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THE DETERMINATION OF THE EFFICIENCY OF A THERMOCOUPLE WITH RESPECT TO THE THOMSON EFFECT AND THE TEMPERATURE DEPENDENCE OF THE ELECTRIC RESISTIVITY

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The efficiency of a thermoelectric couple with regard to the Thomson effect and the temperature dependence of the electric resistivity is derived by introducing a derived relation between the Sebeck and Peltier coefficient and by linearization of the differential equation of heat transfer of the thermocouple. These expressions for the efficiency show, besides the possibility of finding the optimal conditions, that the up-to-date criterion — the figure of merit z — is not unique.

I. ON THE DIFFERENTIAL EQUATION OF THE TEMPERATURE PROBLEM OF A WORKING THERMOCOUPLE

The general expression for the efficiency of a thermocouple consisting of n and p arms of the lengths $l_n = l_p = l$, whose surface is perfectly insulated is given in [9] or [5] by

$$\eta = R\left(\frac{\Delta V}{R + r'}\right)^2 \left[Q_p + \lambda_n S_n \left.\frac{\mathrm{d}T}{\mathrm{d}x}\right|_{\substack{n\\x=l}} + \lambda_p S_p \left.\frac{\mathrm{d}T}{\mathrm{d}x}\right|_{\substack{p\\x=l}}\right]^{-1} \tag{1}$$

where R is the load resistance, ΔV the thermoelectric voltage of the couple, $r' = r_n + r_p$ is the total resistance of the couple, Q_p is the Peltier heat absorbed at the warm end of the couple, λ_n and λ_p are the thermal conductivities of the n and p arms, respectively, S_n and S_p are the crosssections areas of the arms, $\left.\frac{\mathrm{d}T}{\mathrm{d}x}\right|_{\substack{n\\x=l}}$ and $\left.\frac{\mathrm{d}T}{\mathrm{d}x}\right|_{\substack{p\\x=l}}$ are the temperature gradients in the n and p arms, respectively, at the warm end (for $x = l$). We assume that the temperature gradient has the direction of the positive x coordinate. In order to substitute the expression for $\mathrm{d}T/\mathrm{d}x$ in Eq. (1) it is necessary to find the analytical solution of the differential equation for heat transfer in the arms. Now we investigate the temperature field of a homogenous sample of the n type with a constant crosssectional area S along the length of the sample under circumstances which are similar to those of a working thermocouple. Therefore it will be assumed that the device works at temperatures T for which $T_1 < T < T_2$ and that the thermal insulation in the device is perfect. That is, it will be assumed that there is no heat transfer between the heat resorvoirs, except trough the thermoelectric arms n and p and that there is no heat transfer through the lateral surfaces of the arms. The heat-balance condition in an arbitrary element of the sample of the length $\mathrm{d}x$ gives the corresponding differential equation

$$S\frac{\mathrm{d}}{\mathrm{d}x}\left(\lambda\frac{\mathrm{d}T}{\mathrm{d}x}\right)\mathrm{d}x + Sw\,\mathrm{d}x = c(x)s(x)S\,\mathrm{d}x\frac{\mathrm{d}T}{\mathrm{d}t}, \tag{2}$$

where w is the thermal productivity of the sources, $c(x)$ the specific heat, $s(x)$ the specific mass of the sample and t the time. In a steady state when $\mathrm{d}T/\mathrm{d}x = 0$, Eq. (2) becomes

$$S\frac{\mathrm{d}}{\mathrm{d}x}\left(\lambda\frac{\mathrm{d}T}{\mathrm{d}x}\right)\mathrm{d}x + Sw\,\mathrm{d}x = 0\,. \tag{3}$$

Eq. (3) with respect to Eq. (16) of paper [1] for covalent semiconductors (when $r = 0$) in the nondegenerate case for a homogenous sample can be written

