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CRITICAL PARAMETERS OF THERMAL DIFFUSIVITY AND THERMAL CONDUCTIVITY OF $NaNO_3$ CRYSTALS

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In the present paper an analysis of thermal diffusivity and thermal conductivity measurements is done. It is shown that thermal diffusivity and thermal conductivity characteristics near the critical point can be expressed by $A|T_i - T_c|^n$. From the values of the parameters A, n (computed by the least square method) it follows that the thermal diffusivity

$$k = 1.06|T_i - T_c|^{0.15} \qquad \text{for } T_i < T_c$$
$$k = 0.96(T_i - T_c)^{0.15} \qquad \text{for } T_i > T_c$$

and the thermal conductivity

$$\lambda = 2.79|T_i - T_c|^{-0.04} \qquad \text{for } T_i < T_c$$
$$\lambda = 3.20(T_i - T_c)^{-0.35} \qquad \text{for } T_i > T_c$$

I. INTRODUCTION

From the collected papers of Fischer [1] and Smith [2] dealing with phase transitions it follows that a great deal of attention is being payed to anomaly quantities measurements near the critical point. The aim of experimental measurements is to prove the theoretical models explaining phase transitions. Ising's model of the ferromagnetic state is the best theoretical model. Ising's model was solved exactly for a two-dimensional lattice and there exist numerical solutions for some types of three-dimensional lattices [3, 4, 5]. Anomaly quantities (specific heat, density, compressibility) near the critical point behave according to Ising's model as

$$X_i = A|T_i - T_c|^n, \tag{1}$$

X_i — the value of the anomaly quantity at the temperature T_i; T_c — critical temperature; A, n — critical parameters. To prove the validity of (1) for a certain material, it is necessary to determine parameters, the A, n near the critical point. The nearer we are to the critical point the more difficult it

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becomes to obtain a sufficient number of specific heat, thermal diffusivity and thermal conductivity values necessary for the computation of the parameters A, n.

In a previous paper [6] specific heat critical parameters of $NaNO_3$ crystals near the critical point were determined.

The aim here is to obtain critical parameters A, n for thermal conductivity and thermal diffusivity values. It will be shown that both quantities behave as (1) near the critical point. The pulse method [10] enables to obtain a sufficient number of thermal conductivity and thermal diffusivity values very near the critical point, which makes it very suitable for our measurements.

II. EVALUATIONS OF CRITICAL EXPONENTS

Let us suppose that for thermal conductivity and thermal diffusivity values we can write the relation

$$X_i = A|T_i - T_c|^n . \qquad (1')$$

It is possible to rewrite relation (1) in the form:

$$\log X_i = \log A + n \log D_i , \qquad (2)$$

where $D_i = |T_i - T_c|$.

The parameters A, n were calculated from relation (2) by the least square method for 5 sets of thermal diffusivity measurements of $NaNO_3$ crystals [7] in the temperature region 200—275 °C ($T_i < T_c$). They were characterized by the mean value

$$n = 0.14 \pm 0.014 \qquad A = 1.20 \pm 0.094$$

Thermal conductivity values were obtained from experimental values of specific heat and thermal diffusivity, supposing constant density. In the temperature region 200—275 °C they were characterized by the mean value:

$$n = -0.05 \pm 0.025 \qquad A = 2.78 \pm 0.047 .$$

For $T_i > T_c$ there was only one set of measurements with a sufficient number of values in the critical region. The following values of the parameters A, n were obtained for the thermal conductivity:

$$n = -0.35 \qquad A = 3.20 ,$$

for the thermal diffusivity

$$n = 0.15 \qquad A = 0.96 .$$

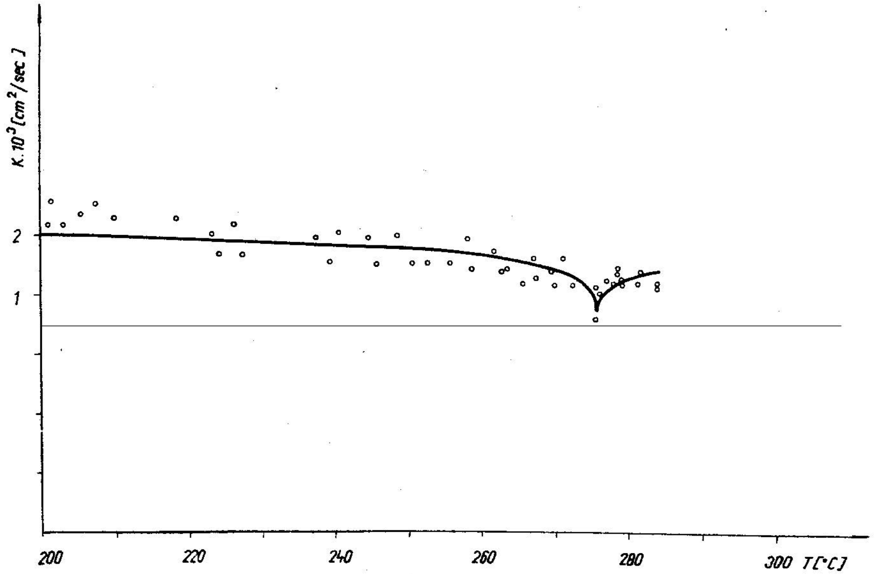

Fig. 1. Theoretical values of thermal diffusivity $k = 1.06|T_i - T_c|^{0.15}$ for $T_i < T_c$ and $k = 0.96(T_i - T_c)^{0.15}$ for $T_i > T_c$ (full line) comparing with experimental values (dotted line).

