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THE STUDY OF THE FREQUENCY SPECTRUM OF FERROCERAMIC TRANSDUCERS

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In the first part of the presented paper some experiments are described, the aim of which was to determine the modes of vibrations of the main resonances in a spectrum of a circular ferroceramic transducer, poled perpendicularly to a basic plane and driven by an electric field on the electro-descovering all the planes of the base. Besides measuring the frequency characteristics, nodal patterns were visualized from which the vibrating modes were estimated.

The second part of this paper deals with the question of how the first experience with materials which have a high electromechanical coupling coefficient forces us to abandon the classical solutions obtained from two-dimensional equations, which appeared valid in the case of materials with a low coupling.

I. INTRODUCTION

Poled ferroelectric ceramics are very often used now as a material for the production of piezoelectric transducers. Especially ceramics made on the base of solid solutions of $PbZrO_3 - PbTiO_3$ have suitable properties and replaced at present barium titanate $BaTiO_3$ having a higher temperature of the Curie point.

The advantage of piezoelectric ceramics is especially their high electromechanical coupling coefficient, which can be even changed by a suitable technology of production. The ceramics make the production of large size and shape transducers possible, which is not the case with crystalline materials. Using ceramics makes it possible to put piezoelectric transducers to new uses and will be a contribution to a number of applications in which other piezoelectric materials or other types of transducers such as the magnetostrictive ones have been used.

The research on the technology of ceramics is devoted to the improving of their properties and their reproducibility in serial production. Great advance has been already made in this field by major world producers. It enables us to begin a detailed research on those properties which are important for the application. Because of a big range of production parameters, the number of experimental facts is still not adequate. The theory for this type of transducers has not been developed either as it is impossible to use methods which are normally used for materilas with a low electromechanical coupling.

A necessary assumption in all problems dealing with vibrations of transducers is a knowledge of elastic, electrical and piezoelectrical coefficients which form the electroelastic matrix, or at least the knowledge of electromechanical coupling coefficients for different modes of vibrations.

The resonance methods for determining those coefficients are based on measuring the resonance and antiresonance frequencies of different modes on samples of various shapes. The necessary condition for determining these frequencies correctly is a detailed knowledge of the frequency spectrum because we must know to which mode of which type of vibrations each of the resonances corresponds. This is a difficult problem when there is a great number of resonances.

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II. FREQUENCY CHARACTERISTICS MEASUREMENT

For the purpose of analysing resonances in a frequency spectrum, the frequency dependence on the transducer current with a constant level of the driving signal 1 V was measured. On the basis of a recommendation in the IRE Standard 1961 [1] the scheme showed in Fig. 1 was used.

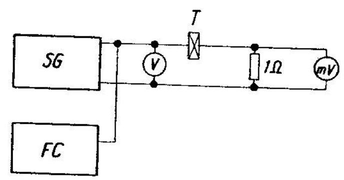


Fig. 1. The measurement of transducer characteristics. SG — signal generator TT 0250, FC — frequency counter TESLA BM 445 E, T — transducer, mV — milivoltmeter TESLA BM 348.

The frequency characteristics were measured point by point for a single driving signal frequency. Special attention was paid to finding the maximum and minimum values of current and determining the corresponding frequencies, which were in further considerations taken for resonance or antiresonance frequencies. The determining of minima was more difficult because they are not so sharply expressed as the maxima. Only the part of spectrum determined by the first three radial modes and a fundamental thickness dilatational mode was measured.

A great number of transducers of our and foreign production have been tested. The examples of characteristics are shown in Figs. 2. and 3. All the transducers had below the frequency of the thickness-dilatational resonance a number of isolated resonances, the frequency distance of which decreased with an increasing frequency. The amplitude of the current was in some cases of resonances of some transducers comparable with the resonance near the thickness resonance, in most cases however it was less and was 10—60 % of the maximum amplitude. In the region of the theoretical thickness-dilatational resonance there exist a number of narrow, very strong resonances and to one of them corresponds a total maximum of current.

III. VISUALIZATION OF NODAL PATTERNS

The nodal patterns were visualized by means of powdered materials which deposit on the vibrating transducer alog the nodal curves or along the curves which connect the points with the minimum amplitudes of strains. The choice of the powder depends on the state of the surface of the transducer and the mode of vibrations. Lycopodia, polishing powders and epoxy resins have been used.

The transducers were lying freely on a layer of foam rubber, so that in this way only the lower base was weighed down by the weight of the transducer.

The maximum open circuit voltage given by the applied generator was 300 V but the driven signal had in the case of the maximum load even higher overtones. The formation of some patterns was very difficult especially in the case of adjacent resonances.

