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EFFECT OF ELECTRONIC INTERVALLEY TRANSITIONS ON ULTRASOUND ABSORPTION AND ELASTIC CONSTANTS OF n-Ge AND n-Si

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The paper deals with the absorption of sound and with the change of elastic constants due to the interaction of the sound with conduction electrons in manyvalley semiconductors with nonspherical surfaces of constant energy. The interaction of the sound with the conduction electrons is introduced by means of (tensor) deformation potential constants. The nonequilibrium electron gas is described by the Boltzmann transport equation. The collision term is approximated by means of the relaxation times for the intravalley and intervalley transitions. The screening of the interaction potential by the conduction electrons is considered, too. The propagation of the acoustic wave is described by the wave equation in which the interaction with the electrons is introduced. The general formulae for the absorption coefficient and the changes of the elastic constants are applied to n-Ge and n-Si. Cases of the high and law sound frequencies are discussed.

I. INTRODUCTION

Information on electron-phonon interaction can be obtained from the measurements of the absorption coefficient and changes of the elastic constants as the function of the electron (hole) concentration. Experiments of this kind were carried out on Si and Ge [1—4] and interpreted in terms of the many-valley models of the energy surfaces [1—7]. There are also many papers dealing with the determination of the absorption coefficient and changes of the elastic constants in the one-valley models (see [7] and references therein). We give in this paper a derivation of the absorption coefficient and the changes of the elastic constants due to the electron-phonon interaction in the many-valley semiconductors taking into account both intervalley and intravalley transitions. Our method is based on the wave equation for the sound wave, in which the interaction with electrons by means of the deformation potentials is introduced and in accordance with Mertsching [9], who determined the acoustoelectric effect in manyvalley semiconductors, we described non-equilibrium electron gas by the Boltzmann equation. The general formulae for the absorption coefficient and the elastic constants, which are derived in the paper are then applied to n-Ge and n-Si. We discuss the cases of the high ($ql_\alpha \gg 1$) and the low frequencies ($ql_\alpha \ll 1$). In the case of the high frequencies we distinguish two cases: a strong and a weak screening of the interaction potentials. In the case of the low frequencies we consider only the strong screening.

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II. FORMULATION OF THE PROBLEM

We shall consider an acoustic wave propagating through a crystal characterized by the displacement vector $\mathbf{u}$, which in a point $\mathbf{r}$ and at a moment t has the form

$$\mathbf{u}(\mathbf{r}, t) = \mathbf{u}_0 \exp\left[\mathrm{i}(\omega t - \mathbf{q} \cdot \mathbf{r})\right], \tag{1}$$

where $\mathbf{u}_0$ is the amplitude of the sound, ω is the angular frequency and $\mathbf{q}$ is the wave vector. In the approximation of the elastic continuum small deformations are described by the tensor

$$u_{ij} = \frac{1}{2}\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right), \tag{2}$$

where x_i are components of the position vector and u_i are components of the displacement vector (1). Because of the deformation, the energies of the electrons in the crystal are changed. The dependence of the energy of the electrons, which are near the bottom of the conductivity band, can be expressed by means of the deformation potential

$$E^\alpha(\mathbf{k}) = E_0^\alpha(\mathbf{k}) + \Theta_{ij}^{(\alpha)} u_{ij} \equiv E_0^\alpha(\mathbf{k}) + V_\alpha, \tag{3}$$

where $E_0^\alpha(\mathbf{k})$ is the energy of the electrons in the undeformed crystal, $\Theta_{ij}^{(\alpha)}$ is the deformation potential tensor of the α-valley, $\mathbf{k}$ is the wave vector of the electron and V_α is the deformation potential in the α-valley. In the relation (3) and in the following the summation is understood with respect to the repeated indices. In Ge and Si of the n-type the electronic energy spectrum consists of a few equivalent minima (valleys), symmetrically distributed in the Brillouin zone. Round each of these minima the energy is

$$E_0^\alpha(\mathbf{k}) = \frac{\hbar^2}{2}\left(\frac{k_x^2}{m_1} + \frac{k_y^2}{m_2} + \frac{k_z^2}{m_3}\right), \tag{4}$$

where the axes x, y, z are directed along the principal axes of the energy elipsoids (for Ge and Si the equienergy surfaces are spheroids, so that $m_1 = m_2 = m_t$ and $m_3 = m_l$). In the k-space we introduce the substitution

$$k_x = k'_x m_1^{1/2}, \quad k_y = k'_y m_2^{1/2}, \quad k_z = k'_z m_3^{1/2}. \tag{4a}$$

