charge states of reaction channels with the resonance production:

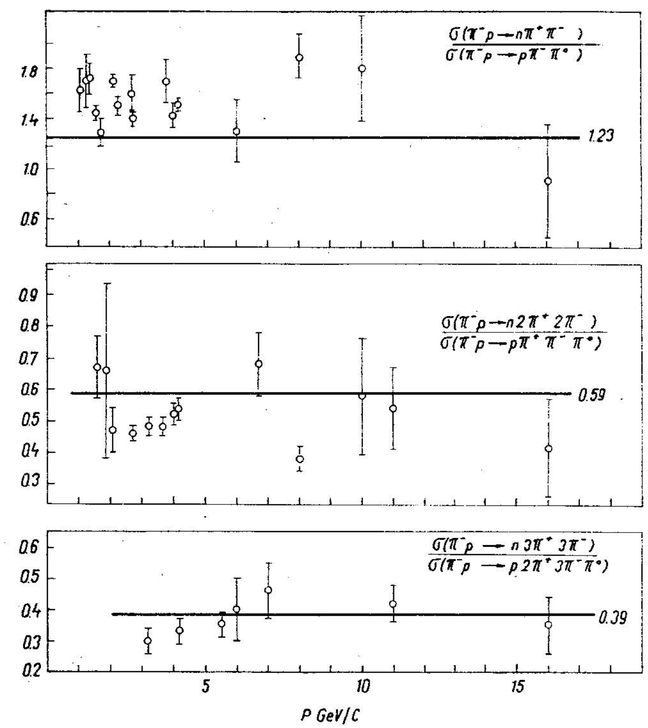

Fig. 1. Comparison of experimental data with the predictions of the statistical isospin model (solid lines).

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5) Note that only a few refs. [12, 14, 17] deal with the measurements of cross sections for $\pi^- p \to n 2\pi^0$ reactions and the measurement results in the coinciding energy region of π^--mesons turned out to be in disagreement. We did not use cross section ratios (2) when estimating model prediction accuracy. Besides we did not use the measurements of cross section ratios of reactions (1), (3) and (6) at the 2.1 GeV energy, which contribution to χ^2 at $\Delta y_T = 0$ is more than 10-fold greater than the average contribution to χ^2 from other points.

$\pi^- p \to N\varrho + (0,\ 1,\ 2,\ 3,\ 4)\pi$, $\pi^- p \to \Delta\pi + (0,\ 1,\ 2,\ 3,\ 4)\pi$ and $\pi^- p \to \Delta\varrho + (0,\ 1,\ 2,\ 3)\pi$
were calculated using a NISCO programme.

These probabilities are listed in Table 4.[6)]

Table 5 presents experimental ratios of cross sections (1), (5) and (8) and model predictions with (y_{Tr}) and without a resonance consideration (y_T).

y_{Tr} are the weighted average values of relative probabilities for different channels of the given reaction listed in Table 4. The experimental probabilities of resonance production served as a weight (Table 3).

It is seen from Table 5 that the consideration of resonances for the reactions (1), (5) and (8) does not improve the model prediction accuracy.

Since the experimental probabilities of resonance production used in the model have been determined mainly with a 20—30 % accuracy, the model predictions are obtained with great errors.

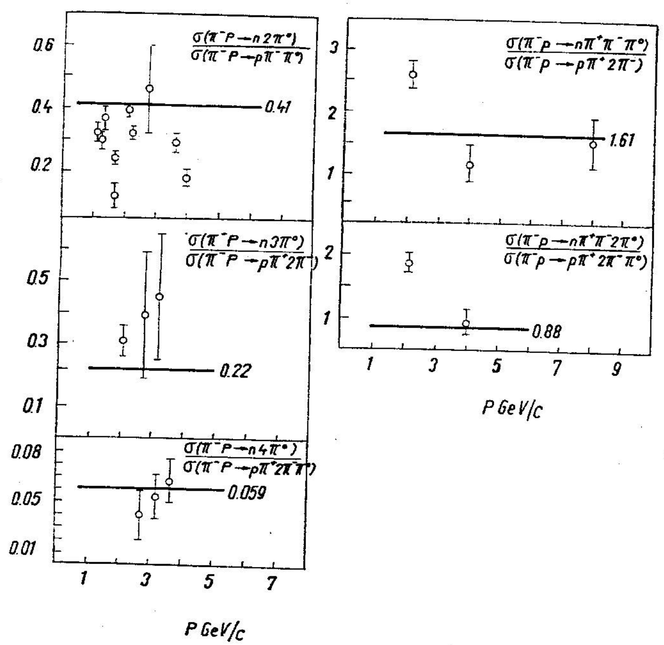

Fig. 2. Comparison of experimental data with the predictions of the statistical isospin model (solid lines).

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6) Including resonances with zero isospin (η, ω, f) does not alter relative probabilities presented in Table 1.

