Letters to the Editor

REMARK TO THE CASSON EQUATION

SYLVIA PULMANNOVÁ*, Bratislava

Casson's law [1] for the relation between shear stress and velocity gradient was originally derived for the rheological model of a suspension of rigid spherical particles, among which the attractive forces act. Owing to these forces, the particles form rod-like aggregates of different lengths. This equation was experimentally verified for different materials: printing inks [1], molten chocolate [2], human blood [3, 4].

Casson assumed that the axial ratio j (that is the ratio of the rod length to the particle diameter) of the rod-like aggregates is a linear function of the $y = (\eta_0 D)^{-1/2}$ (η_0 is the viscosity of the suspending liquid and D is the velocity gradient), that is

$$j = \alpha + \beta y . \tag{1}$$

This relation is a generalization of the relations $j = j_0$, where j_0 is the axial ratio of a single particle, which is valid for the great velocity gradients, and $j = \beta(\eta_0 D)^{-1/2}$, which is valid for low values of D.

The resultant relation between F and D is then

$$F^{1/2} = k_0 + k_1 D^{1/2} , \tag{2}$$

where F is the shear stress.

A special generalized form of (1), which has also practical meaning, is

$$j = \alpha + \beta y^r , \tag{3}$$

from which we can obtain, in an analogical way as that used by Casson

$$F^{r/2} = k_0 + k_1 D^{r/2} . \tag{4}$$

Let us substitute into the equation

$$\eta = \eta_0(1 - c) + \eta_0 a j c ,$$

(c is the volume concentration), obtained by Casson, relation (3) instead of (1). Thus we obtain

$$\eta = \eta_0(1 - c) + \eta_0 a \alpha c + \frac{a\beta c}{D^{r/2}} \eta_0^{(1-r/2)} . \tag{5}$$

* Ústav teórie merania SAV, Bratislava, Dúbravská cesta.

Relation (5) can be applied only to very diluted suspensions. Casson generalized it in the following way:

Let us consider a small addition of the solid material to the suspension, the added volume being δc to every $(1 - \delta c)$ volume unit of the original suspension. Suppose that the new suspension is a very diluted suspension with the volume concentration δc of the original suspension. The whole concentration of the new suspension is $c^* = c(1 - \delta c) + \delta c$, that is $\delta c = dc/(1 - c)$, where $dc = c^* - c$. The viscosity of the original suspension η can be considered as the viscosity of the suspending medium of the new suspension. By (5) the viscosity of the new suspension is

$$\eta' = \eta[1 + (a\alpha - 1)\delta c] + \frac{a\beta\delta c}{D^{r/2}} \eta^{(1-r/2)} \tag{6}$$

and for $d\eta = \eta' - \eta$ we have

$$d\eta = \left[\eta(a\alpha - 1) + \frac{\eta^{(1-r/2)}}{D^{r/2}} a\beta\right] \frac{dc}{(1 - c)} . \tag{7}$$

Then we obtain

$$\frac{d\eta}{A\eta + B\eta^{(1-r/2)}} = \frac{dc}{1 - c} , \tag{8}$$

when $a\alpha - 1 = A$ and $a\beta/D^{r/2} = B$.

By integrating this equation with the boundary condition $\eta = \eta_0$ at $c = 0$, we obtain

$$\eta^{r/2} = \left[\frac{\eta_0}{(1 - c)A}\right]^{r/2} + \frac{B}{A}\left[\left(\frac{1}{1 - c}\right)^{Ar/2} - 1\right] . \tag{9}$$

We obtain the relation between F and D multiplying by $D^{r/2}$:

$$F^{r/2} = k_1 D^{r/2} + k_0 ,$$

where

$$k_1 = \left[\frac{\eta_0}{(1 - c)A}\right]^{r/2} , \quad k_0 = \frac{a\beta}{A}\left[\left(\frac{1}{1 - c}\right)^{Ar/2} - 1\right] . \tag{10}$$

For example, for $r = 2$, we obtain the equation

$$F = k_1 D + k_0 , \tag{11}$$

that is the Bingham relation for the plastic flow [5].

REFERENCES

[1] Casson N., *A flow equation for pigment-oil suspensions of the printing ink type*. In: *Rheology of disperse systems*. Ed. C. C. Mill, Pergamon Press, New York—London 1959.

[2] Steiner E. H., Rev. Intern. de la Chocolaterie *13* (1958), No. 7.

[3] Scott-Blair G. W., Nature *186* (1960), 708.

[4] Merril E. W., Margetts W. G., Cokelet G. R., Gillibard E. R., *The Casson equation and rheology of blood near zero shear*. Proc. 4th Congr. Rheol., Part 4, 135. Ed. A. L. Copley, Interscience, New York 1963.
[5] Reiner M., *Phenomenological Macrorheology*. In: *Rheology, Theory and Applications*, Vol. 1. Ed. F. R. Eirich, Acad. Press., New York 1956.

Received January 11th, 1971