Then the energy is

$$E_0^\alpha(\mathbf{k}') = \frac{\hbar^2 k'^2}{2}. \tag{4b}$$

The density of the states in the volume $dk_x\, dk_y\, dk_z$ equals

$$\frac{dk_x dk_y dk_z}{8\pi^3} = (m_1 m_2 m_3)^{1/2} \frac{dk'_x dk'_y dk'_z}{8\pi^3}.$$

Using (4b) we obtain for the density of the states in the unit volume

$$g(E) = \frac{\sqrt{2}}{2\pi^2} \frac{m_{ef}^{3/2}}{\hbar^3} \sqrt{E}, \tag{5}$$

where

$$m_{ef} = Z^{2/3}(m_1 m_2 m_3)^{1/3} \tag{6}$$

is the "effective mass of the density of states" and Z is the number of the equivalent valleys. (For n-Ge $Z = 4$ and for n-Si $Z = 6$). From the theory of elasticity the equation of motion is well-known

$$\varrho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ik}}{\partial x_k}, \tag{7}$$

where σ_{ik} is the stress tensor and ϱ is the density of crystals. The stress tensor is determined by the adiabatic change of the internal energy of the crystal at an infinitesimal deformation

$$\sigma_{ik} = \left(\frac{\partial W}{\partial u_{ik}}\right)_{S=\text{const.}}, \tag{8}$$

where W is the interval energy and S the entropy of the unit volume. The internal energy of the system consists of the elastic energy $W_{el}(u_{ij})$ and of contributions of individual electrons $W_e(\mathbf{k})$

$$W = W_{el}(u_{ij}) + W_e(\mathbf{k}). \tag{9}$$

The lowest approximation for W_{el} is given by Hooke's law, the elastic energy is a quadratic function of the strain tensor

$$W_{el}(u_{ij}) = W_{el}(0) + \tfrac{1}{2} \lambda_{ijkl} u_{ij} u_{kl}, \tag{10}$$

where λ_{ijkl} is the tensor of elastic constants. The contribution of electrons by means of (3) is given by the relation

$$W_e(\mathbf{k}) = \sum_{\mathbf{k},\alpha} E_0^\alpha(\mathbf{k})\, F_\alpha(\mathbf{k}, \mathbf{r}, t) + F_\alpha(\mathbf{k}, \mathbf{r}, t)\, \Theta_{ij}^\alpha u_{ij}], \tag{11}$$

where $F_\alpha(\mathbf{k}, \mathbf{r}, t)$ is the distribution function of electrons of the α-valley. It measures the electron density with the wave vector $\mathbf{k}$ at the position $\mathbf{r}$ and at the time t. The entropy of the nonequilibrium gas of electrons is

$$S = k_0 \sum_{\mathbf{k},\alpha} [(F_\alpha - 1) \ln(1 - F_\alpha) - F_\alpha \ln F_\alpha], \tag{12}$$

where k_0 is the Boltzmann constant. Hence it follows that the derivation (8) has to be done at a constant F_α. Now we can rewrite the equation (4) in the form

$$\varrho \frac{\partial^2 u_i}{\partial t^2} = \lambda_{ijkl} \frac{\partial^2 u_k}{\partial x_j \partial x_l} + \frac{\partial}{\partial x_j} \sum_{\mathbf{k},\alpha} \Theta_{ij}^\alpha F_\alpha(\mathbf{k}, \mathbf{r}, t). \tag{13}$$

We shall consider nondegenerate electron gas which is described by the Boltzmann distribution function in the equilibrium state

$$f_{0\alpha}(\mathbf{k}) = \exp\left\{\frac{1}{k_0 T}[\eta - E_0^\alpha(\mathbf{k})]\right\}, \tag{14}$$

where the parameter η is given by the relation

$$\exp\left(\frac{\eta}{k_0 T}\right) = \frac{4\pi^3 \hbar^3 N_{0\alpha}}{(2\pi m_{ef} k_0 T)^{3/2}}. \tag{15}$$