Table 2

Values y_T and y_e, the number of experimental points, χ_e^2 and $P(\chi_e^2)$ for the cross-section ratios under investigation

Cross-section ratio	Number of exper. points	y_T	y_e	χ_e^2	$P(\chi_e^2)$ %
1.	15	1.23	1.49 ± 0.03	23.4	9
2.	9	0.41	0.33 ± 0.02	61.0	1
3.	3	1.67	2.00 ± 0.20	15.3	1
4.	3	0.22	0.32 ± 0.04	0.6	90
5.	14	0.59	0.47 ± 0.01	19.2	15
6.	2	0.88	1.4 ± 0.4	17.4	1
7.	4	0.06	0.06 ± 0.01	4.9	30
8.	7	0.39	0.34 ± 0.03	5.1	60

Table 3

Experimental probabilities (%) for resonance production in reactions (1), (5) and (8)

a) boson resonances:

Final state	non-resonant channel	η	ϱ^0	ϱ^-	ω	f
$n\pi^+\pi^-$	40 ± 10		45 ± 9			10 ± 3
$p\pi^-\pi^0$	55 ± 10			35 ± 10		
$n2\pi^+2\pi^-$	20 ± 10		25 ± 5			
$p\pi^+2\pi^-\pi^0$	30 ± 5	2 ± 1	20 ± 3	17 ± 3	12 ± 3	
$n3\pi^+3\pi^-$	1 ± 15		46 ± 7			14 ± 5
$p2\pi^+3\pi^-\pi^0$	12 ± 10	8 ± 2	24 ± 3	15 ± 5	30 ± 10	

b) Baryon resonances and pair resonance production:

Final state	$\Delta^{++}(1236)$	$\Delta^+(1236)$	$\Delta^-(1236)$	$\varrho^0\Delta^-$	$\varrho^0\Delta^+$	$\omega\Delta^{++}$
$n\pi^+\pi^-$		2.5 ± 1.0	2.5 ± 1.0			
$p\pi^-\pi^0$		10 ± 3				
$n2\pi^+2\pi^-$		8 ± 3	40 ± 10	4 ± 1	3 ± 1	
$p\pi^+2\pi^-\pi^0$	20 ± 5					
$n3\pi^+3\pi^-$		11 ± 2	28 ± 13			
$p2\pi^+3\pi^-\pi^0$						11 ± 3

In conclusion, it can be stated that the statistical isospin model — when resonances are not considered — predicts cross sestion ratios for the reaction $\pi^- p \to N + (2, \ldots, 6)\pi$ within 1—16 GeV energy range with the accuracy of about 10 %.

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Table 4
Calculated relative probabilities for reactions (1), (5) and (8) with the consideration of various resonance production

a) boson resonances:

Final state	Non-resonant channel	η	ϱ^0	ϱ^-	ω	f
$n\pi^+\pi^-$	0.467		0.444			1
$p\pi^-\pi^0$	0.378			0.556		
$n2\pi^+2\pi^-$	0.21		0.153			
$p\pi^+2\pi^-\pi^0$	0.358	0.556	0.133	0.187	0.56	0.378
$n3\pi^+3\pi^-$	0.096		0.068			0.21
$p2\pi^+3\pi^-\pi^0$	0.248	0.277	0.120	0.078		

b) Baryon resonances and pair resonance production:

Final state	Δ^{++}	Δ^+	Δ^-	$\varrho^0\Delta^-$	$\varrho^0\Delta^+$	$\omega\Delta^{++}$
$n\pi^+\pi^-$		0.096	0.467			
$p\pi^-\pi^0$		0.193				
$n2\pi^+2\pi^-$		0.043	0.188	0.144	0.036	
$p\pi^+2\pi^-\pi^0$	0.158					
$n3\pi^+3\pi^-$		0.20	0.080			
$p2\pi^+3\pi^-\pi^0$						0.178

Table 5
Comparison of model predictions with resonances taken into account (y_{Tr}) and without resonances (y_T) to experimental ratios (y_e) for reactions (1), (5) and (8)

Ratio number	y_e	y_T	y_{Tr}
1.	1.49 ± 0.03	1.23	1.18 ± 0.25
5.	0.47 ± 0.01	0.59	0.59 ± 0.12
8.	0.34 ± 0.02	0.39	0.38 ± 0.10

REFERENCES

[1] Fermi E., Phys. Rev. *92* (1953), 452.
[2] Zalewski K., Danysz J., Nucl. Phys. *B 2* (1967), 249.
[3] Bartke J., Nucl. Phys. *82* (1966), 673.
Bartke J., Czyzewski O., Nucl. Phys. *B 5* (1968), 582.
[4] Grishin V. G., et al., Preprint JINR, P 1-5431, Dubna 1970.
[5] Elbert J. W. et al., Nucl. Phys. *B 19* (1970), 85.
[6] Finkelstein J., Kajántie K., Nuovo Cim. *A 56* (1968), 659.
[7] Satz H., Phys. Rev. Lett. *19* (1967), 1453.
[8] Zalewski K., Acta Phys. Polon. *28* (1965), 207.
[9] Беленький С. З. и др., Усп. физ. Наук *62* (1957), 1.
[10] Milburn R. H., Rev. Mod. Phys. *27* (1955), 1.

