Fyz. čas. SAV 22 (1972), No. 1–2

DIFFRACTION SCATTERING OF DEUTERONS WITH 2.45 GeV/c MOMENTUM IN NUCLEAR PHOTOEMULSION

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The basic experimental data about the primary beam and about the cases of elastic scattering have been obtained by means of measuring multiple scattering and ionization. Having made a correction for the Coulomb scattering we have obtained the cross-section of the diffraction scattering of deuterons on the nuclei of the photographic emulsion $\sigma_{el} = 792 \, ^{+103}_{-78}$ m barn. Reducing this value for 1 nucleon we get the value $\sigma_{el}/nucl = 64 \pm 7$ m barn. The values of the differential cross-sections obtained experimentally have been compared with the square amplitude of diffraction scattering $|f_d|^2$. A good agreement of the theoretical and experimental data can be seen in the region of very small angles.

I. INTRODUCTION

It is known that in accordance with the „black" target model for kinetic energy $T > 600$ MeV the total effective cross-section of the proton — nucleus interaction depends only slightly on the momentum of an impinging particle. We cannot make the same conclusion for the deuteron — nucleus interaction. The cross-section of the deuteron — nucleus interaction Al^{27} for the energy $T = 710$ MeV σ_t (Al) $= 1.63 \pm 0.09$ barn [1] differs considerably from the total cross-sections of deuterons with nuclei Al^{27} and C^{12} for the energy $T = 650$ MeV mentioned in paper [2], where σ_t (Al) $= 345 \pm 5$ m barn and σ_t (C) $= 185 \pm 5$ m barn. The difference between the total cross-sections is caused mainly by a different value of the cross-section of elastic scattering (in paper [1] σ_{el} (Al) $= 620 \pm 60$ m barn and in papers [2] and [3] σ_{el} (Al) $= 287 \pm 19$ m barn, σ_{el} (C) $= 131 \pm 15$ m barn).

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Trying to contribute to the solution of this problem in this paper we have been investigating the cross-section of elastic scattering of deuterons with $T = 1.2$ GeV on atomic nuclei of a photographic emulsion.

II. EXPERIMENTAL PROCEDURE

By means of an electrostatic separator and of magnetic lenses a deuteron beam with the momentum 2.43 GeV/c was separated from the particles emitted from the inner target of the synchrophasotron of the High Energies Laboratory JINR in Dubna. The beam had the maximum density of 2×10^4 tracks/cm^2 and the half-thickness of 7 mm. This beam was used to expose an emulsion of $20 \times 10 \times 5$ cm^3 built-up of layers of a nuclear photographic emulsion of the NIKFI BR-2 type of the thickness of 450 μ. The chamber edge was exposed to a beam of π^- mesons with the momentum of 4 GeV/c [4] in order to make the calibration of ionization possible. The photographic emulsions were exposed in the photographic laboratories of JINR. All measurements were performed with the ZEISS KSM 1 microscope.

The scattering of the primary particle was detected by measuring track coordinates. The track coordinates y (projection onto the emulsion plane) and the track coordinates z (perpendicular to the emulsion plane) were measured with the cell length $t = 500$ μ. The measured tracks satisfied the following requirements:

a. the scattering of the primary tracks was less than $\pm 2°$ in the emulsion plane;

b. the angle between the track and the track projection onto the emulsion plane was $\Delta \in (-1°20', 0°40')$. The determination of this interval was based on the distribution Δ from 294 primary tracks (Fig. 1);

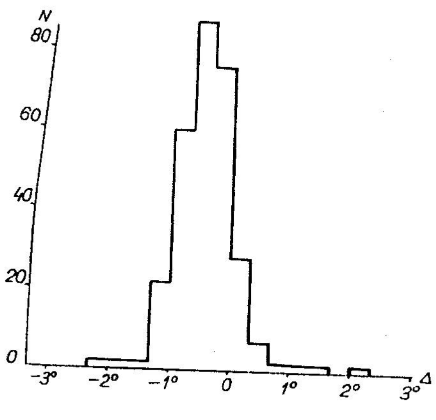

Fig. 1. Statistic distribution of the dip angle Δ of 294 primary tracks. The dip angle $\bar{\Delta} = -0°25'$.

c. the distance between all measured points of the tracks and the emulsion surface was more than 10 μ;

d. the projection length of the track in the emulsion was more than 6 mm.