The nonequilibrium distribution function F_α will be determined by solving the Boltzmann transport equation

$$\frac{\partial F_\alpha}{\partial t} + \mathbf{v} \cdot \frac{\partial F_\alpha}{\partial \mathbf{r}} + \frac{d\mathbf{k}}{dt} \cdot \frac{\partial F_\alpha}{\partial \mathbf{k}} = -\left(\frac{\partial F_\alpha}{\partial t}\right)_{\text{coll.}}, \tag{16}$$

where

$$\mathbf{v} = \frac{1}{\hbar} \frac{\partial E(\mathbf{k})}{\partial \mathbf{k}} \tag{17}$$

is the group velocity of the electron with the wave vector $\mathbf{k}$. The time change of the $\mathbf{k}$ vector is

$$\frac{d\mathbf{k}}{dt} = -\frac{e}{\hbar}\left(\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{H}\right) - \frac{1}{\hbar} \frac{\partial V_\alpha}{\partial \mathbf{r}}. \tag{18}$$

$$\langle\ldots\rangle = \frac{\int(\ldots)f_{0\alpha}k^2\mathrm{d}k}{\int f_{0\alpha}k^2\mathrm{d}k} = \frac{\sqrt{2}}{N_{0\alpha}\pi^2}\frac{m_{el}^{3/2}}{\hbar^3}\int_0^\infty(\ldots)f_{0\alpha}\sqrt{E}\,\mathrm{d}E. \tag{38}$$

For the sake of simplicity we introduce the only intervalley transition time τ_z. $\tau_{\alpha\beta}$ can be replaced by the transition time among the nearest neighbouring valleys. From the equation (37), taking into account that it is enough to respect from $\boldsymbol{E}_1$ only the longitudinal component (since the magnetic field accompanying the acoustic wave can be neglected, we have rot $\boldsymbol{E} \approx 0$) then we obtain

$$N_{\alpha 1} = -\frac{N_{0\alpha}}{k_0T}\left[\left(\frac{\mathrm{i}eE_1}{q} + V_\alpha\right)\langle 1-(1+\mathrm{i}\omega\tau)I_\alpha\rangle + \right.$$
$$\left. + \left(ZV_\alpha - \sum_\beta V_\beta\right)\left\langle\frac{\tau}{\tau_z}I_\alpha\right\rangle + \left\langle\frac{\tau}{\tau_z}I_\alpha\right\rangle\sum_\beta N_{\beta 1}\right]\left[1-\left\langle\left(1-\frac{Z\tau}{\tau_z}\right)I_\alpha\right\rangle\right]^{-1}. \tag{39}$$

We shall assume that the sound wave propagates in the direction of the z axis, which we identify with one of the principal crystal directions, that is with the direction in which the pure longitudinal or transverse wave can propagate. Then the displacement has the form $u_i = e_i u$, where e_i are unit polarization vector components and

$$u = u_0 \exp\left[\mathrm{i}(\omega t - qz)\right].$$

Thus the equations (13) and (30) can be rewritten in the form

$$\varrho\frac{\partial^2 u}{\partial t^2} = \lambda\frac{\partial^2 u}{\partial z^2} + \frac{\partial}{\partial z}\sum_{\boldsymbol{k},\alpha}\Theta_\alpha F_{\alpha 1}(\boldsymbol{k},\boldsymbol{r},t) \tag{40}$$

$$\varepsilon\frac{\partial E_1}{\partial z} = -4\pi e\sum_\alpha N_{\alpha 1}, \tag{41}$$

where

$$\lambda = \lambda_{izkz}e_ie_k, \quad \Theta_\alpha = \Theta_{iz}^\alpha e_i, \quad \varepsilon = \varepsilon_{zz}.$$

By substituting (39) into (41) we shall find the expression for E_1

$$-\frac{\mathrm{i}eE_1}{q} = \left\{V + \frac{1}{A}\sum_\alpha v_\alpha\frac{1-\left\langle I_\alpha\left(1-\frac{Z\tau}{\tau_z}\right)\right\rangle - \mathrm{i}\omega\langle\tau I_\alpha\rangle}{\left\langle 1-\left(1-\frac{Z\tau}{\tau_z}\right)I_\alpha\right\rangle}\right\}\times$$

$$\times\left\{1-\frac{q^2L^2}{A}\left(1-\sum_\alpha\left\langle\frac{\tau}{\tau_z}I_\alpha\right\rangle\left\langle 1-\left(1-\frac{Z\tau}{\tau_z}\right)I_\alpha\right\rangle^{-1}\right)\right\}^{-1}, \tag{42}$$