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\lambda\frac{\mathrm{d}T}{\mathrm{d}x}\right) + \frac{3}{2}\frac{k}{e}j\frac{\mathrm{d}T}{\mathrm{d}x} + \varrho j^2 = 0, \tag{4}$$

where j is the density of the electric current and ϱ the electric resistivity. The differential equation (4) could be obtained directly by putting equal to zero Eq. (16) from paper [1], which expresses the total heat released by the electric current in an element of a nonhomogenous semiconductor in the case of a temperature gradient, i. e. the equation

$$q = \frac{j^2}{\sigma_{el}} + \frac{\mathrm{d}}{\mathrm{d}x}\left(\lambda\frac{\mathrm{d}T}{\mathrm{d}x}\right) + T\frac{j}{e}\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{r+2}{r+1}kT\frac{F_{r+1}(\mu^*)}{F_r(\mu^*)}\frac{1}{T} - \frac{\mu}{T}\right),$$

where $\sigma_{el} = 1/\varrho$, r is related to the mean free path by the expression $l = l_0(T)\varepsilon^r$, $F_r(\mu^*)$ is the Fermi integral, μ is the chemical potential and $\mu^* = \mu/kT$ for the given conditions. As the mean free path for covalent semi-

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conductors $l' \sim 1/T$, see [2] or [3], the expression of the electrical resistivity Eq. (10) of paper [1] for the mentioned conditions can be written

$$\varrho_T = \partial T^{3/2}\,, \tag{5}$$

or in a general form

$$\varrho_i = \partial T_i^s \tag{5a}$$

for ∂, which does not depend on temperature, can be written

$$\partial = \frac{\varrho_i}{T_i^s}\,. \tag{6}$$

With regard to Eq. (5a) and Eq. (6), Eq. (4) becomes

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\lambda\frac{\mathrm{d}T}{\mathrm{d}x}\right) + \frac{3}{2}\frac{k}{e}j\frac{\mathrm{d}T}{\mathrm{d}x} + \frac{\varrho_i}{T_i^s}T^s j^2 = 0\,. \tag{7}$$

About the impossibility of finding an analytical solution of Eq. (7) when the productivity of the sources is a function of temperature, deals paper [4]. The similar differential equation

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\lambda\frac{\mathrm{d}T}{\mathrm{d}x}\right) - \frac{J}{S}\tau\frac{\mathrm{d}T}{\mathrm{d}x} + \left(\frac{J}{S}\right)^2\varrho = 0 \tag{8}$$

has an exact solution only in the case when some special assumptions concerning the forms of $\lambda(T)$, $\varrho(T)$ and $\tau(T)$ are satisfied [5], i. e. $\lambda\varrho/\tau = A\int\tau\,\mathrm{d}T + B$, where A and B are arbitrary constants.

These assumptions are not fulfilled even in the case when λ and τ are constants and $\varrho \sim T^s$ at $s \neq 1$. Eq. (8), in which τ is the Thomson coefficient, has for $\mathrm{d}T/\mathrm{d}x$ an analytical solution in the form of a convergent series in the case when λ, τ and ϱ are constant and further assumptions are satisfied. This solution can be written [5]

$$\lambda S\frac{\mathrm{d}T}{\mathrm{d}x}\bigg|_{x=1} = Q - \tfrac{1}{2}q_J - \tfrac{1}{2}q + \ldots \text{ where } Q = \frac{\lambda S\Delta T}{l}\,,$$

$$q = J\tau\Delta T\,,\quad q_J = J^2\varrho\frac{1}{S}\,,\quad T = T_2 - T_1\,.$$

The denominator of the expression for efficiency in Eq. (1), which means the whole heat added to the warm junction per unit time, neglecting the further terms of the series which do not exceed 5 per cent of the foregoing terms can be written $Q_p + Q - \frac{1}{2}q_J - \frac{1}{2}q_\tau$. The uncomfortable term containing the