From the measured values of the coordinates y_i, z_i their first differences

$$P^y_{i+1} = y_i - y_{i+1} \qquad P^z_{i+1} = z_i - z_{i+1} \tag{1}$$

and their second differences

$$D^y_{i+2} = P^y_{i+1} - P^y_{i+2} = y_i - 2y_{i+1} + y_{i+2} \tag{2}$$

$$D^z_{i+2} = P^z_{i+1} - P^z_{i+2} = z_i - 2z_{i+1} + z_{i+2}$$

were computed.

As a matter of fact the mean angle of the multiple Coulomb scattering $\langle\alpha\rangle$, which is proportional to the mean value of the absolute value of the second differences $\langle|D^y_i|\rangle$ corrected for spurious scattering and errors of measuring D_n, was measured by measuring the coordinates y_i with the cell length t

$$\langle\alpha\rangle = \frac{57.3 D_c}{t} = \frac{57.3}{t}\sqrt{\langle|D^y_i|\rangle^2 - D_n^2}\,. \tag{3}$$

The momentum of the particle can be determined by the formula

$$p\beta\ [\text{MeV/c}] = \frac{K}{\langle\alpha\rangle}\left(\frac{t}{100}\right)^{1/2}, \tag{4}$$

where K is the scattering constant. For deuterons with the momentum of 2.43 GeV/c ($\beta = 0.79$) we chose $K = 26.6$ [5]. The mean value from the absolute values of the second differences $\langle|D^y_i|\rangle$ was computed for each track. Their distribution is shown in Fig. 2. The momentum of the particles whose track value $\langle|D^y_i|\rangle$ was higher than 0.38 μ is $p < 2$ GeV/c. These particles are considered as secondary and, therefore, were not taken into the set of the primary tracks, which were dealt with further on.

The measuring of the ionization of the primary tracks disclosed that apart from deuterons with the mean ionization 35.5 ± 0.2 of grains/100 μ the beam contained impurities with considerably lower mean ionization, approximatly equal to the ionization of π^- mesons with the momentum of 4 GeV/c, i. e. 27.6 ± 0.5 of grains/100 μ [4]. They are particles with the relative velocity $\beta = 1$, while the relative velocity of the primary deuterons $\beta = 0.79$. The presence of particles with a higher value of β and the same momentum in the primary beam influences the distribution $\langle|D^y_i|\rangle$ (Fig. 2), which is extended to higher values of $p\beta$.

It was found in paper [6] that the beam impurities with a lower mean ionization were not uniformly distributed. They increase from the beam centre to its edges.

The measured tracks were grouped according to the distance from the beam centre. The mean value of the track $\langle l_i \rangle$ and the probabilities that the particles are impurities p_i [6] were computed for each interval. If n_i represents the

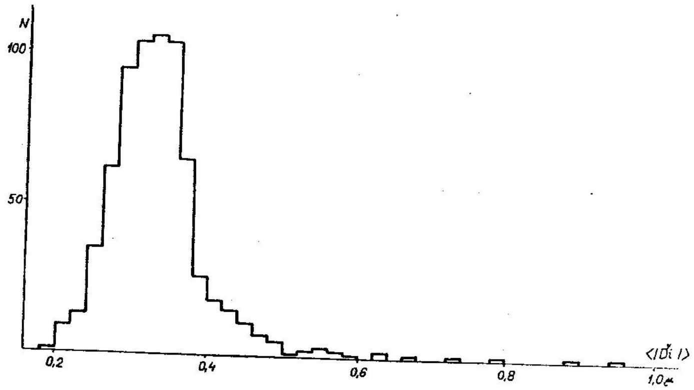

Fig. 2. Statistic distribution of the mean values from the absolute values of the second differences $\langle |D_i^y| \rangle$ on all the tracks. The value $\langle |D^y| \rangle = 0.38\ \mu$ is considered to be the maximum value for the primary tracks.

number of the tracks in an interval i, then the total length of the impurity tracks l' is equal to

$$l' = \sum_i n_i p_i \langle l_i \rangle . \tag{5}$$

The values n_i, p_i and $\langle l_i \rangle$ are tabulated in Table 1.