During repeated measurements striking changes of the nodal patterns were observed. They had the following reasons:

1. The frequency of the driving generator changed a little. It is obvious from this fact that in some cases, especially near the thickness resonance a relatively small change of frequency ($5 \times 10^{-4} f$) can result in a striking change of a nodal pattern. See Fig. 4, (f = 192.100 c/s and f = 192.200 c/s).
2. The nodal pattern, however, has been changed even in the case when the frequency tolerance was in the region of $10^{-5} f$ of the driving frequency. It could be accouted for by the change of temperature of the sample which caused the change of the velocity of propagation of elastic waves and in this way a shift of the resonance frequency.
3. Some changes of the nodal pattern were caused by the change of the driving voltage. It could be caused by the following reasons:
   a) The non-linearity of the properties of ferroceramics.
   b) An increasing influence of the coupled vibrations in the case of a higher driving signal.

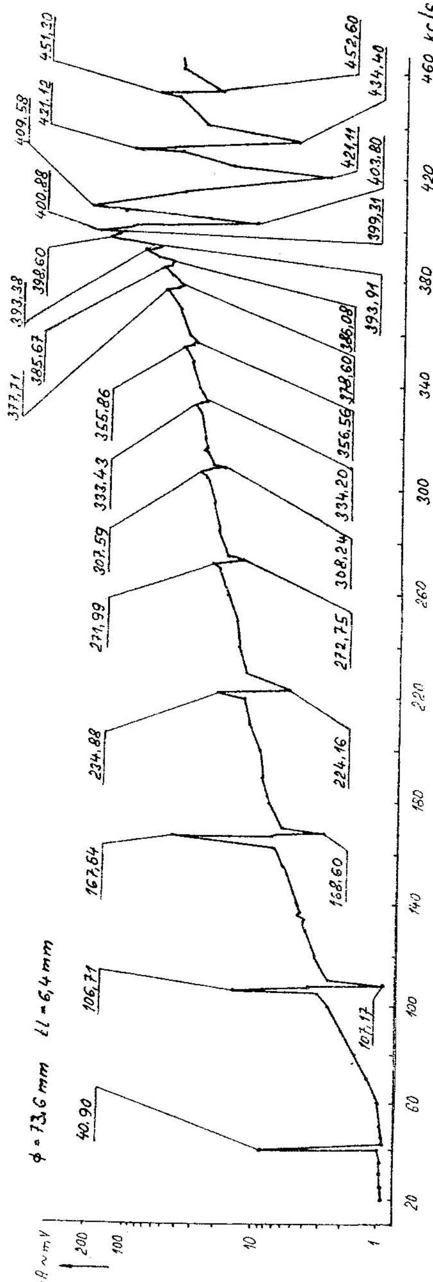


Fig. 2. The characteristic of transducer No. 2.

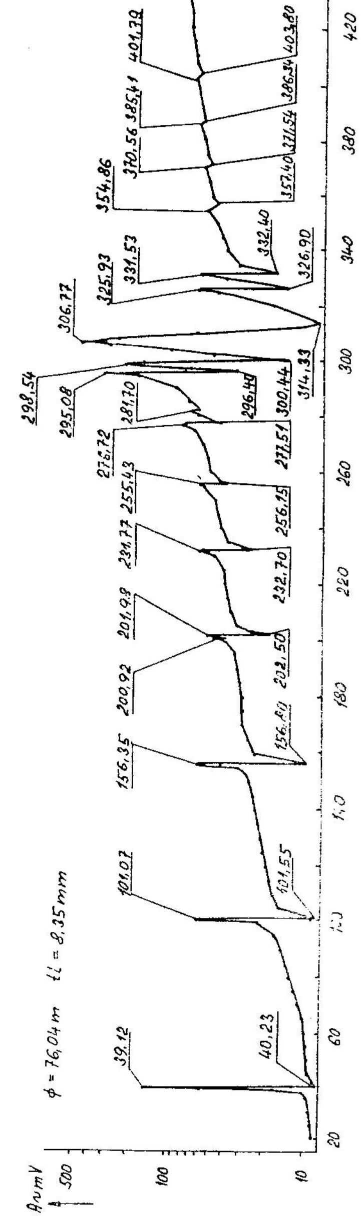


Fig. 3. The characteristic of transducer No. 3.

