Fyz. čas. SAV 21 (1971), No. 2-3

THE USE OF THERMOELASTIC EFFECTS FOR THE MEASUREMENT OF THE ULTRASOUND ABSORPTION COEFFICIENT

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The aim of the paper is to discuss the method of measurement of the coefficient of ultrasound absorption in solids using the heat energy developing due to the absorption during the ultrasound propagation. Using as basis the equation of the heat-flow, the author derives a relation describing the connection between the value of the absorption coefficient and the position of the temperature maximum found along the investigated specimen during the measurement. The experimental measurements were made for unsoftened PVC (novodur) and for texgumoid, using the ultrasound frequency of 16.8 kHz. The studied specimens were of cylindrical shape.

I. INTRODUCTION

If ultrasound propagates in a medium with non-zero absorption, a transformation of the acoustic energy into thermal energy takes place. The latter is used mainly to raise the temperature of the medium with the ultrasound. The amount of this heat energy and hence the increase in temperature are given by the properties of the medium, namely by its absorption coefficient.

The ultrasound heat effects may be used for the measurement of basic acoustic quantities. We mention as an example temperature sensitive sound detectors [1] inserted in the form of thin heated wires in an acoustic field in liquids and gases, where they change their heat resistance due to the influence of the heat which allows the measurement of ultrasound propagation velocity.

In the paper the author presents a method of measurement of the absorption coefficient of solids based on the thermoelastic effect. If the tested specimen is given a suitable shape (e. g. the shape of a cylinder) and if certain boundary conditions are satisfied, the value of the absorption coefficient is given by the position of the temperature maximum on the surface of the studied specimen measured in the direction of the ultrasound propagation [2]. The author presents the deduction of the relation of the absorption coefficient and of the position of the maximum of the temperature distribution measured along the specimen as well as the results obtained by the solution of this relation. These theoretical results have been used for the measurement of the absorption coefficient of two dielectric materials at the frequency of 16.8 kHz.

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II. THEORETICAL PART

To establish temperature distribution the differential equation of heat-flow is to be used as a point of departure. With regard to the conditions of the experiment, the choice of the specimens is restricted to the shape of a homogeneous cylinder having the length l and the diameter $d \ll l$. This cylinder is heat-isolated from its environment and its axis of symmetry coincides with the coordinate axis x. Further, it is assumed, that the temperature $T(x, t)$ may be taken as equal at all points of the bar cross-section, with the same coordinate x and at the same time t.

If k is the coefficient of thermal conductivity, ϱ the density and γ the specific heat of the cylinder, the equation of heat-flow may be written in the following form [3]

$$\frac{\partial T}{\partial t} = a^2 \frac{\partial^2 T}{\partial x^2} + f(x, t), \qquad (1)$$

where $a^2 = k/\varrho\gamma$, $f(x, t) = F(x, t)/\varrho\gamma$, the function $F(x, t)$ being called the density of heat sources determining the amount of heat generated in a unit volume of the cylinder per unit of time.

Let the initial and boundary conditions be given by the equations

$$\begin{aligned} T(x, 0) &= 0, \\ T(0, t) &= \mu_1(t), \\ T(1, t) &= \mu_2(t). \end{aligned} \qquad (2)$$

Using the conditions of (2) a unique solution of Eq. (1) may be found in the form [3]

$$T(x, t) = \int_0^t \int_0^l \left\{ \frac{2}{l} \sum_{n=1}^{\infty} e^{-(n\pi a/l)^2 (t-\tau)} \sin \frac{n\pi x}{l} \sin \frac{n\pi\xi}{l} \right\} f(\xi, \tau) \, d\xi d\tau + \\ + \mu_1(t) + \frac{x}{l} [\mu_2(t) - \mu_1(t)], \qquad (3)$$

where

$$\bar{f}(x, t) = f(x, t) - \left(\frac{\partial U}{\partial t} - a^2 \frac{\partial^2 U}{\partial x^2}\right),$$

$$U(x, t) = \mu_1(t) + \frac{x}{l}\,[\mu_2(t) - \mu_1(t)].$$

Eq. (3) describes the temperature distribution all over the length of the studied specimen, if the form of the function $\bar{f}(x, t)$ is known. The function $f(x, t)$ may be found by the energetic analysis of phenomena taking place in the specimen in the course of the measurement. The form of the functions $\mu_1(t)$ and $\mu_2(t)$ depends on the conditions existing on the boundaries of the sample.

Let the simplest case be given by the conditions $\mu_1(t) \equiv \mu_2(t) \equiv 0$. In this case $T(x, t)$ is given by the function $f(x, t)$ alone. Let its form be found for the case of a plane ultrasound wave propagating along the sample. If the sample is characterized by a non-zero absorption, then the increasing coordinate x is accompanied by the decreasing intensity of the wave and the absorbed energy turns into heat energy. Let α be the intensity coefficient of the absorption. The absorption relationship for the intensity I of the wave may be written in the form

$$I = I_0 e^{-\alpha x},$$

where I_0 is the ultrasound intensity at the beginning of the sample.