The multiple scattering was measured on 701 tracks with the total length

Table 1

square [mm²]	n_i	p_i	$\langle l \rangle_i$ [cm]	l_i' [cm]
87 + 86	20	0.38	3.56	27.06
85	66	0.26	3.06	52.50
84	61	0.135	3 12	27.70
83	137	0.072	2.64	26.03
82	143	0.058	3.04	25.20
81	86	0.056	3.08	14.85
80	93	0.033	3.18	9.76

of 2002.9 cm. Having eliminated 95 secondary tracks belonging to the particles with the movement $p < 2$ GeV/c, we obtained the length of the primary tracks $l_p = 1822.8$ cm. Further measuring was performed on the set of these 606 tracks.

According the formula (5) we computed the length belonging to the particles of impurity with a higher relative velocity. Then we subtracted the resulting value $l' = 183.1$ cm from the length of the primary tracks, and so we obtained the length of the primary tracks of deuterons $l_d = 1639.7$ cm.

The scattering of the primary particle into small angles was identified by means of the second differences D_i^y, D_i^z. The quotient of the first difference P_i^y and the cell length t is an angle in the point x_i formed by the axis x given by the motion of the microscope table and by the projection of the track in the emulsion plane — $\varphi_i = P_i^y/t$. The angle between the particle track in the emulsion and its projection onto the emulsion plane, i. e. the so-called dip-angle Δ for the point x_i equals $\Delta_i = (P_i^z/t)\ s$, where s is the shrinkage factor of the emulsion.

If a particle is scattered in the point x_{i+1}, the angle of scattering in the emulsion plane $\varphi = \varphi_i - \varphi_{i+1} = D_{i+1}^y/t$ and the change of the dip-angle equals $\delta = \Delta_i - \Delta_{i+1} = s \times D_{i+1}^z/t$. The values t and s are constant, and therefore the scattering angle of the particle in the point x_i is proportional to the second differences D_i^y, D_i^z.

In the case when the particle is scattered between the points x_i and x_{i+1}, the scattering of the track manifests itself in the form of high values of the second differences D_i^y, D_{i+1}^y and D_i^z, D_{i+1}^z. Both successive values of the second differences have the same sign. In this case the angles of scattering φ and δ are computed from the difference of angles in the points x_i and x_{i+2}

$$\varphi = \varphi_i - \varphi_{i+2} = \frac{P_i^y - P_{i+2}^y}{t} \tag{6}$$

$$\delta = \Delta_i - \Delta_{i+2} = \frac{P_i^z - P_{i+2}^z}{t}\ s . \tag{7}$$

The solid angle of scattering ϑ is given by the formula

$$\cos\vartheta = \cos\varphi \cos\Delta_1 \cos\Delta_2 - \sin\Delta_1 \sin\Delta_2$$

and for small angles

$$\vartheta^2 = \varphi^2 + \delta^2 . \tag{8}$$

For further analysis all values of the second differences $> 2\ \mu$ were chosen from 606 primary tracks with the total length 1822.8 cm. In the case when the scattering occurred in the point x_i, in which the coordinates y_i, z_i were measured, the scattering was identified from the values of $\varphi = 4$ mrad and

$\delta = 8.6$ mrad. It was possible that the scattering occured in the interval between the measured points. In the least favourable case, if the scattering occured in the centre of the interval x_i, x_{i+1}, it was identified from the values of $\varphi \geqslant 8$ mrad and $\delta \geqslant 17.2$ mrad. The difference of the lower limit of the detectability of scattering for the angles of φ and δ is caused by the emulsion shrinkage.