where

$$v_\alpha = V_\alpha - V, \quad V = \frac{1}{Z}\sum_{\alpha=1}^{Z}V_\alpha \tag{43}$$

and

$$L^2 = \frac{1}{\chi^2} = \frac{\varepsilon k_0 T}{4\pi e^2 N_{0\alpha}}$$

is the square of the Debye screening length of the nondegerate electron gas of the concentration $N_{0\alpha} = N_0/Z$. The quantity A is

$$A = \sum_{\alpha=1}^{Z}A_\alpha = \sum_{\alpha=1}^{Z}\frac{\langle 1-(1+\mathrm{i}\omega\tau)I_\alpha\rangle}{1-\left\langle\left(1-\frac{Z\tau}{\tau_z}\right)I_\alpha\right\rangle}. \tag{44}$$

From the equation of motion (40) the absorption coefficient of the sound amplitude can be now determined as an imaginary part of the wave vector $\boldsymbol{q}$ (assuming that $q \gg \alpha$) and the effective elastic constants can be determined as its real part

$$2\lambda\alpha = \mathrm{Im}\left(\sum_{\alpha=1}^{Z}\frac{\mathrm{i}}{u}\Theta_\alpha N_{\alpha 1}\right) \tag{45}$$

$$\lambda' = \frac{\varrho\omega^2}{q^2} = \lambda + \frac{1}{q}\mathrm{Re}\left(\sum_{\alpha=1}^{Z}\frac{\mathrm{i}}{u}\Theta_\alpha N_{\alpha 1}\right). \tag{46}$$

We shall not write down the expression for α a λ', which could be obtained by substituting for $N_{\alpha 1}$, but we shall discuss the expressions (45) and (46) for n-Ge and n-Si in the following section.

IV. SPECIAL CASES

In this section we shall apply the relations (47) and (48) to n-Ge and n-Si. In this case, the deformation potential (3) is of the form

$$V_\alpha = (\Theta_\alpha\delta_{ij} + \Theta_u a_i^{(\alpha)}a_j^{(\alpha)})u_{ij}, \tag{49}$$

where $a_i^{(\alpha)}$ are the components of the unit vector from the centre of the Brillouin zone to the centre of the α-valley, Θ_d is the so-called dilatation constant and Θ_u characterizes the narrowing in the transverse direction. The relation (49) with

$$\boldsymbol{u} = \boldsymbol{e}u = \boldsymbol{e}u_0 e^{i(\omega t - \boldsymbol{q}.\boldsymbol{r})}$$

can be rewritten in the form

$$V_\alpha = \Theta_\alpha(-iqu),$$

where

$$\Theta_\alpha = (\Theta_d + \tfrac{1}{3}\Theta_u)(\hat{\boldsymbol{q}} \cdot \boldsymbol{e}) + \Theta_u \varphi_\alpha$$

$$\varphi_\alpha = (\hat{\boldsymbol{q}} \cdot \boldsymbol{a}_\alpha)(\boldsymbol{a}_\alpha \cdot \boldsymbol{e}) - \tfrac{1}{3}(\hat{\boldsymbol{q}} \cdot \boldsymbol{e}). \tag{50}$$

$\boldsymbol{e}$ is the unit polarization vector of the sound and $\hat{\boldsymbol{q}} = \boldsymbol{q}/|\boldsymbol{q}|$ is the unit vector along the sound wave propagation direction. In n-Ge the valleys lie along the body diagonal ($Z = 4$)

$$\boldsymbol{a}_1 = \frac{1}{\sqrt{3}}(1, 1, 1); \quad \boldsymbol{a}_2 = \frac{1}{\sqrt{3}}(-1, 1, 1); \quad \boldsymbol{a}_3 = \frac{1}{\sqrt{3}}(1, -1, 1);$$

$$\boldsymbol{a}_4 = \frac{1}{\sqrt{3}}(1, 1, -1); \quad (\boldsymbol{a}^{(\alpha)} \equiv \boldsymbol{a}_\alpha)$$

and in n-Si they lie along the coordinates ($Z = 6$)

$$\boldsymbol{a}_1 = (\pm 1, 0, 0); \quad \boldsymbol{a}_2 = (0, \pm 1, 0); \quad \boldsymbol{a}_3 = (0, 0, \pm 1).$$