Thomson heat is always neglected in calculations, which leads to introducing the figure of merit z, although it follows from Eq. (17) of paper [1] that for a nondegenerated gas the Thomson coefficient becomes $\tau = 3k/2e = 129.3 \times 10^{-6}$ V/deg, that means it is a value comparable to α. At a concentration optimal from the point of view of the maximum value of figure of merit z at which $\mathrm{d}z/\mathrm{d}n = 0$ [6], we have $\alpha = 2k/e = 172 \times 10^{-6}$ V/deg. By neglecting the temperature dependence of the thermal conductivity at the given thermal interval, Eq. (7) can be written

$$\lambda\frac{\mathrm{d}^2T}{\mathrm{d}x^2} + j\frac{3k}{2e}\frac{\mathrm{d}T}{\mathrm{d}x} + \frac{\varrho_i}{T_i^s}T^s j^2 = 0\,. \tag{9}$$

Burstein found an analytical solution for $\mathrm{d}T/\mathrm{d}x$ for a similar equation by introducing a phenomenal Thomson coefficient in the case $\varrho(T) = \varrho_0 + \varrho_1 T$ [7]. Owing to the fact thas this solution contains a product of an exponential and a hyperbolical function with relatively complicated arguments, it would be impossible to find the optimal conditions [8]. We could obtain in this case a useful form of the expression for the efficiency by putting $\tau = 0$ and assuming the temperature independences of α, ϱ and λ. An approximate solution in a concrete case could be found be means of a digital computer [9].

II. THE DETERMNANTION OF THE EFFICIENCY AND OPTIMAL CONDITIONS OF A WORKING THERMOCOUPLE FOR THE PARTICULAR METHODS OF LINEARIZATION OF EQ. (7).

1. Linearization in the case of constant ϱ_n and λ

In this case the differential equation (7) becomes

$$\frac{\mathrm{d}^2T}{\mathrm{d}x^2} + \frac{3}{2}\frac{k}{e}\frac{j}{\lambda}\frac{\mathrm{d}T}{\mathrm{d}x} + \frac{\varrho_n}{\lambda}j^2 = \frac{\mathrm{d}^2T}{\mathrm{d}x^2} + a\frac{\mathrm{d}T}{\mathrm{d}x} + c = 0\,, \tag{10}$$

where the meaning of the constants a and c is given by this equation. The solution of Eq. (10) is

$$T(x) = C_1\exp(-ax) - \frac{c}{a}x + C_2\,,$$

where C_1 and C_2 are constants of integration. With regard to the boundary conditions $T(0) = T_1$ and $T(l) = T_2$, we have

$$\left.\frac{dT}{dx}\right|_{\substack{n\\x=l}} = \frac{\Delta T + \frac{c}{a} l}{1 - \exp(-la)} a \exp(-la) - \frac{c}{a}.$$

Introducing the foregoing expression and using the relation $j_n = \Delta V/(R + r')S_n$ we obtain

$$\left.\frac{dT}{dx}\right|_{\substack{n\\x=l}} \lambda_n S_n = \frac{\frac{\Delta V}{R + r'}\left(\frac{3}{2}\frac{k}{e}\Delta T + \frac{\Delta V}{R + r'}\frac{\varrho_n}{S_n} l\right)}{\exp\left(\frac{\Delta V}{R + r'}\frac{1}{S_n}\frac{l}{\lambda_n}\frac{k}{e}\frac{3}{2}\right) - 1} - \frac{2}{3}\frac{e}{k}\frac{\Delta R}{R + r'}\lambda_n\varrho_n \tag{11}$$

and an analogous expression for the p arm. ϱ_n in Eq. (10) is defined by $r_n = \varrho_n l/S_n$, where r_n is the ohmic resistance of the n arm in a working thermocouple, i. e. in the case of a steady temperature field. Theoretically

$$\varrho_n = \frac{1}{l}\int_0^l \varrho(T)\, dx,$$

where $T = T(x)$ is the temperature course in the sample. In order to compare the expression for the efficiency derived in this paper with the so far known ones, containing the figure of merit η_z, and the Peltier coefficient π_{n-p} is expressed by α, it is necessary to find an analogous relation between α and π_{n-p}. Regarding the mentioned circumstances and Eqs. (12) and (18a) of paper [1] we obtain the relation

$$\pi_{n-p} = \left[\alpha - 3\left(\frac{T_1}{\Delta T}\ln\frac{T_2}{T_1} - 1\right)\frac{k}{e}\right] T_2, \tag{12}$$