Then we examined the neighbourhood values of the high value of the second difference observed. If there was found a high value of the second difference with the reverse sign, the case was eliminated as a distorsion, also in cases when there were many big deviations with alternating signs. 67 cases caused by distorsion were eliminated out of 193 cases of the high value of the second difference. The scattering angle ϑ was computed for the other cases according to the formulae (6), (7) and (8). The error in determining thus ϑ is relatively big in the range of very small angles ($\vartheta < 20$ mrad). Therefore, the scattering angle was measured again by means of a more precise coordinate method described in paper [7] for all cases of scattering angles $\vartheta > 5$ mrad. The error of determining the scattering angle ϑ is smaller when this method is used. Its mean value $\langle \sigma_\vartheta \rangle = 2.9$ mrad [7].

Considering the fact that the Coulomb scattering dominates the diffraction scattering in the range of <10 mrad, we determined the angular distribution for $\vartheta \geqslant 10$ mrad. The cases of the scattering angles $\vartheta < 7$ mrad were eliminated and the scattering angles ϑ from the interval of 7—10 mrad were taken into account only for geometrical corrections.

The mean value of ionization was measured on primary tracks I_0 in eighty cases of the scattering angle $\vartheta \geqslant 7$ mrad. In 5 cases the values were $I_0 < 30$ grains/100 μ. These cases were eliminated as impurity particles. Distribution of the mean values of the ionization of primary tracks is shown in Fig. 3.

By means of measuring the multiple scattering and the mean value of ionization of secondary tracks, 9 cases of inelastic scattering, or the so-called

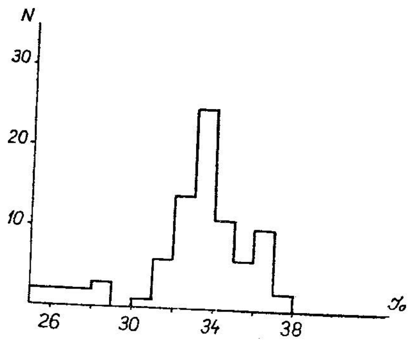

Fig. 3. Statistic distribution of the mean values of the ionization I_0 measured on the primary tracks of the cases of scattering onto the angle $\vartheta > 7$ mrad. For the particles of impurities $I_0 < 30$ grains/100μ.

stripping, were eliminated. On the whole, 66 cases of elastic deuteron scattering with the scattering angles of $\vartheta = 7$—50 mrad were found.

III. ANGULAR DISTRIBUTION AND CROSS SECTION OF ELASTIC SCATTERING

The projection of the scattering angle ϑ onto the plane perpendicular to the direction of the primary track α is given by the relation $\sin \alpha = \delta/\vartheta$ for small values of angles of φ and δ. We assume that the scattering is axially symmetric, i. e. that the angular distribution in the plane perpendicular to the direction of the primary particles (the so-called target diagram) is isotropic.

Our target diagram was made from 66 cases of scattering (Fig. 4) to ascertain how many cases of scattering were left out. It can be seen in Fig. 4 that the

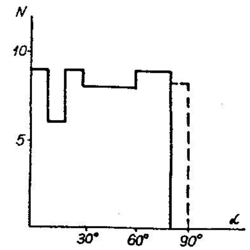

Fig. 4. The target diagram of the cases of elastic scattering of deuterons onto the photographic emulsion nuclei.

angular distribution α is practically isotropic apart from the interval $\alpha = 80$—$90°$. We did not identify any case in this interval. The correction for the target diagram comprises 1/8 of all the cases, i. e. 8.25 cases.

N_i cases of scattering were measured in the intervals $(\vartheta_i, \vartheta_i + \Delta\vartheta)$. Having made the correction for the target diagram we got N_i' cases in these intervals, where

$$N_i' = (1 + 8.25/\sum_i N_i) N_i .$$

Considering that the precision of the measuring of the angles φ and δ ($\sigma_\varphi = 1.15$ mrad and $\sigma_\delta = 3.78$ mrad) is not the same, we made also the geometrical correction of the angular distribution N_i' [7] and so we obtained the distribution N_i''. The cross-sections of the elastic scattering for the angular intervals $\Delta\sigma_i$ were computed in accordance with the formula: $\Delta\sigma_i = 1/\sum_k n_k \langle l \rangle_i$, where $\langle l \rangle_i$ is the mean free path for the scattering onto the intervals $(\vartheta_i, \vartheta_i + \Delta\vartheta)$ and n_k is the number of atoms of some element in 1 cm^3 of the photographic emulsion.