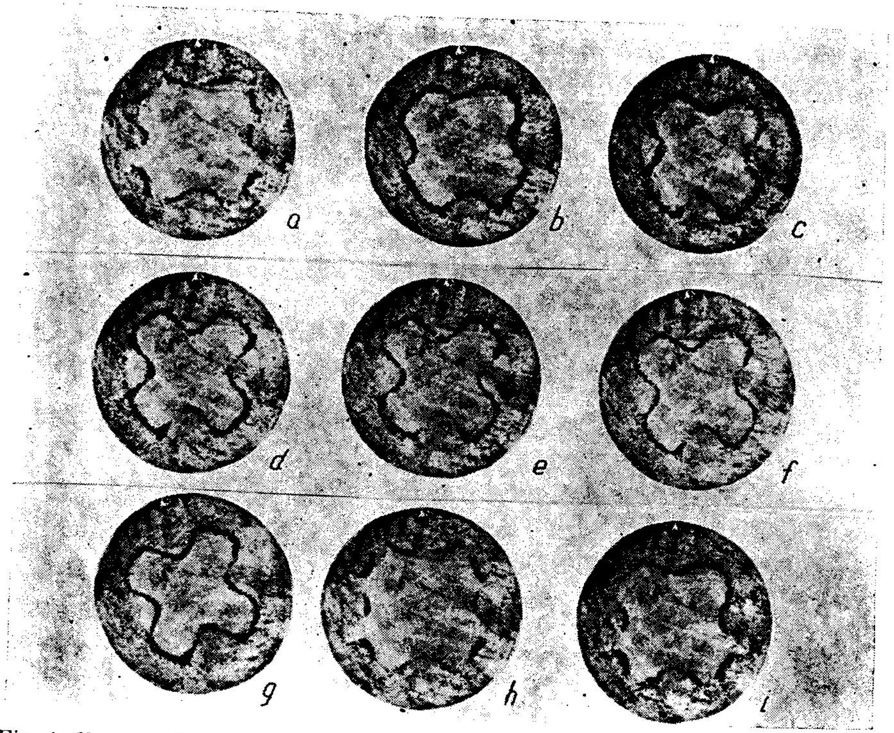


Fig. 4. Changes of nodal patterns (transducer ⌀ 75.0 mm, th. 11.2 mm). a. 191422 c/s, b. 191600 c/s, c. 191800 c/s, d. 191870 c/s, e. 191900 c/s, f. 192000 c/s, g. 192100 c/s, h. 192200 c/s, i. 192300 c/s.

c) The harmonic content of the driving signal.
These questions will be studied later.

IV. EXPERIMENTAL RESULTS

1. Radial vibrations

To illustrate the presence of the radial modes in the spectrum let us use the nodal patterns of three transducers: The transducers No 1, No 2 and No 3 had the dimensions ⌀ 75 mm, ⌀ 73.6 mm, ⌀ 76 mm and the thicknesses of 3.1 mm, 6.24 mm and 8.35 mm, respectively.

The nodal patterns observed in the case of the lowest resonance frequency are shown in Fig. 5. In case of all the three transducers as well as in a number of others of somewhat different sizes the first resonance corresponded to the

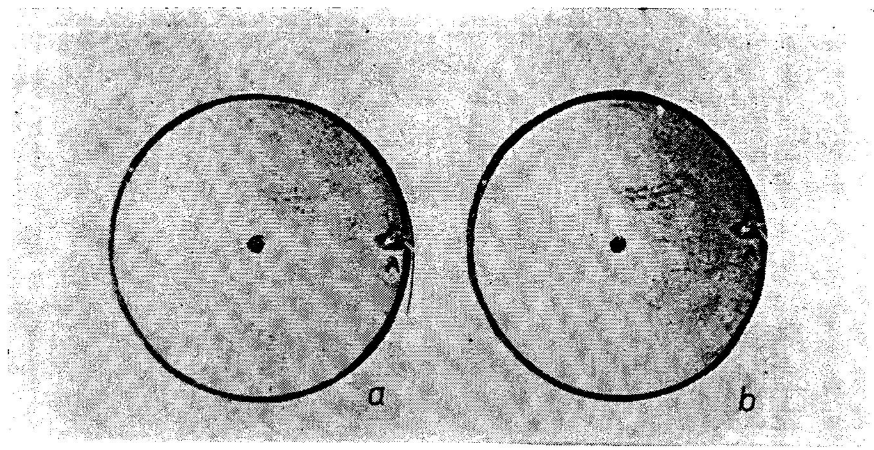


Fig. 5. The first radial modes (transducer No. 3). a. f_r = 39123 c/s, b. f_a = 40186 c/s.

lowest radial mode, which has one nodal point in the centre of the transducer.

Fig. 6 shows nodal patterns corresponding to the second pronounced maximum of the current characteristic. It is the radial mode.

The nodal pattern corresponding to the third maximum of the current characteristic is shown in Fig. 7. It is the third radial mode influenced by one of the coupling modes. It can be seen from the fact that the nodal circles are deformed. In some cases the existence of coupling modes results in doubling some of the circles.

The nodal patterns of the 4th and the 6th radial modes are shown in Fig. 8.

2. Vibrations near the thicknes-dilatational resonance

From the 2nd and 3rd pictures it is obvious that near the theoretical resonant frequency of thickness-dilatational modes several sharp resonances can be found and one of them is the absolute maximum of the characteristic. It can be said that no isolated thickness-dilatational resonance occurs in the case of transducers with a diameter to thickness ratio less than 15.