Since $1/Z \sum_{\alpha=1}^{Z} \boldsymbol{a}_\alpha \boldsymbol{a}_\alpha = \frac{1}{3}\overset{\rightrightarrows}{1}$ ($\overset{\rightrightarrows}{1}$ is the unit tenzor), we obtain $\sum_{\alpha=1}^{Z} \varphi_\alpha = 0$. Consequently, for v_α defined by the relation (43), we have

$$v_\alpha = -iqu\Theta_u\varphi_\alpha. \tag{51}$$

The values φ_α for the principal direction of n-Ge and n-Si are given in Table 1. Table 1 gives also the elastic constants characterizing the acoustic wave propagasing in an particular direction, and with a given polarization. For the reciprocal mass tensor we have

$$\mathbf{M}_\alpha^{-1} = \frac{1}{m_t}\overset{\rightrightarrows}{1} - \left(\frac{1}{m_t} - \frac{1}{m_l}\right)\boldsymbol{a}_\alpha\boldsymbol{a}_\alpha \tag{52}$$

$$\frac{1}{Z}\sum_{\alpha=1}^{Z} \mathbf{M}_\alpha^{-1} = \frac{1}{m}\overset{\rightrightarrows}{1},$$

Table 1

$\hat{\boldsymbol{q}}$	$\boldsymbol{e}$	n-Ge				n-Si			λ
		φ_1	φ_2	φ_3	φ_4	φ_1	φ_2	φ_3	
$(1, 0, 0)$	$(0, 1, 0)$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	0	c_{44}
$\frac{1}{\sqrt{2}}(1,1,0)$	$(0, 0, 1)$	$\frac{\sqrt{2}}{3}$	0	0	$-\frac{\sqrt{2}}{3}$	0	0	0	c_{44}
	$\frac{1}{\sqrt{2}}(1, -1, 0)$	0	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}(c_{11} - c_{12})$
$\frac{1}{\sqrt{3}}(1,1,1)$	$\frac{1}{\sqrt{2}}(1, -1, 0)$	0	$-\frac{\sqrt{6}}{9}$	$\frac{\sqrt{6}}{9}$	0	$\frac{1}{\sqrt{6}}$	$-\frac{1}{\sqrt{6}}$	0	$\frac{1}{3}(c_{11} - c_{12} + c_{44})$
$(1, 0, 0)$		0	0	0	0	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	c_{11}
$\frac{1}{\sqrt{2}}(1, 1, 0)$		$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$	$\frac{1}{2}(c_{11} + c_{12} + 2c_{44})$
$\frac{1}{\sqrt{3}}(1, 1, 1)$		$\frac{2}{3}$	$-\frac{2}{9}$	$-\frac{2}{9}$	$-\frac{2}{9}$	0	0	0	$\frac{1}{3}(c_{11} + 2c_{12} + 4c_{44})$

where

$$\frac{1}{m} = \frac{1}{3}\left(\frac{1}{m_l} + \frac{2}{m_t}\right) = \frac{1}{m_t}\frac{2 + a}{3}, \quad a = \frac{m_t}{m_l}. \tag{53}$$

Further we shall use the following denotations

$$m\mathbf{M}_\alpha^{-1} \cdot \hat{\boldsymbol{q}} = \boldsymbol{r}_\alpha, \quad \boldsymbol{q} \cdot \boldsymbol{r}_\alpha = x_\alpha, \quad e_\alpha = \sqrt{\frac{2Ex_\alpha}{m}}\,\tau, \tag{54}$$

where for $\boldsymbol{r}_\alpha$ and x_α, owing to (52), it follows

$$\boldsymbol{r}_\alpha = \frac{3}{2 + a}[\hat{\boldsymbol{q}} - (1 - a)(\hat{\boldsymbol{q}} \cdot \boldsymbol{a}_\alpha)\boldsymbol{a}_\alpha], \quad x_\alpha = \frac{3}{2 + a}[1 - (1 - a)(\hat{\boldsymbol{q}} \cdot \boldsymbol{a}_\alpha)^2].$$

The values of the quantities $\boldsymbol{r}_\alpha$ and x_α are tabulated in Mertsching's paper [9].

For spherically symmetric surfaces, where $a = 1$ and $x_\alpha = 1$, l_α equals $\tau \sqrt{2E/m}$, that is l_α is the mean free path of electrons.

