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ELECTROMAGNETIC INTERACTIONS AND FORM FACTORS[1]

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Some problems connected with purely electromagnetic phenomena are reviewed and the application of electromagnetic interactions as a tool for getting information about the structure of strongly interacting particles is discussed.

I. ELECTROMAGNETIC INTERACTIONS

Quantum electrodynamics enables us to calculate electromagnetic processes within a quantum field theory assuming a local interaction between the electromagnetic current $j_\mu(x)$ and the electromagnetic field $A_\mu(x)$, with an interaction Lagrangian given by

$$L_{int}(x) = ej_\mu(x)A^\mu(x).$$

We all know that this model is unsatisfactory and leads in nearly all calculations to an infinite answer, and that we therefore have to deal with divergent expression in the so-called renormalization procedure. The conclusion is that these divergences are due to our ignorance of what really happens at very small distances. If we understand how to cut off the infinite integrals in quantum electrodynamics at some limiting small distance, or equivalently at some high momentum transfer $q^2 \sim \Lambda^2$, then we get finite answers and no renormalization would be required at all.

Let us therefore review the status of quantum electrodynamics especially at high momentum transfers. A comparison of the present theory with experiments leads us to either of the following conclusions:

a) Theory and experiment coincide. In this case it is possible to specify an upper limit of a fundamental length for a „finite"theory.

b) There are discrepancies between theory and experiments, then we know the limits of the applicability of the present theory.

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In order to discuss the various experimental tests in a quantitive way, it is necessary to have in mind some picture, how to modify QED at short distances. Let me mention two possiblilities:

a) The electron and muon may have a finite size, they do not behave like point charges. This effect can be taken into account by means of a form factor, for example

$$G(q^2) = 1 - \frac{q^2}{\Lambda^2} \approx \frac{\Lambda^2}{q^2 + \Lambda^2}.$$

Such a modification would remove the self-mass infinity.

b) Arbitrary modifications of the photon propagator

$$\frac{1}{q^2} \to \frac{1}{q^2} \frac{\Lambda^2}{q^2 + \Lambda^2}$$

or of the fermion propagator.

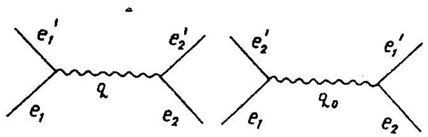
These prescriptions all violate some fundamental principles like unitarity and gauge invariance. Therefore limits quoted for the cutoffs Λ are only useful in the sense that by this cutoff a possible deviation of a measured cross section from the predicted one might be parametrized for instance in the form

$$\sigma_{exp} = \sigma_{QED}(1 + q^2/\Lambda^2).$$

Since tests are only significant if strong interactions are completely absent, we have to consider electromagnetic reactions involving electrons and muons only, like electron-electron-, electron-positron scattering, electron-positron pair-production and bremsstrahlung of electrons and muons [1].

1. $e^-e^- \to e^-e^-$, $e^+e^- \to e^+e^-$, $e^+e^- \to \mu^+\mu^-$

The lowest order diagram for the Møller scattering contains space-like virtual photons.



Thus this experiment can therefore be regarded as a test of space-like photon-propagators and of the electron vertex function.

A modified Møller cross-section can be obtained by multiplying the two amplitudes by a formfactor

$$G_\Lambda(q^2) = (1 + q^2/\Lambda^2)^{-1},$$

$$\frac{d\sigma}{d\Omega}(\Theta, \Lambda) = C(E)\left[\frac{16E^4 + q_0^4}{q^4} G_\Lambda^2(q^2) + \frac{32E^4}{q^2 q_0^2} G_\Lambda(q^2) G_\Lambda(q_0^2) + \right.$$
$$\left. + \frac{16E^4 + q^4}{q_0^4} G_\Lambda^2(q_0^2)\right](1 + \delta),$$

$$q^2 = -4E^2 \sin^2\frac{\Theta}{2}, \qquad q_0^2 = -4E^2 \cos^2\frac{\Theta}{2},$$

where δ includes the radiative corrections.

The Princeton-Stanford collaboration [2] produced remarkable data on $e^- - e^-$ scattering with colliding electron beams at 550 MeV. The data give $\Lambda^{-2} = (0.06 \pm 0.06)\,(\text{GeV/c})^{-2}$, consistent with $G_\Lambda(q^2) = 1$, and therefore consistent with QED. This experiment implies for the electron vertex a cutoff

$$\Lambda_e > 4\ \text{GeV/c}$$

and for the photon propagator a cutoff $\Lambda_\gamma > 4$ GeV/c.