From the nodal patterns in Fig. 9 it can be concluded that some resonances near the theoretical thickness-dilatational resonance have a complicated shape, i. e. that there exists a coupled mode. In these cases the points on the bases do not vibrate with the same amplitude and phase as we expect in the case of thickness modes. That means that in this case the model which replaced the vibrating transducer by a source of so called ,,piston vibrations" cannot be used for problems concerning a "near field". Our experiments proved that

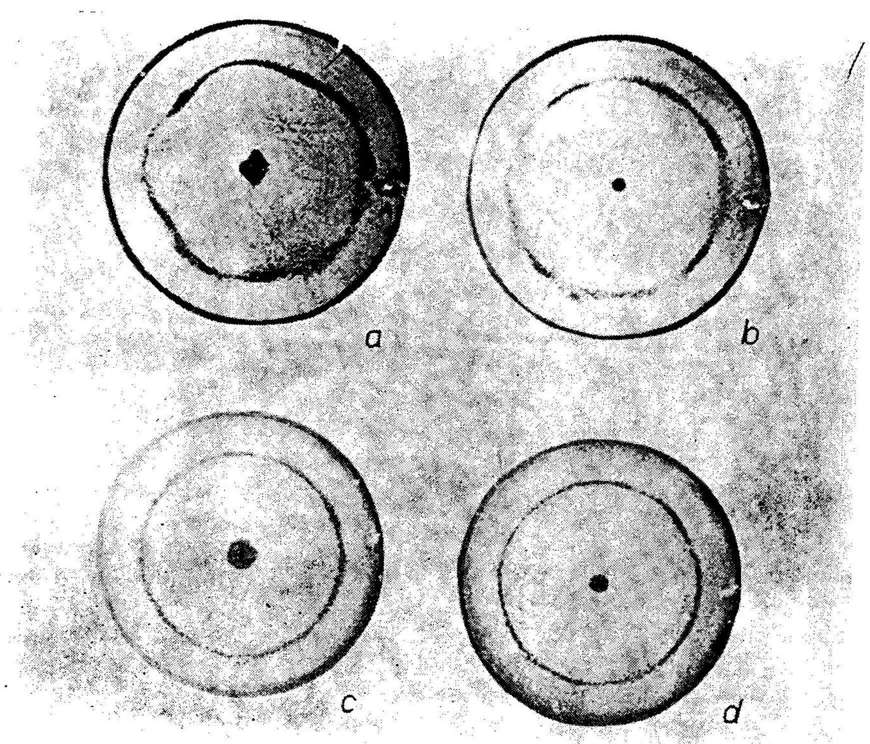


Fig. 6. The second radial modes. a. transducer No. 3, f_r = 100726 cls, b. f_a = 101264 c/s, c. transducer No. 2, f_r = 107519 c/s, d. f_a = 108256 c/s.

for the description of the behaviour of the transducer near the thickness resonance or for its design, the simple idea of the resonance occurring at the frequency at which the half-wavelength equals the thickness is not sufficient. It is necessary to take into account the effect of all dimensions and the piezo-electric effect on the final vibration.

3. Flexular vibrations

For the study of the ferroceramic plate the theory of isotropic plates can be applied. It can be done under the assumptions which are used in the case of studying the spectrum of vibrations. For driving the flexural vibrations of a free vibrating transducer is is nesessary to bring about the actions of a force couple which must be done by a proper electrode configuration. As it follows from the theory the electrodes covering the whole faces of the bases