Since the sound velocity w is much lower than the mean thermal velocity of electrons (for temperatures roughly higher than 1 °K) we have

$$\omega\tau \ll q l_\alpha;$$

then I_α given by (34) can be written as

$$I_\alpha \approx \frac{\operatorname{arctg} q l_\alpha}{q l_\alpha} - \frac{i\omega\tau}{1 + q^2 l_\alpha^2}. \tag{55}$$

We shall also make use of the fact that the intervalley transitions are less frequent than the intervalley transitions

$$\frac{Z\tau}{\tau_z} < 1.$$

Now we introduce the effective relaxation time

$$\frac{1}{\tau_\alpha} = \frac{1}{\langle\tau\rangle}\left\langle \frac{Z\tau}{\tau_z} + \frac{q^2 l_\alpha^2}{3} \right\rangle. \tag{56}$$

The reason why τ_α is called the effective relaxation time will be seen from the following expressions, which will be derived for the two limiting cases: a) $q l_\alpha \ll 1$, $qL \ll 1$ and b) $q l_\alpha \gg 1$.

a) $q l_\alpha \ll 1$, $qL \ll 1$. In this limit we obtain

$$2\lambda\alpha = \operatorname{Im} B \tag{57}$$

$$\lambda' - \lambda = \frac{1}{q}\operatorname{Re} B,$$

where

$$B = -\frac{N_0}{Z k_0 T}\Theta_u q \left\{ \sum_\alpha \Theta_\alpha \varphi_\alpha \left(1 - \frac{i\omega\tau_\alpha}{1 + i\omega\tau_\alpha'}\right) + \right.$$

$$\left. + i \sum_\alpha \frac{\omega\tau_\alpha\varphi_\alpha}{1 + i\omega\tau_\alpha'} \frac{\sum_\alpha \Theta_\alpha \langle l_\alpha^2\rangle \frac{\tau_\alpha}{1 + i\omega\tau_\alpha'}}{\sum_\alpha \frac{\langle l_\alpha^2\rangle \tau_\alpha}{1 + i\omega\tau_\alpha'}} \right\}, \tag{58}$$

and

$$\tau_\alpha' = \frac{\tau_\alpha}{\langle\tau\rangle}\left\langle \tau - \frac{Z\tau^2}{\tau_z} \right\rangle.$$

However, the expression (58) will be much simpler if we assume that the intravalley transition are much more frequent than the intervalley transitions, that is

$$\frac{Z\tau}{\tau_z} \ll 1, \tag{59}$$

so that $\tau_\alpha' \approx \tau_\alpha$ and if we express α and λ' for the transverse wave. For the transverse wave

$$\sum_\alpha \frac{\varphi_\alpha}{1 + i\omega\tau_\alpha} = 0. \tag{60}$$

Therefore, for the absorption coefficient of the transverse wave we obtain

$$\alpha = \frac{N_0\omega}{Z k_0 T \varrho w^3}\Theta_u^2 \sum_\alpha \frac{\varphi_\alpha^2 \omega\tau_\alpha}{1 + (\omega\tau_\alpha)^2}, \tag{61}$$

having used the approximate relations $\omega \approx wq$ and $\lambda \approx \varrho w^2$, w is the velocity of the corresponding acoustic wave. Similarly we obtain for the elastic constants

$$\lambda' - \lambda = -\frac{N_0}{Z k_0 T}\Theta_u^2 \sum_\alpha \frac{\varphi_\alpha^2}{1 + (\omega\tau_\alpha)^2}. \tag{62}$$

Now it can be seen from (61) and (62) why τ_α is called the effective relaxation time.

If we use the φ_α values from Table 1, we obtain for the change of the elastic constants $\Delta\lambda = \lambda' - \lambda$ from relation (58)

$\frac{1}{2}(\Delta c_{11} - \Delta c_{12}) = 0, \quad \Delta c_{11} = 0$ for n-Ge

and

$\Delta c_{44} = 0, \quad \frac{1}{3}(\Delta c_{11} + 2\Delta c_{12} + 4\Delta c_{44}) = 0$ for n-Si.

Consequently, in n-Ge only c_{44} changes and in n-Si both c_{11} and c_{12} change in such a manner that $\Delta c_{11} = -2\Delta c_{12}$. For Δc_{44} we obtain in the limit $\omega\tau_\alpha \ll 1$

$$\Delta c_{44} = -\frac{1}{9}\Theta_u^2 \frac{N_0}{k_0 T} \quad (n\text{-Ge}). \tag{63}$$

This result agrees with the one obtained by Keyes (quoted in [3]).