where

$$Q_P = -Sj\pi_{n-p}. \tag{13}$$

On substituting the expression from Eqs. (11), (12) and (13) and putting $R/r = m$, Eq. (1) becomes

$$\eta = \frac{\Delta T}{T_2}\frac{m}{m+1}\left\{1 - 3\left(\frac{T_1}{\Delta T}\ln\frac{T_2}{T_1} - 1\right)\frac{k}{e}\frac{1}{\alpha} + \frac{1}{T_2}\frac{2}{3}\frac{e}{k}\frac{\lambda_n\varrho_n}{\alpha} + \right.$$

$$+\frac{\Delta T}{T_2}\frac{\frac{3}{2}\frac{k}{e}\frac{1}{\alpha} + \left[(m+1)\left(1 + \frac{S_n}{S_p}\frac{\varrho_p}{\varrho_n}\right)\right]^{-1}}{\exp\left[\frac{\Delta T}{(m+1)\left(1 + \frac{S_n}{S_p}\frac{\varrho_p}{\varrho_n}\right)}\frac{\alpha}{\varrho_n\lambda_n}\frac{3}{2}\frac{k}{e}\right] - 1} - \frac{1}{T_2}\frac{2}{3}\frac{e}{k}\frac{\lambda_p\varrho_p}{\alpha} +$$

$$\left. + \frac{\Delta T}{T_2}\frac{\frac{3}{2}\frac{k}{e}\frac{1}{\alpha} + \left[(m+1)\left(1 + \frac{S_p}{S_n}\frac{\varrho_n}{\varrho_p}\right)\right]^{-1}}{\exp\left[\frac{\Delta T}{(m+1)\left(1 + \frac{S_p}{S_n}\frac{\varrho_n}{\varrho_p}\right)}\frac{\alpha}{\varrho_p\lambda_n}\frac{3}{2}\frac{k}{e}\right] - 1}\right\}^{-1}. \tag{14}$$

However, the examination of Eq. (14) shows that the efficiency depends even in this most simple case on α, λ and ϱ unlike in the so far known clasical expression for η_z [6], in which the figure of merit occurs. We will find the value of $u = S_n/S_p$, which maximizes η. This condition will be satisfied if the sum of the terms in the parenthesis of Eq. (14) will be minimal. By setting the derivative of this sum with respect to u equal to zero we obtain for u a complicated and for a further application useless expression, which can be simplified and leads to the same optimal values for u as in the clasical case putting

$$\lambda_n\varrho_n = \lambda_p\varrho_p. \tag{15}$$

The terms corresponding to the n and p arms in Eqs. (1) and (14) are equivalent, therefore the condition (15) does not imply loss of generality when we consider the influence of these material constants on the expression for the efficiency, but makes this one easier to survey. Therefore the condition in (15) makes it possible to compare the expression for efficency Eq. (14) with the classical expression for η_z, as in both cases the same value of u is optimal from the standpoint of maximum values of both efficiencies. With regard to Eq. (15) we obtain

$$u = \frac{S_n}{S_p} = \left(\frac{\lambda_p\varrho_n}{\lambda_n\varrho_p}\right)^{1/2} = \frac{\lambda_p}{\lambda_n} = \frac{\varrho_n}{\varrho_p} \tag{16}$$

as the solution of the foregoing condition, in order to maximize the value of the efficiency expressed in Eq. (14). On substituting Eq. (16) and putting $m = 1$ which is in agreement with the condition of the maximal ouput, i. e. satisfies the equation

$$\frac{d}{dm} R\left(\frac{\Delta V}{R + r'}\right)^2 = \frac{d}{dm}\frac{(\Delta V)^2 r' m}{r'^2(m+1)^2} = 0.$$