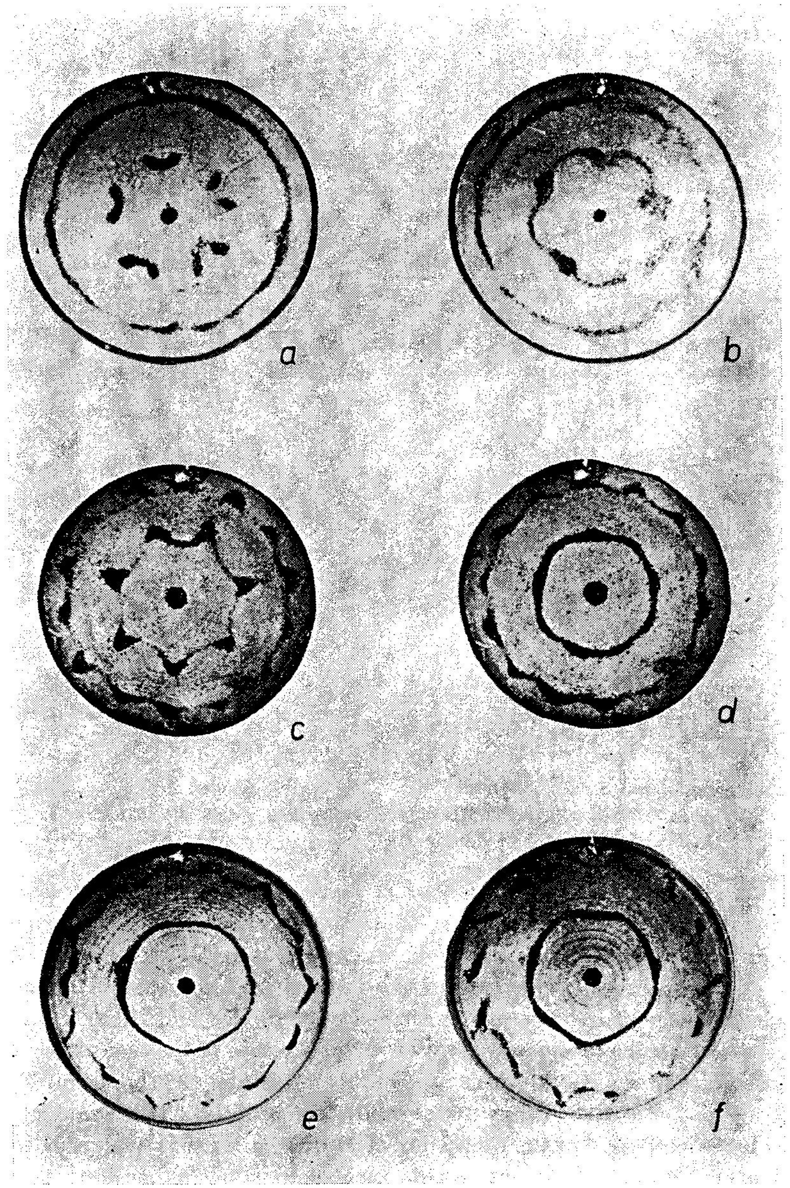


Fig. 7. The third radial modes. a. transducer No. 3, face A, f = 156693 c/s, b. face B, c. transducer No. 2, face A, f = 168696 c/s, d. face B, f = 169396 c/s, e. transducer No. 1. face A, f = 178257 c/s, f. face B.

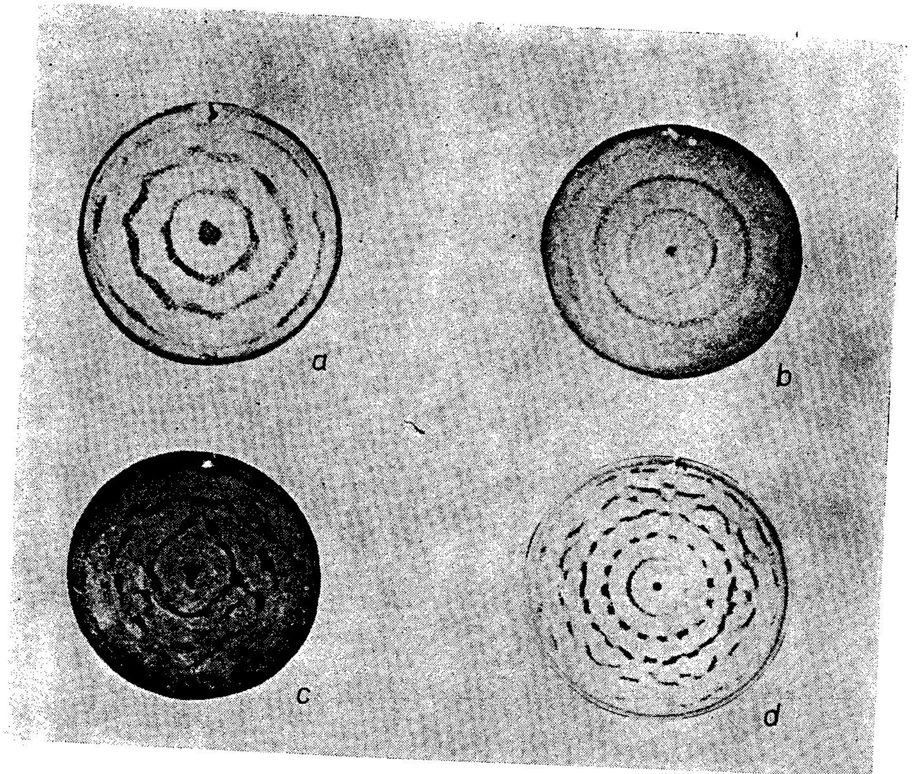


Fig. 8. The nodal patterns of high radial modes. a. transducer No. 2, $f = 225334$ c/s, b. $f = 227513$ c/s, c. $f = 226192$ c/s, d. transducer No. 3, $f = 370380$ c/s.

are not suitable for the excitation of flexural vibrations. Figs. 10. and 11., however, prove that flexural vibrations existed in the described experiments.

An interesting effect was observed on some patterns of type F(2.1). It is shown in Fig. 10. A small turning of the pattern took place even in the case of a small change of frequency. The turned pattern was sometimes less sharp.

An interesting phenomenon was observed in the case of the transducer No 2. Besides the pattern F(3.0) there appeared sometimes at the frequency of 14.166 c/s patterns with three nodal radii (see Fig. 10). The same pattern turned by 60° appeared at the frequency of 21.092 c/s. Since it was very difficult to drive both these patterns it is not possible to say whether both types of vibrations are the same. As for the pattern at the frequency of 14.116 c/s it is not possible to say whether it is the deformed mode F(3.0) or F(3.1) but at the frequency of 21.092 c/s it is probably the mode F(3.0). Another pattern with