Table 1

T [°C]	$10^3 \times k_e$ [cm²s⁻¹]	$10^3 \times k_t$ [cm²s⁻¹]	T [°C]	$10^3 \times k_e$ [cm²s⁻¹]	$10^3 \times k_t$ [cm²s⁻¹]
200.9	2.16	2.01	262.3	1.33	1.55
201.4	2.55	2.00	263.0	1.38	1.54
203.2	2.16	2.00	264.9	1.15	1.50
205.4	2.34	1.99	266.6	1.59	1.46
207.4	2.46	1.98	267.1	1.22	1.44
209.7	2.27	1.97	269.1	1.35	1.38
218.3	2.23	1.93	269.4	1.11	1.37
223.1	1.97	1.90	270.6	1.55	1.32
224.0	1.65	1.90	271.9	1.10	1.26
225.9	2.12	1.89	274.9	1.06	0.76
227.2	1.63	1.88	275.1	0.56	0.68
237.2	1.90	1.82	275.5	0.94	0.87
239.1	1.50	1.80	276.5	1.18	1.02
240.3	1.98	1.79	277.5	1.12	1.11
244.2	1.90	1.76	277.9	1.32	1.13
245.4	1.46	1.75	278.0	1.37	1.13
248.3	1.92	1.73	278.2	1.18	1.15
250.5	1.47	1.70	278.3	1.16	1.15
252.4	1.47	1.68	278.8	1.16	1.18
255.4	1.46	1.65	280.8	1.14	1.25
257.8	1.76	1.62	281.2	1.33	1.27
258.3	1.39	1.61	283.3	1.07	1.32
261.3	1.67	1.56	283.5	1.14	1.32

In Fig. 1 theoretical values of $k = 1.06|T_t - T_c|^{0.15}$ for $T_t < T_c$ and $k = 0.96(T_t - T_c)^{0.15}$ for $T_t > T_c$ (full line) and experimental values (dotted line) in both temperature regions are shown. Necessary values for Fig. 1 are in Table 1. Experimental and theoretical values of $\lambda = 2.79|T_t - T_c|^{-0.04}$ for $T_t < T_c$ and $\lambda = 3.20(T_t - T_c)^{-0.35}$ for $T_t > T_c$ are shown in Table 2.

Table 2

T [°C]	$10^3 \times \lambda_t \frac{\text{cal}}{\text{cm.s.deg}}$	$10^3 \times \lambda_c \frac{\text{cal}}{\text{cm.s.deg}}$	T [°C]	$10^3 \times \lambda_t \frac{\text{cal}}{\text{cm.s.deg}}$	$10^3 \times \lambda_c \frac{\text{cal}}{\text{cm.s.deg}}$
206.9	2.34	2.63	271.9	2.67	2.47
222.9	2.37	2.45	274.9	3.08	3.10
223.9	2.37	2.51	277.9	2.21	2.76
249.9	2.44	2.29	281.0	1.72	1.35
253.9	2.46	2.47	284.6	1.46	1.17
256.9	2.48	2.35	289.4	1.27	1.40
258.9	2.49	2.68	291.4	1.22	1.42
263.9	2.53	2.10	283.9	1.16	1.48
264.9	2.54	2.45	298.4	1.07	1.40
268.9	2.59	2.66	303.4	1.11	0.72
270.9	2.64	3.40			

To proove the validity of our results, the Wilcoxon nonparametric test [8] of agreement of theoretical and experimental values was done. Results of the Wilcoxon test confirm the behaviour of the thermal conductivity and the thermal diffusivity near the critical point as in (1).

For critical indices of the specific heat α, the thermal diffusivity n_k, the thermal conductivity n_λ and the density β there follows from our measurements near the critical point the relation:

$$n_\lambda = \alpha + n_k + \beta .$$

III. CONCLUSION

It was found that the thermal conductivity and the thermal diffusivity of $NaNO_3$ crystals near the critical point can be expressed by the relation (1).

The critical exponent of thermal conductivity was investigated also by Kellner [9] but near the critical point of argon. As there are few experimental values, the critical exponent n is in the interval

$$-1/3 < n < 1/3 .$$

Our critical exponent of thermal conductivity is also in the same interval.

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