Eq. (14) becomes

$$\eta_{\varrho=\varrho_n} = \frac{1}{2}\frac{\Delta T}{T_2}\left[1 - 3\left(\frac{T_1}{\Delta_T}\ln\frac{T_2}{T_1} - 1\right)\frac{k}{e\alpha} + \right.$$

$$\left. + \frac{\Delta T}{T_2}\frac{3\frac{k}{e\alpha}+\frac{1}{2}}{\exp\left(\frac{3}{8}\frac{\alpha\Delta T}{\lambda}\frac{k}{e}\right)-1} - \frac{1}{T_2}\frac{4}{3}\frac{e}{k}\frac{\lambda\varrho}{\alpha}\right]^{-1}. \qquad (17)$$

The so far known expression of efficiency, which is in agreement with the same circumstances but without the supplementary condition (15), in the case when the Thomson heat is neglected becomes after some simplification

$$\eta_z = \frac{1}{2}\frac{\Delta T}{T_2}\left[1 + \frac{2}{zT_2} - \frac{1}{4}\frac{\Delta T}{T_2}\right]^{-1}, \qquad (18)$$

where the figure of merit z is given by $z = \alpha^2[(\lambda_n\varrho_n)^{1/2} + (\lambda_p\varrho_p)^{1/2}]^{-2}$ in our circumstances $z = \alpha^2/4\lambda\varrho$ and

$$\eta_z = \frac{1}{2}\frac{\Delta T}{T_2}\left[1 + \frac{8}{T_2}\frac{\lambda\varrho}{\alpha^2} - \frac{1}{4}\frac{\Delta T}{T_2}\right]^{-1}. \qquad (18a)$$

It can be seen that η_z Eq. (18) depends at the same temperature conditions only on the figure of merit z, so far has remained the criterion of the applicability of a thermoelectric material. However, the examination of Eq. (17) reveals that several different values of $\eta_{\varrho=\varrho_n}$ correspond to a fixed z. From this point of view Eq. (17) removes the nonuniqueness by using Eq. (18). The expression in the square brackets of Eq. (17) can be with regard to the second law of thermodynamics greater or equal to one. This can be assumed as a physical reason of validity of the relation between λ, ϱ, α, T_1 and T_2. From Eq. (18a) this relation becomes

$$\frac{1}{T_2}\left(\frac{8\lambda\varrho}{\alpha^2} - \frac{1}{4}\Delta T\right) \geqslant 0 .$$

2. Linearization by means of an exponential function

On substituting into the modified right-hand side of Eq. (9)

$$\frac{d^2T}{dx^2} + \frac{j}{e}\frac{k}{\lambda}\frac{3}{2}\frac{dT}{dx} = -\frac{j^2}{\lambda}\frac{\varrho_i}{T_i^s}T^s, \qquad (19)$$

$T = T_0 \exp(vx)$, where T_0 and v computed from the boundary conditions $T_{x=0} = T_1$, $T_{x=l} = T_2$ gives

$$T = T_1 \exp\left(\frac{x}{l}\ln\frac{T_2}{T_1}\right), \qquad (20)$$

which is the solution of a onedimensional heat transfer without inside sources under the assumption that the thermal insulation is perfect and that $\lambda \sim 1/T$ [6]; then the linearized differential equation (19) can be written

$$\frac{d^2T}{dx^2} + \frac{j}{\lambda}\frac{k}{e}\frac{3}{2}\frac{dT}{dx} = -\frac{j^2}{\lambda}\frac{\varrho_i}{T_i^s}T_1^s \exp\left(s\frac{x}{1}\ln\frac{T_2}{T_1}\right). \qquad (21)$$

By introducing the constants a, b, and c, we obtain $d^2T/dx^2 + a\, dT/dx = -b \exp(cx)$. The solution of Eq. (21) with regard to the boundary conditions gives

$$\left.\frac{dT}{dx}\right|_{x=l}^{n} = a\frac{\Delta T + \frac{b}{c^2+ac}\exp(cl-1)}{\exp(al) - 1} - \frac{b\exp(cl)}{c+a}. \qquad (21a)$$

In order to simplify further calculation with regard to $\varrho_{in}/T_{in}^s = \partial$, we compute T_{in} corresponding to $\varrho_{in} = \varrho_n$ from equation

$$\varrho_n = \frac{1}{l}\int_0^l \varrho(T)\,dx = \frac{\varrho_{in}}{T_{in}^s}\frac{1}{l}\int_0^l [T(x)]^s\,dx . \qquad (22)$$