Recent experiments on electron-positron elastic scattering performed at Orsay [3] are also testing space-like photon propagators, giving a cut-off of

$$\Lambda_\gamma > 2.5\ \text{GeV/c}.$$

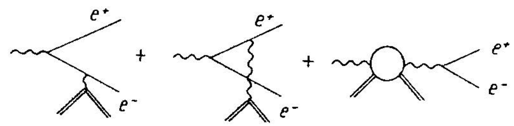
An experiment on e^-e^+ annihilation into a muon pair measured at Orsay [4] gives a photon propagator limit

$$\Lambda_\gamma > 1.7\ \text{GeV/c},$$

but for the time-like region.

2. Electron-positron pair-production

This process is described by the two Bethe—Heitler graphs and by a Compton graph



In an experiment symmetric with respect to e^+ and e^- the influence of the Compton term can be kept small. At large angles and energies the electron propagator assumes large space-like values. Parametrizing the result according to

$$\sigma_{exp}/\sigma_{QED} = 1 + \frac{M^4_{e^-e^+}}{\Lambda^4}$$

($M_{e^-e^+}$ ... the invariant mass of the final state, which is proportional to the mass squared of the off-shell fermion), the DESY-MIT collaboration [5] obtains a cut-off parameter

$$\Lambda > 1.6 \text{ GeV/c}.$$

Figure 1 shows the result of the quoted experiment. Experiments carried out at Harvard, Daresbury, Cornell and CEA are also in agreement with theory [6].

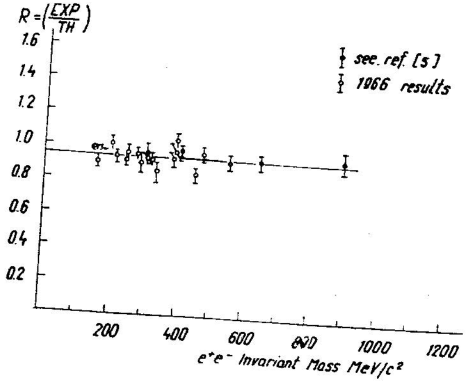
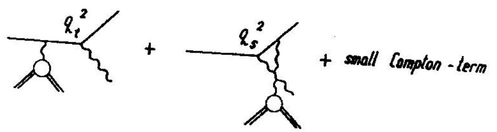


Fig. 1. Electron-positron large angle pair experiment, DESY-MIT Collaboration [5].

3. Bremsstrahlung of electrons and muons at large angles

In this process both time-like (q_t^2) and space-like (q_s^2) values of the electron or muon propagator contribute, according to the diagrams



This experiment gives therefore information supplementary to pair-production experiments. From experiments done at Cornell [7] and Harvard [8] a cutt-off for the fermionpropagators

$$\Lambda > 1.6 \text{ GeV}$$

can be obtained (see Figure 2).

Therefore we can conclude that all these experiments I just mentioned for momentum transfers now available agree with quantum electrodynamics, or in other words, QED id valid down to distances of

$$\sim 4 \times 10^{-15} \text{ cm}.$$

Contrary to experiments at high momentum transfers experiments at low momentum transfers can be performed with very high precision. Now, let us see what low momentum transfer QED can tell us, and I will concentrate only on the g-factor of the muon.

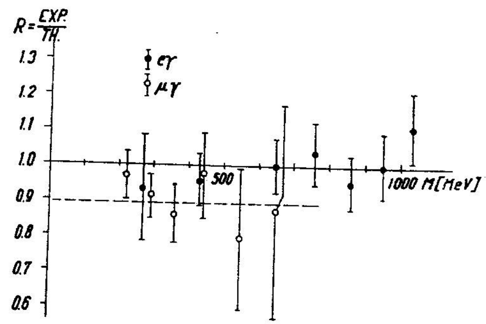


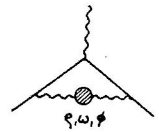
Fig. 2. Large angle Bremsstrahlung [7, 8]. M is the invariant mass of the final systems.

The g-factor of the muon is modified by higher order corrections in QED. Measurements of $(g-2)/2$, the muon anomaly, can test these corrections.

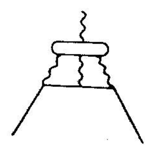
With $1/\alpha = 137.0360 \pm 0.00015$ the theoretical value for $(g-2)/2$ of the muon is

$$\left.\frac{g-2}{2}\right|_{\mu,\, th} = (116587.5 \pm 2.7) \times 10^{-8} = \frac{\alpha}{2\pi} + 0.76578\,\frac{\alpha^2}{\pi^2} + (49 \pm 25)\,\frac{\alpha^3}{\pi^3}.$$

In this value part of the hadronic contributions according to the diagram



and photon-photon scattering contributions in the form of the diagram



have been included [9].

Compared with the most recent experimental value from a muon storage ring experiment performed at CERN [10],

$$\left.\frac{g-2}{2}\right|_{\mu,\,exp} = (116616 \pm 31) \times 10^{-8},$$

again no discrepancy between theory and experiment can be stated.