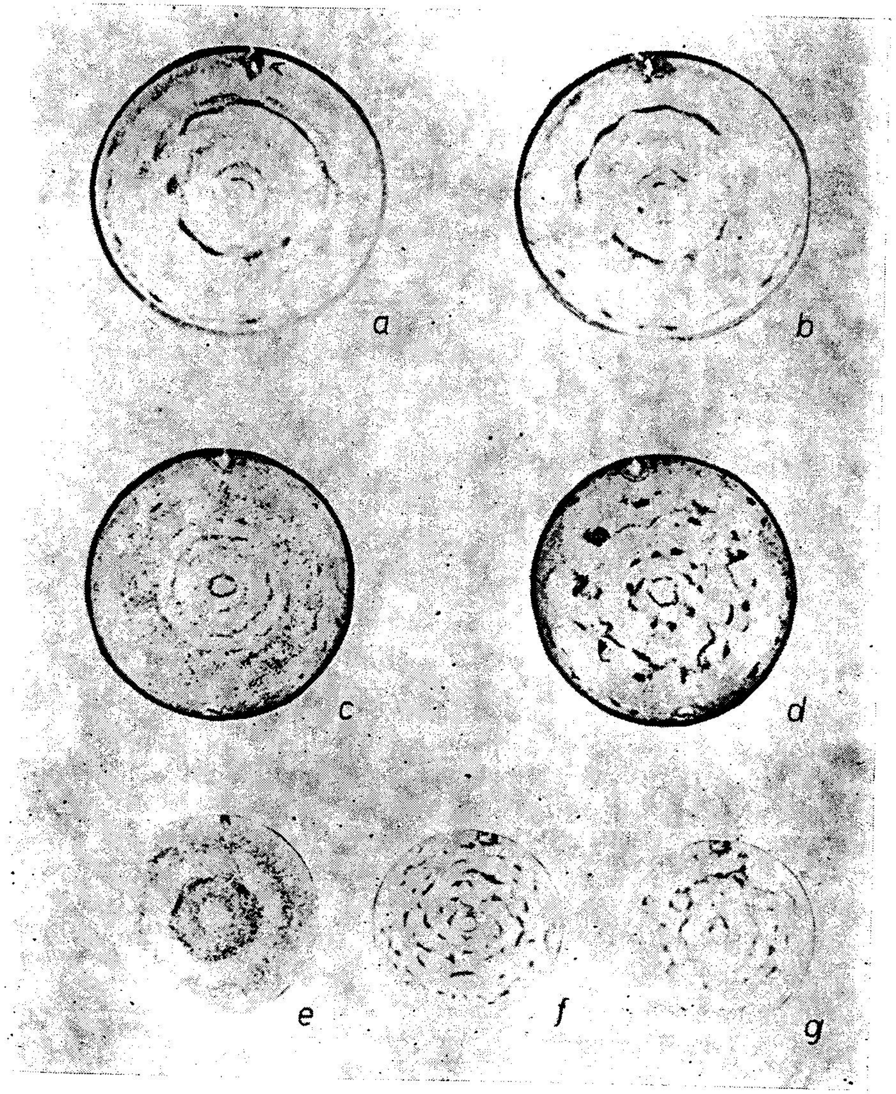
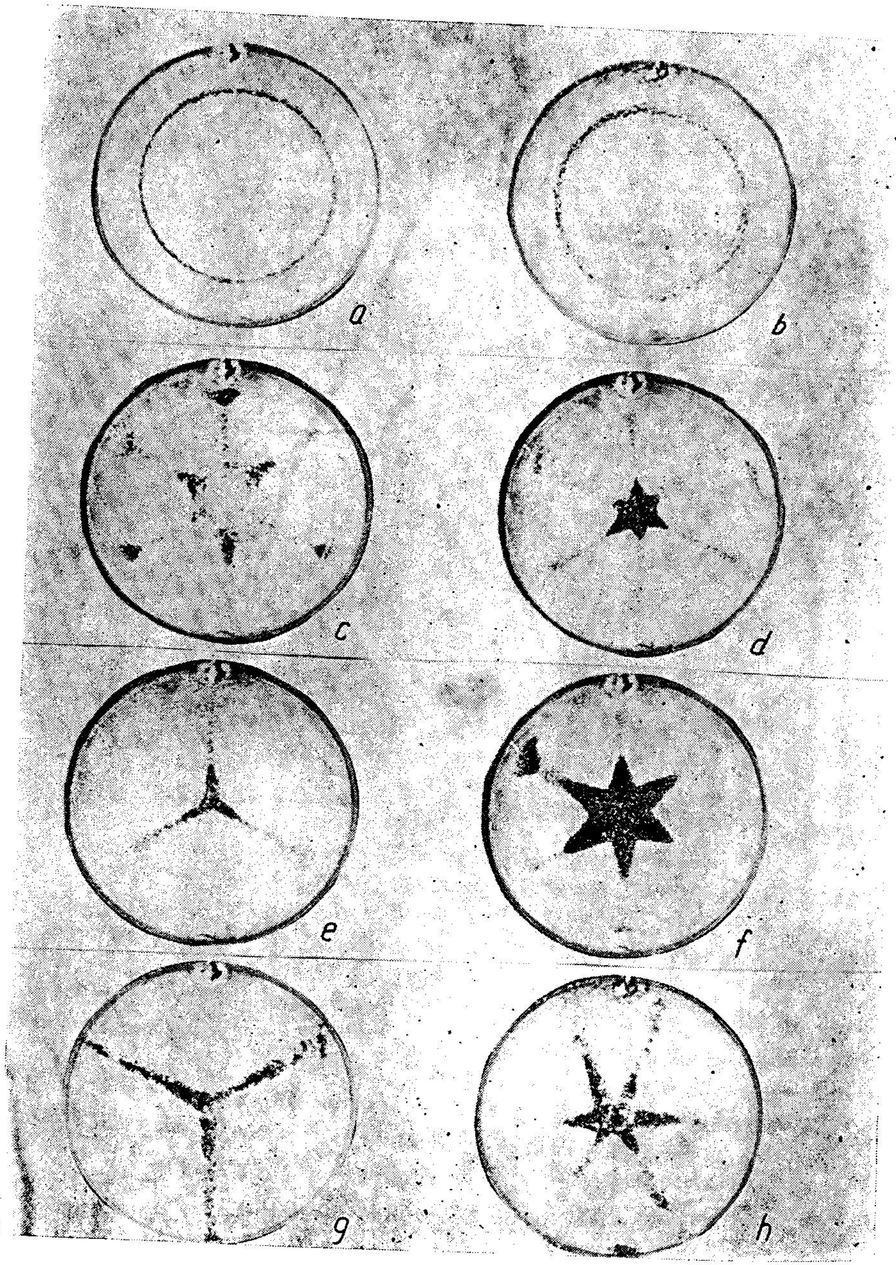