On substituting Eq. (20) for T into Eq. (22) we have

$$\varrho_n = \frac{1}{l}\frac{\varrho_{in}}{T_{in}^s}\int_0^l T_1^s \exp\left(s\frac{x}{l}\ln\frac{T_2}{T_1}\right)dx = \varrho_{in}\left(\frac{T_1}{T_{in}}\right)^s\frac{1}{s}\frac{(T_2/T_1)^s - 1}{\ln(T_2/T_1)}. \qquad (23)$$

From Eq. (23) T_{in}, corresponding to $\varrho_{in} = \varrho_n$, is given by

$$T_{in} = T_1\left[\frac{1}{s}\frac{(T_2/T_1)^s - 1}{\ln(T_2/T_1)}\right]^{1/s}. \qquad (24)$$

The value of T_{in} can be determined by an experimentally found value of r_n from the equation

$$\varrho_n = r_n\frac{S_n}{l} = \frac{\varrho_{jn}}{T_{jn}^s}T_{in}^s,$$

which implies

$$T^s_{in} = r_n \frac{S_n}{l} \frac{T^s_{jn}}{\varrho_{jn}}, \tag{24a}$$

where ϱ_{jn} is the electric resistivity of the n sample at the temperature T_j, $\eta_\varrho(T)$ is maximized with respect to S_n/S_p at $m = l$, $T_{in} = T_{ip}$, $\lambda_n\varrho_n = \lambda_p\varrho_p$ in a similar way if Eq. (16) is satisfied. Under these circumstances we obtain

$$\eta_{\varrho(T)} = \frac{1}{2}\frac{\Delta T}{T_2}\left\{1 - 3\left(\frac{T_1}{\Delta T}\ln\frac{T_2}{T_1} - 1\right)\frac{k}{e}\frac{1}{\alpha} - \frac{1}{T_2}\left(\frac{T_2}{T_1}\right)^s\left[2s\ln\frac{T_2}{T_1} + \frac{\alpha\Delta T}{\lambda}\frac{3}{4}\frac{k}{e}\right]^{-1} + \frac{\Delta T}{T_2}\frac{k}{e}\frac{3}{\alpha}\frac{\frac{\alpha}{\lambda\varrho}\frac{\Delta T}{4}\left[\left(\frac{T_2}{T_1}\right)^s - \left(\frac{T_1}{T_i}\right)^s\right]\left[\left(2s\ln\frac{T_2}{T_1}\right)^2 + \frac{\alpha\Delta T}{\lambda\varrho}\frac{k}{e}\ln\frac{T_2}{T_1}\right]^{-1}}{\exp\left(\frac{\alpha\Delta T}{\lambda}\frac{3}{8}\frac{k}{e}\right) - 1}\right\}^{-1}. \tag{25}$$

This expression for the efficiency of a thermocouple is different from the one known so far for η_z in the following points:

1. The Thomson heat has not been neglected.
2. The temperasure dependence of the electric resistivity has not been neglected.
3. The temperature dependence of the thermal condtctivity was taken into account in a special case.
4. At the given temperature conditions $\eta_{\varrho(T)}$ is not a function of the figure of merit z but depends separately on α and $\lambda\varrho$, respectively so that $\eta_{\varrho(T)} = \eta(T_1, T_2, \alpha, \lambda, \varrho)$, unlike $\eta_z = \eta(T_1, T_2, z)$. From the form of the figure of merit $z = \alpha^2/4\lambda\varrho = i^2\alpha^2/i^2 4\lambda\varrho = \alpha_i^2/4\lambda_i\varrho_i$ it is seen that the same η_z but in general different values of $\eta_{\varrho(T)}$ corresponds to an arbitrary i. Eq. (25) for $\eta_{\varrho(T)}$ — analogical as in the foregoing case Eq. (17) — shows the non-uniqueness of the determination of the efficiency by means of Eq. (18) or Eq. (18a) for η_z.