The recoil of the nucleus was not observed in our cases of deuteron scattering to small angles. Such were the cases of scattering on compound nuclei, and, therefore, the number of hydrogen atoms in 1 cm³ of the photographic emulsion is not included in the sum n_k.

The mean free path in all intervals of the angle ϑ and the corresponding cross-sections $\Delta\sigma_i$ are shown in Tab. 2.

Table 2

$\Delta\vartheta$ [mrad]	$\langle l\rangle_i$ [cm]	$\Delta\sigma_i^c$ [m barn]	$\Delta\sigma_i$ [m barn]	$\frac{\Delta\sigma_i^d}{\Delta\Omega}$ [barn sr⁻¹]
10–20	55.9 ± 10.3	56	366^{+82}_{-58}	329^{+87}_{-61}
20–30	87.4 ± 20.2	1	235^{+72}_{-44}	149^{+46}_{-28}
30–40	230.3 ± 86.3	0	89^{+53}_{-24}	40^{+24}_{-11}
40–50	666 ± 410	0	31^{+55}_{-12}	11^{+17}_{-4}

The cross-section of $\Delta\sigma_i$ is the sum of cross-sections of the diffraction and the Coulomb scatterings. As it was impossible to distinguish the Coulomb scattering from the diffraction one experimentally, we evaluated the theoretical value of the corss-section of the Coulomb scattering $\Delta\sigma_i^c$. The elastic scattering amplitude of particles with the momentum $p = \hbar k$ on nucleus with the radius R_j and the charge Z_j given in papers [8] and [9] in the form

$$f(\tilde{\vartheta}) = ik\left[\frac{RJ_1(kR\tilde{\vartheta})}{k\tilde{\vartheta}} + 2\,in\,\frac{J_0(kR\tilde{\vartheta})}{(k\tilde{\vartheta})^2}\right], \tag{9}$$

where $\tilde{\vartheta}$ is the scattering angle in the center of the mass system $R = R_j + R_d$, $n = mZ_dZ_je^2/\hbar^2k$ and m is the reduced deuteron mass. Formula (9) is derived on condition that $n \ll 1$, which is fulfilled satisfactorily in our case. For light nuclei ($\bar{Z}_j = 7$, $\bar{A} = 14$) $n = 0.034$ and for heavy nuclei ($\bar{Z} = 41$, $\bar{A} = 94$) $n = 0.23$. The first member of formula (9) is a diffraction scattering amplitude, the second is a Coulomb scattering amplitude. The differential cross section of the Coulomb scattering equals

$$\frac{d\sigma^c}{d\Omega} = |f_c(\tilde{\vartheta})|^2 = 4n^2\,\frac{J_0^2(kR\tilde{\vartheta})}{k^2\tilde{\vartheta}^4}. \tag{10}$$

The cross-sections of the Coulomb scattering $\Delta\sigma_i^c$ (Tab. 2) were computed by means of a numerical integration of formula (10).

$$\Delta\sigma_i^c = 2\pi\int_{\vartheta_i}^{\vartheta_i+\Delta\vartheta} |f_c(\tilde{\vartheta})|^2 \sin\vartheta\, d\vartheta. \tag{11}$$

The cross-sections of the diffraction scattering $\Delta\sigma_i^d$ were computed from the difference between $\Delta\sigma_i - \Delta\sigma_i^c$.