Fig. 9. The nodal patterns near the theoretical thickness resonance. a. transducer No. 3, face A, $f = 312847$ c/s, b. face B, c. transducer No. 2, face A, $f = 413293$ c/s, d. $f = 420720$ c/s, e. transducer ⌀ 50 mm, th. 11.2 mm, $f = 188721$ c/s, f. transducer ⌀ 47.8 mm, th. 5.2 mm, face A, $f = 404331$ c/s, g. face B.



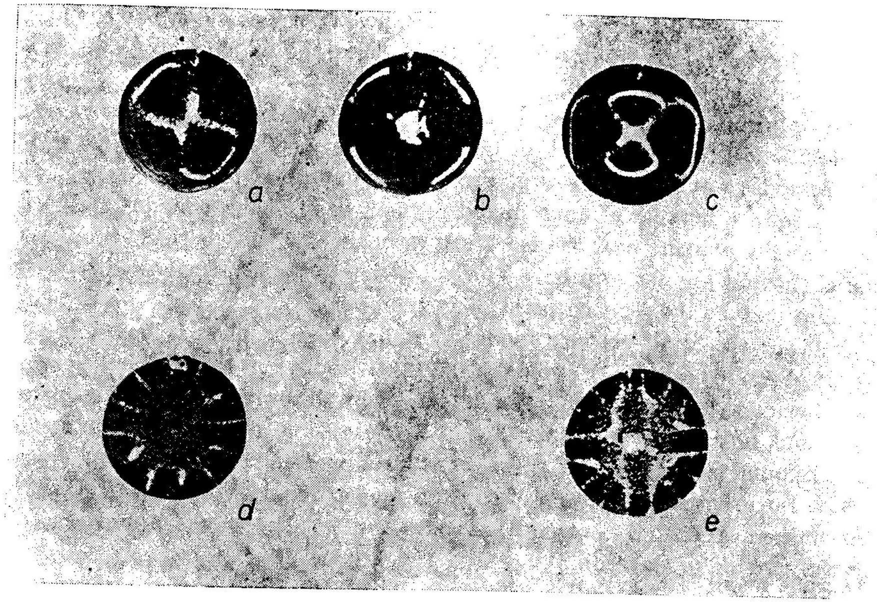


Fig. 11. The nodal patterns of flexural modes (transducer ⌀ 20 mm, th. 1 mm).
a. $f = 58398$ c/s, b. $f = 116520$ c/s, c. $f = 132462$ c/s, d. $f = 7227$ c/s, e. $f = 120231$ c/s.

three diameters has been found at the frequency 10.544 c/s. These patterns, however, diffused after a short time especially in the case of a higher driving signal. The contribution of higher overtones in a driving signal to its origin has not yet been explained but it is not possible to eliminate its influence. We can conclude from the given patterns that the circular ferroceramic transducers with the electrodes covering all the planes of the bases can produce some modes of flexural vibrations. When designing radially or thicknesswise vibrasing transducers, the possibility of the existence of coupled or unwanted flexural modes must be taken into account.

V. PRESENT DAY THEORY OF VIBRATING FERROCERAMIC TRANSDUCERS

According to the theory based on the solution of differential equations derived originally for the case of an infinitely thin plate, the spectrum of

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Fig. 10. The nodal patterns of flexural modes (transducer No. 2). a. 8550 c/s, face A, b. 8550 c/s, face B, c. 10544 c/s, face A, d. 14116 c/s, face A, e. 14116 c/s, face A, b. 8550 c/s, face B, c. 105444 c/s, face A, d. 14116 c/s, face A, e. 14116 c/s, face A, f. 14116 c/s, face B, g. 21092 c/s, face A, h. 21239 c/s, face A.

vibrations of circular discs of ferroceramics, poled perpendicularly to the plane of base having electrodes covering all their plane, should have only radial and thickness-dilatational modes.