For Δc_{11} in the limit $\omega\tau_\alpha \ll 1$ we obtain

$$\Delta c_{11} = -\frac{2}{9}\Theta_u^2\frac{N_0}{k_0T} \qquad (n\text{-Si}). \tag{64}$$

This result is twice smaller than that derived by Mason [3].

b) $ql_\alpha \gg 1$. For the sound absorption coefficient and the changes of elastic constants in the case of strong screening of the interaction potentials by the conduction electrons, that is $qL \ll 1$, we obtain

$$\alpha = \sqrt{\frac{\pi m}{2}}\frac{N_0\omega\Theta_u}{Z(k_0T)^{3/2}\varrho w^2}\left[\sum_{\alpha=1}^{Z}\Theta_\alpha\frac{\varphi_\alpha}{\sqrt{x_\alpha}} + \left(\Theta_d + \frac{1}{3}\Theta_u\right)(\hat{\mathbf{q}}\cdot\mathbf{e})\sum_{\alpha=1}^{Z}\frac{\varphi_\alpha}{\sqrt{x_\alpha}}\right] \tag{65}$$

$$\lambda' - \lambda = -\frac{N_0\Theta_u^2}{Zk_0T}\sum_\alpha\varphi_\alpha^2. \tag{66}$$

If the screening of the interaction potentials by the conduction electrons can be neglected, that is if $qL \gg 1$, we obtain the expressions for the sound absorption coefficient and the changes of elastic constants

$$\alpha = \sqrt{\frac{\pi m}{2}}\frac{N_0\omega}{Z(k_0T)^{3/2}\varrho w^2}\sum_{\alpha=1}^{Z}\frac{\Theta_\alpha^2}{\sqrt{x_\alpha}} \tag{67}$$

$$\lambda' - \lambda = -\frac{N_0}{Zk_0T}\sum_{\alpha=1}^{Z}\Theta_\alpha^2. \tag{68}$$

The expression (66) gives the same change of elastic constants as we obtained in the case a) for $\omega\tau_\alpha \ll 1$. However, if we do not consider screening the results are different. So for n-Ge in this case ($qL \gg 1$) we obtain

$$\Delta c_{11} = \Delta c_{12} = -(\Theta_d + \tfrac{1}{3}\Theta_u)^2\frac{N_0}{k_0T} \qquad (n\text{-Ge},\quad qL \gg 1) \tag{69}$$

and Δc_{44} (n-Ge) is the same as in the case a), the formula (63).

For n-Si we obtain

$$\Delta c_{44} = 0; \quad \Delta c_{12} = \tfrac{1}{3}\Theta_u^2\frac{N_0}{k_0T} + \Delta c_{11}; \tag{70}$$

$$\Delta c_{11} = -[\tfrac{2}{9}\Theta_u^2 + (\Theta_d + \tfrac{1}{3}\Theta_u)^2]\frac{N_0}{k_0T} \qquad (n\text{-Si},\ qL \gg 1).$$

V. CONCLUSION

Starting from the wave equation (13) in which the interaction of the displacement with the electrons is introduced, which latter are described in the nonequilibrium state by the Boltzmann transport equation (29), we have derived the sound absorption coefficient and the changes of elastic constants. The general approach given in sections II and III was then applied to the nondegenerate n-Ge and n-Si. We derived the absorption coefficient and the changes of elastic constants due to the interaction of the acoustic wave with the electrons in the two limiting cases $ql_\alpha \ll 1$ and $ql_\alpha \gg 1$. In the former case we considered the strong screening ($qL \ll 1$) but in the latter case we distinguished two possibilities, the strong screening and the weak or no screening ($qL \gg 1$) of the interaction potentials by the conduction electrons. We mainly concentrated on the derivation of the changes of elastic constants when considering both the intervalley and the intravalley transitions. Our result (in the case of strong screening) for the change of the c_{44} elastic constant of n-Ge agrees with that derived by Keyes (see [3]), whereas for the change of c_{11} of n-Si, which changes together with c_{12} in such a way that $\Delta c_{11} = -2\Delta c_{12}$, we obtained the result which is twice smaller than that derived by Mason [3]. The authors of paper [4] suggested that the intervalley and intravalley transitions of electrons would contribute additively to the changes of elastic constants. We considered both these transitions but we obtained no additivity of these effects.

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