The dependence of the differential cross-section of the diffraction scattering on the scattering angle ϑ is shown in Fig. 5. The theoretical curve is computed as the square of the diffraction scattering amplitude

$$\frac{d\sigma^d}{d\Omega} = |f_d(\tilde{\vartheta})|^2 = \frac{R^2J_1^2(kR\tilde{\vartheta})}{\tilde{\vartheta}^2}. \tag{12}$$

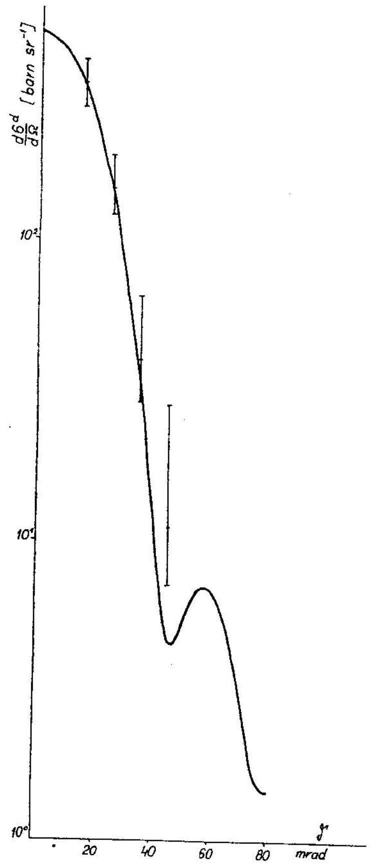

Fig. 5. The differential cross-section of he elastic scattering. ϑ is the scattering tangle in the laboratory system.

As the experimental values of the differential cross-section were in good agreement with the theory in the range of small angles ($\vartheta < 40$ mrad), the cross-section of the diffraction scattering in the interval of $\vartheta = 0-10$ mrad was determined with the help of the numerical integration of equation (12). The diffraction scattering amplitude $|f_d(\vartheta)|$ is small for $\vartheta > 50$ mrad. The cross-section of the diffraction scattering σ_{el} was approximated by the sum of $\Delta\sigma_i^d$ for $\vartheta \in (0{,}50)$ mrad.

$$\sigma_{el} = \sum_i \Delta\sigma_i^d = 792\,^{+103}_{-78}\ \text{mbarn}$$

$$\sigma_{el}/\text{nucl} = 64 \pm 8\ \text{mbarn}.$$

IV. CONCLUSION

The above described method of scattering identification and of the exact measurements of scattering angles enabled us to measure the differential cross-section of the diffraction scattering in the range of very small angles. In this range, i. e. $\vartheta \in (10\ \text{mrad}, 40\ \text{mrad})$ the differential cross-sections determined experimentally are in good agreement with the theory. The differential cross-section of the elastic scattering is <10 m barn sr^{-1} for higher angles. In our statistics it represents less than 1 case for $\Delta\vartheta = 10$ mrad. Considerably more statistics are necessary if conclusions are to be arrived at for the range of $\vartheta > 40$ mrad. We hope to obtain sufficient statistics from experimental material with a higher value of the primary momentum of deuterons.

If we assume, in agreement with the theory, that $\sigma_{el} \sim A^{2/3}$, then we can compute the cross-section of the elastic scattering onto 1 nucleon $\sigma_{el}/nucl = \sigma_{el}A^{-2/3}$. The obtained values of the cross-section of σ_{el} and $\sigma_{el}/nucl$ were compared with the values given in papers [1], [2] and [3] (Tab. 3). It can be seen from Tab. 3 that our value of the cross-sections of the diffraction scattering is in good agreement with the value given in paper [1].

Finally we wish to thank all those who have helped us in the course of our investigations, above all Dr. K. D. Tolstov from the High Energies Laboratory in JINR for valuable discussions, as well as S. I. Ljubomilov for the photochemical processing of the emulsions. We should like to thank Dr. M. Lojová for providing some experimental material, and also to express our thanks to Miss G. Paňková, Mrs. Špaleková, Miss E. Hrnčiariková for taking part in the measuring procedures. Last but not least, we are grateful to Professor J. Dubinský for his moral support.

Table 3

Paper	σ_{el} [m barn]	A	$\sigma_{el}/nucl.$ [m barn]
Bisheva et al. [1]	620 ± 62	27	69 ± 7
Jafar et al. [2]	287 ± 19	27	32 ± 2
Dulton et al. [3]	131 ± 15	12	25 ± 3
our values	844 ± 103	48.4	64 ± 8

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Received December 1st, 1970