In an elementary volume $S\Delta x$, where S is the cross-section of the cylinder, the absorbed energy is

$$\Delta W = S\,[I_0 e^{-\alpha x} - I_0 e^{-\alpha(x+\Delta x)}],$$

which may be written in the first approximation in the form

$$\Delta W = S I_0 e^{-\alpha x} \alpha \Delta x.$$

The density of the heat source $F(\xi, \tau)$ is given by the relation

$$F(\xi) = \frac{\Delta W}{S \Delta \xi} = I_0 \alpha e^{-\alpha \xi},$$

hence

$$f(\xi) = \frac{I_0 \alpha}{\gamma \varrho} e^{-\alpha \xi} = i_0 e^{-\alpha \xi}.$$

On the basis of the preceding relations the function $T(x, t)$ may be written in the form

$$T(x, t) = \frac{2 i_0}{\pi a^2} \sum_{n=1}^{\infty} \frac{1 + (-1)^{n+1} e^{-\alpha l}}{n} \left[1 - e^{-(n\pi a/l)^2 t}\right] \frac{\sin \frac{n\pi x}{l}}{\alpha^2 + \left(\frac{n\pi}{l}\right)^2}.$$

Let us consider the case of the steady state, which from the mathematical point of view corresponds to the condition $t \to \infty$. If this condition is adopted for $T(x, t)$, then

$$T(x, t) = \frac{2 i_0}{\pi a^2} \sum_{n=1} \frac{1 + (-1)^{n+1} e^{-\alpha l}}{n} \frac{\sin \frac{n\pi x}{l}}{\alpha^2 + \left(\frac{n\pi}{l}\right)^2}. \tag{4}$$

The numerical calculation of (4) was performed by the digital computer URAL-2. The results applying to the sample length $l = 10$ cm are plotted in Fig. 1, where the shift of the temperature maximum towards the beginning of the sample is visible, for a growing coefficient of absorption. Fig. 2 gives a chart allowing to find the value of the absorption coefficient immediately from the position of the temperature maximum.

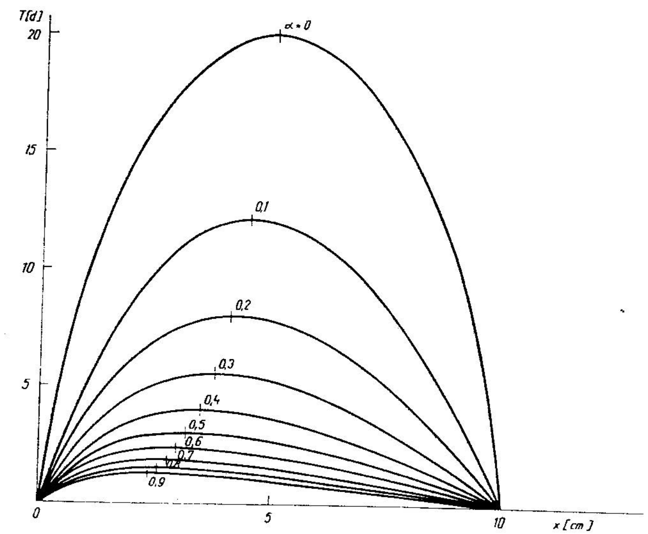

Fig. 1. Theoretical distribution of temperature $T(x)$ for the intensity absorption coefficient $\alpha = 0;\ 0.1;\ 0.2;\ \ldots$

Curves shown in Figs. 1 and 2 correspond to zero boundary conditions only. If these conditions cannot be satisfied, it is necessary to make sure that the functions $\mu_1(t)$, $\mu_2(t)$ are at least of the same character in all the studied samples. In that case the shift of the temperature maximum accompanying an increase of the absorption coefficient is kept unchanged even if the numerical values do not correspond fully to the results shown in Figs. 1 and 2. In such a case a sample with a known absorption coefficient should be used as a normal.

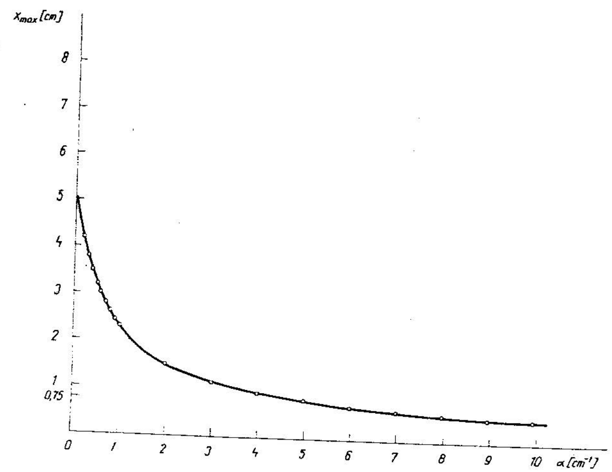

Fig. 2. Relation between a temperature maximum position x_{max} and the intensity absorption coefficient α.