[11] Kittel W., Lauscher P., *Normalized Isospin Coefficients* (NISCO), CERN Program Library, W 700.
[12] Bulos F. et al., Phys. Rev. *187* (1969), 1827. (0.654-1.247 GeV/c).
[13] Bacon T. C. et al., Phys. Rev. *157* (1967), 1263.(1.08—1.7 GeV/c).
[14] Crouch H. R. et al., Phys. Rev. Lett. *21* (1968), 845. (1.4—4.0 GeV/c).
[15] Alitti J. et al., Nuovo Cimento *29* (1963), 519. (1.59 GeV/c).
[16] Allen D. D. et al., Nuovo Cimento *58* (1968), 701. (1.7 GeV/c).
[17] Caroll A. S. et al., Phys. Rev. *177* (1969), 2047. (1.71—2.46 GeV/c).
[18] Christon R. et al., Phys. Rev. *143* (1966), 1105. (1.89 GeV/c).
[19] Carmony D. D. et al., Proceedings 1962 ICHEP at CERN. (2.03 GeV/c).
[20] Boyd J. H. et al., Phys. Rev. *166* (1968), 1458.
West E. et al., Phys. Rev. *149* (1966), 1089.
Satterblom P. H. et al., Phys. Rev. *134* (1964), B 207. (2.1 GeV/c).
[21] Reynolds B. G. et al., Phys. Rev. *184* (1969), 1424. (2. 26 GeV/c).
[22] Miller D. H. et al., Phys. Rev. *153* (1967), 1423.
Klein P. R. et al., Phys. Rev. *150* (1966), 1123. (2.7 GeV/c).
[23] Alitti J. et al., Nuovo Cimento *35* (1965), 1, 713.
Alitti J. et al., Phys. Lett. *21* (1966), 354. (2.75 GeV/c).
[24] Allen R. H., Lind V. G., Bull. Amer. Phys. Soc. *13* (1968), 589. (3.2 GeV/c).
[25] Chung S. U. et al., Phys. Rev. *165* (1968), 1491. (3.2—4.2 GeV/c).
[26] Lee Y. Y. et al., Phys. Rev. *159* (1967), 1156. (3.63 GeV/c).
[27] Гришин В. Г. и др., Ядерная физика *10* (1967), 1204.
Гришин В. Г. и др., Ядерная физика *11* (1970), 1242.
Bondar L. et al., Nuovo Cimento *31* (1964), 485, 729. (4 GeV/c).
[28] Eisner R. P. et al., Phys. Rev. *164* (1967), 1699. (4.16 GeV/c).
[29] Luke R. A., Lund V., Bull. Amer. Phys. Soc. *13* (1968), 589. (4.2 GeV/c).
[30] Bomse F. et al., Phys. Rev. *162* (1967), 1328. (5.5 GeV/c).
[31] Suen K. F. et al., Phys. Rev. *D 1* (1970), 54.
Crennel D. J. et al., Preprint B NL 12726. (6 GeV/c).
[32] Miyashita S. et al., Phys Rev. *D 1* (1970), 771. (6.7 GeV/c).
[33] Cason N. M. et al., Phys. Rev. *148* (1966), 1282.
Milgram M. S. et al., Nucl. Phys. *B 18* (1970), 1. (7 GeV/c).
[34] Lamsa J. W. et al., Phys. Rev. *166* (1968), 1395.
Cason N. M. et al., Phys. Rev. *D1* (1970), 851.
Allard J. F. et al., Nuovo Cimento *50 A* (1967), 106.
Barish S. J. et al., Phys. Rev. *184* (1969), 1375.
Poirier J. A. et al., Phys. Rev. *163* (1967), 1462. (8 GeV/c).
[35] Bellini G. et al., Nuovo Cimento *53 A* (1968), 788. (8.16 GeV/c).
[36] Fleury P. at al., Proceedings 1962 ICHEP at CERN, 597.
Biswas N. N. et al., Phys. Rev. *134* (1964), B 901. (10 GeV/c).
[37] Caso C. et al., Nuovo Cimento *62 A* (1969), 755.
Conte F. et al., Nuovo Cimento *51 A* (1967), 175.
Caso C. et al., Nuovo Cimento *47 A* (1967), 675.
Daronian P. et al., Preprint CERN 68-7. (11 GeV/c).
[38] Joffredo M. L. et al., Phys. Rev. Lett. *21* (1968), 1212.
Brandenburg C. W. et al., Nucl. Phys. *B 16* (1970), 287. (13.20 GeV/c).
[39] Junkmann B. et al., Nucl. Phys. *B 8* (1968), 471.
Honecker R. et al., Nucl. Phys. *B 13* (1969), 586. (16 GeV/c).

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