3. The linearization by means of $T = T_1 + \frac{\Delta T}{l} x$

This linearization gives a differential equation, which has an analytical solution (by the method of variation of constants) only for the integer s. The expression for the efficiency is maximized by the same optimal conditions as in the foregoing cases and has the form $\eta(T_1, T_2, \alpha, \alpha/\lambda\varrho)$, but is more complicated already at $s = 2$ than in the foregoing cases.

4. The comparison of the values of the single efficiencies and the analysis of the applicability of the expression for η_z.

In order to compare the factors at $\frac{1}{2}\Delta T/T_2$ in the different expressions of efficiencies η_z, $\eta_{\varrho=\varrho n}$ and $\eta_{\varrho(T)}$ according to Eqs. (18a), (17) and (25), i. e. the values $\emptyset = \eta 2T_2/\Delta T$ (at $T_1 = 300°$ K and $T_2 = 400°$ K), a table (see Tab. 1) of those factors was calculated. The places in the table with the same values of figure of merit z are indicated with the same i. There are further in the table the values of $\alpha_1 = \alpha_n + \alpha_p = 4.7 \times 10^{-6}$ V deg^{-1} and $(\lambda\varrho)_1 = 10^{-5}$ V^2 deg^{-1}. The meaning of the particular values in Table 1 is indicated in right upwards.

Table 1

α [Vdeg^{-1}] / $\lambda\varrho$ [V^{2}deg^{-1}]	α_1	$\alpha_1(1.5)^{1/2}$	$\alpha_1(2)^{1/2}$	$\alpha_1(3)^{1/2}$	$\alpha_1(4)^{1/2}$	$\alpha_1(6)^{1/2}$
$(\lambda\varrho)_1$	(4) 0.2193 0.2255 0.210	(6) 0.298 0.3064 0.2814	(7) 0.368 0.373 0.338	(8) 0.466 0.480 0.419		(i) $\emptyset_{zi}$ $\emptyset_{\varrho=\varrho n}$ $\emptyset_{\varrho(T)}$
$1.5(\lambda\varrho)_1$	(2) 0.156 0.1596 0.152	(4) 0.2193	(5) 0.272 0.279 0.257	(7) 0.368 0.366 0.303		
$2(\lambda\varrho)_1$	(1) 0.122 0.1236 0.1193	(3) 0.173 0.175 0.1668	(4) 0.2193	(6) 0.298 0.302 0.277		(8) 0.466 0.474 0.372
$3(\lambda\varrho)_1$		(1) 0.122 0.1215 0.1181	(2) 0.156 0.1576 0.1508	(4) 0.2193 0.2215	(5) 0.272 0.274 0.255	
$4(\lambda\varrho)_1$				(3) 0.173 0.1738 0.1653	(4) 0.2193 0.220 0.207	

The agreement of the values η_z — Eq. (18a) and $\eta_{\varrho=\varrho n}$ — Eq. (17) can be verified in the case of small values of the exponent $\frac{3}{8}\frac{\alpha}{\lambda\varrho}\Delta T\frac{k}{e}$ in Eq. (17); in the present case they are of the order of hundredths, where the $\exp\left(\frac{3}{8}\frac{\alpha}{\lambda\varrho}\Delta T\frac{k}{e}\right) \doteq$ $\doteq 1 + \frac{3}{8}\frac{\alpha}{\lambda\varrho}\Delta T\frac{k}{e}$ if the further terms can be neglected. By neglecting the difference of the terms $3[(T_1/\Delta T)\ln(T_2/T_1) - 1]\,k/e\alpha$ and $\frac{1}{4}\Delta T/T_2$, which are smaller by two orders than the other terms, the sum of the last two terms in the brackets of Eq. (17) becomes $8\lambda\varrho/T_2\alpha^2$, which is in agreement with Eq. (18a). The expression $\eta_{\varrho(T)}$ can in a similar way be transferred to the expression for η_z.

The criterion of applicability of Eq. (18a) for η_z is the possibility transferring Eq. (17) or Eq. (25) to the form of Eq. (18a) in the foregoing way. The values of the corresponding efficiencies are different in those circumstances in which the foregoing transfer is not possible.

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