III. EXPERIMENTAL PART

The measurement of the absorption coefficient was carried out according to the block-scheme shown in Fig. 3. The studied material V was given a cylin-

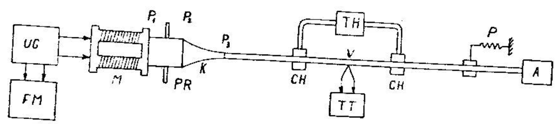

Fig. 3. Block-diagram of the experimental apparatus for the measurement of the absorption coefficient. UG – ultrasonic generator; FM – frequency meter; M – magnetostrictive transducer; PR – half-wave resonator; K – horn; P – spring; TT – thermistor thermometer; P_1, P_2, P_3 – acoustic couplers; V – sample; A – absorber; CH – cooler; TH – thermostat.

drical shape with a cross-section diameter of 5 mm and with the length of several tens of cm. The absorber A at the end of the sample prevented the formation of standing waves. The junctions P_1, P_2 were obtained by hard soldering of each of the parts. The junction P_3 was secured by a spring. The boundary conditions were kept by coolers CH in which water of a constant temperature circulated controlled by a thermostat. The coolers defined a length of 10 cm on the sample, that being the length for which the computation of the series (4) had been carried out.

The choice of the material was influenced by the requirement of a sufficiently high absorption in agreement with the theoretical results shown in Fig. 2. The choice of a particular kind of material was irrelevant, since the primary aim of the investigation was a verification of theoretical results. There are some insulating substances distinguished by their relatively high absorption. Among the materials unsoftened PVC and texgumoid were available in the desired bar shape.

The resonance frequency of the magnetostrictive transducer M was set to 16.8 kHz and the temperature distribution was measured along the measured sample in the steady state. The results are shown in Fig. 4. Both curves show distinct peaks. In the case of the novodur bar the temperature maximum developed at the distance of 1.8 cm, in the case of the texgumoid bar at the distance of 2.6 cm from the beginning of the sample.

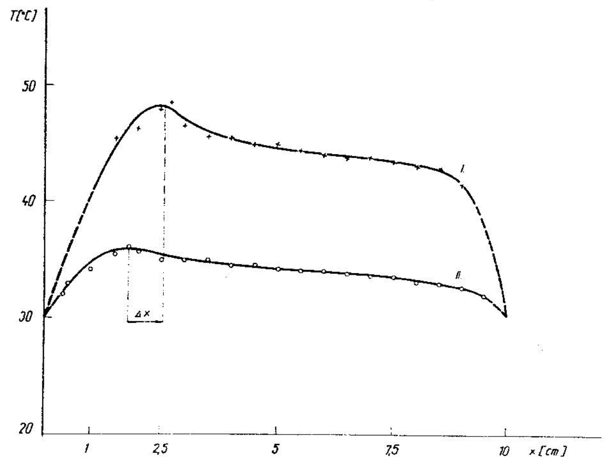

Fig. 4. Experimental results. I – texgumoid, II – novodur.

The position of the temperature peaks corresponds to the amplitude absorption coefficients of 0.77 cm^{-1} and 0.40 cm^{-1}. The amplitude of the acoustic deviation at the input of the sample was about 10^{-6} m. The results of the measurements were reproducible. The size of the temperature pick-up (a bead termistor) allowed to locate the temperature maximum with an accuracy of 10^{-3} m. As the plot in Fig. 2 is not linear, it is obvious that the increase of the accuracy of the absorption coefficient measurement is inversely proportional to the absorption coefficient value.

IV. CONCLUSION

Note that not all conditions assumed at the deduction of the relation (4) were fulfilled. The studied sample was not heat isolated from the environment. With regard to the fact that the aim of the investigation was to verity qualitatively theoretical results and that the temperature difference between the sample and its environment is small, this inaccuracy is justifiable.

The measured temperature plots indicate the existence of relatively high absorption coefficients of the measured samples. Let two factors be mentioned, which, in the opinion of the author, determine decisively this fact.

In the first place it is the difficulty of keeping the boundary conditions of the experiment, which are a necessary condition for the solution (4). A failure to satisfy the boundary conditions in the course of the measurements results in a distortion of the curves shown in Fig. 1 as well as in the shift of the temperature peak.

More relevant, however, is the fact that a measurable effect may be obtained only if ultrasound waves are used whose amplitude cannot be considered as small. The amplitude of deviation measured during the measurements reported in this paper amounted to about 10^{-6} m, which in the case of a plane wave corresponds to an intensity of the wave of the order of 1 W cm^{-2} (for the frequency of 16.8 kHz). It is well known, however, from [4] and [5] that if the intensity is so high, the increase of the absorption coefficient is so great that the reported experimental results are thereby fully justified.

The results of the measurements have confirmed the suitability of the suggested method of measurement of the absorption coefficient in the range of nonlinear waves. For measurement of the absorption coefficient of different samples it is necessary to keep the same boundary conditions as well as the same ultrasound intensity, because if these conditions are not satisfied, a considerable distortion of the relative results might take place. The suggested method is suitable mainly for high absorption materials.

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Received November 10th, 1970