An important role, however, have even the flexural and a number of composite modes, for the excitation of which theoretically other configurations of electrodes would be necessary. From other experimental facts, as e. g. the existence of a number of unpredicted resonances of contour modes and especially modes existing near the thickness-resonance, it is clear that two-dimensional theories are not sufficient. The approximate solution suggested by prof. Mindlin may be the starting point of further work.

Another theory of contour modes that has been applied is based on the works of A. Love and V. Petržílka [2], [3]. It is obvious that it is not possible to determine all the spectrum of radial modes by the only solution of frequency equations of contour modes for $n = 0$ because some modes from the family of compound modes can have radial or at least predominantly radial character.

The Love-Petržílka solution of contour modes does not deal with the whole determined spectrum. Some other modes can be determined approximately by the method of Galerkin.

The exact experimental verification of results for circular transducers made from ferroceramics has not yet been done. Only resonances near the theoretically predicted resonance have been measured. The character of the found resonance has not been studied in details. The checking should be done by optic methods and by visualization of nodal patterns.

It has been found experimentally that the actual spectrum consists of a number of contour modes not predicted by any theoretical solution. It will be necessary to study them in details and to find theoretical solutions including them.

So far the applied theory has shown a most striking disadvantage in the region of the frequency of thickness resonances. While in the case of contour modes the resonance of unpredicted modes are small when compared with the known modes, in the neighbourhood of the theoretical frequency of the thickness modes the resonances are comparable to the main resonance. To find the character of resonances near the resonance of the thickness mode is the main problem, the solution of which has a great importance for practical applications.

First of all it will make possible to create the optimal designs of thickness dilatational vibrating transducers from the point of view their efficiency. Some uncertainties in determining the constants of materials by resonance methods with the use of thickness-dilatational modes could be also removed.

VI. CONCLUSION FROM THE EXPERIMENTAL WORK

The frequency characteristic is a most important source of information of transducer properties. The number of important parameters such as Poisson's number, the velocity of elastic waves propagation, coefficients of electromechanical coupling can be determined from it.

It follows from the experimental experience that it is very difficult to find the corresponding resonance frequencies to different modes when we lack further experimental data.

The calculations made on the basis of approximate formulae are not sufficient because the spectrum consists of a number of resonances especially near the region of the theoretical resonance frequency of thickness dialatational modes. The visualization of the nodal patterns makes possible the investigation of a number of resonances and provides the information about the possible influence of the coupling modes on the vibrating state.

By this method we shall also be able to study the influence of changes of dimensions on the locations of resonances in the frequency spectrum.

From the experiments we can draw the following conclusions:

1. The nodal patterns depend very strongly on frequency. They are sometimes influenced by a change less than $5 \times 10^{-4} f$.
2. There is a strong dependence of the transducer resonance frequency on temperature.
3. Flexural modes have been found in the spectrum of a ferroceramic circular plate with the electrodes covering all the planes of the bases. They existed both independently and coupled with radial and thickness modes. That is why we must take into account the possible coupled flexural modes in the case of design. For practical applications ti would be suitable to study the possibility of efficient driving excitation of the flexural modes in ferroceramics, especially in connection with their usage as ultrasonic generators for radiations into gases.
4. It was experimentally verified that when the described geometry is used, we can drive first three radial modes effectively. The resonances of radial modes have comparable current amplitudes with those near the theoretical frequency of the thickness-dilatational resonance. The measuring of the intensity of ultrasonic waves radiated in the direction of the axis gave interesting results. Transducer vibrations radiate ultrasonic waves into the air with an intensity almost comparable to the intensity of radiation of the thickness--vibrations at the fundamental frequency. A more detailed study of these modes might increase the number of applicable vibrations of circular plates as ultrasonic generators. It can result in the exploitation of the frequency region of 50—150 kc/s. There is a great shortage of efficient and cheap ultrasonic

transducers in this frequency region. And it is this region where the resonances of the radial modes of transducers of normal sizes (1–5) cm, which are produced for common usage as thickness mode sources, lie.

5. From the frequency characteristics and the nodal patterns of modes near the theoretical frequency of the thickness resonance we can conclude that an isolated thickness-dilatational resonance does not exist. In this region there exist a number of vibrations and to one of them the absolute maximum of current characteristic corresponds. The modes in this region have a composite character and the coupled modes play a great role. The complicated shape of nodal patterns proves that in this region there exist modes in which the base of the transducer does not vibrate in the same phase. The idea of replacing the transducer vibrating in this way by a source of so-called „piston-vibrations" is not applicable to considerations dealing with the „near field".

6. It is a most gratifying fact that our Czechoslovak transducers are comparable to the transducers of foreign production. Czechoslovak ferroceramic transducers on the basis of the solid solution of $PbZrO_3$ – $PbTiO_3$ developed in the Research Institute for Electrical Ceramics and manufactured in Tesla Works, Hradec Králové are comparable, for example, to the transducers of the German firm Piezolan (GDR), the quality of which is well-known.

VII. ACKNOWLEDGEMENTS

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