In connection with purely electromagnetic phenomena let me just mention processes with external electromagnetic fields, in which I am personally interested.

Since now very high magnetic fields are available these processes become experimentally performable. We are working especially on Compton-scattering in an external magnetic field [11] and on magnetic bremsstrahlung (or synchrotron radiation). The essential point of these calculations is the use of the exact wave function for an electron bound in an external magnetic field. This fact makes the calculations very complicated, but they are more exact than the normal Born expression. This year autumn the first experiments on magnetic bremsstrahlung will be done at SLAC.

II. FORMFACTORS

Having established the validity of quantum electrodynamics, let me proceed to processes in which strongly interacting particles are involved. The analysis of elastic electron-nucleon scattering has provided us with a wealth of detail about the electromagnetic structure of the nucleons. The basic formula for all the investigations is the Rosenbluth cross-section

$$\sigma(\Theta) = \sigma_M \left[\frac{G_E^2(q^2) + G_M^2(q^2)}{1+\tau} + 2\tau G_M^2(q^2)\,\mathrm{tg}^2\frac{\Theta}{2}\right],$$

with $\tau = q^2/4M^2$ and σ_M is the Mott cross-section, which has been derived under the assumption of the one-photon exchange.

G_E and G_M are the electric and magnetic Sachs formfactors of the nucleon, which are normalized as follows

$$G_E^p(0) = 1 \qquad G_M^p(0) = \mu_p = 2.793$$
$$G_E^n(0) = 0 \qquad G_M^n(0) = \mu_n = -1.913.$$

In order to use the Rosenbluth formula for the determination of the form factors, it is first necessary to test it. It means that we have to answer the question if there is any experimental evidence for including higher orders in α.

The simplest way to examine such higher order effects is to consider the Rosenbluth-plot, in which the ratio

$$\frac{\sigma}{\sigma_M} = a + b\,\mathrm{tg}^2\frac{\Theta}{2}$$

is plotted against $\mathrm{tg}^2\,\Theta/2$ for a fixed q^2. It can be shown that higher order effects manifest themselves in a deviation from the straight line. For values of $q^2 \lesssim 4\ (\mathrm{GeV/c})^2$ no essential deviation has been found [12]. A second method considers interference terms between one- and two-photon exchange amplitudes. These interference terms are changing signs under replacing electron-proton scattering by positron-proton scattering. With A_1 the real one-photon exchange amplitude and A_2 the two-photon exchange amplitude the cross-section is

$$\sigma^{\pm}(\Theta) \sim |\pm \alpha A_1 + \alpha^2 A_2|^2 = \alpha^2 A_1^2 \pm 2\alpha^3 A_1\,\mathrm{Re}\,A_2 + \alpha^4 |A_2|^2.$$

Then we get

$$R = \frac{\sigma(e^+p)}{\sigma(e^-p)} \approx 1 + \frac{4\alpha\,\mathrm{Re}\,A_2}{A_1}.$$

Therefore an inequality between these two cross-sections is an indication for a two-photon exchange. Figure 3 shows that the experimental results are consistent with $R = 1$ for $q^2 \leqslant 5\ (\mathrm{GeV/c})^2$ [13]. Thus the validity of the Rosenbluth formula can be taken for granted.

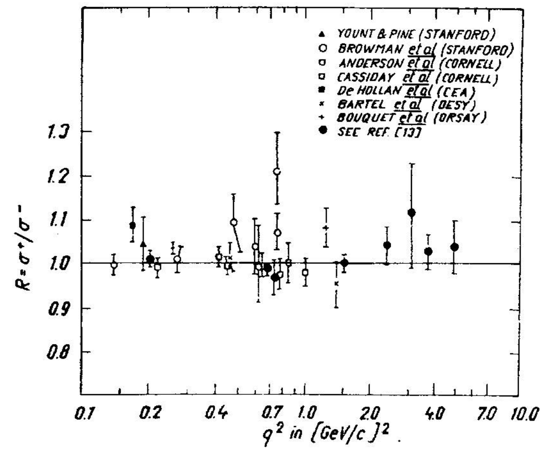


Fig. 3. Experimental data on tests of the two-photon exchange [13].

Deducing the form factors for the proton and neutron, as a first orientation their behaviour can be described by the scaling law

$$G_E^p(q^2) = \frac{G_M^p(q^2)}{\mu_p} = \frac{G_M^n(q^2)}{\mu_n} = G_D \qquad G_E^n = 0$$

and by the dipole formula

$$G_D = \frac{1}{(1 + q^2/0.71)^2}\,.$$

The dipole formula has no theoretical basis, but gives a reasonable good phenomenological fit.

Therefore the data can be conveniently discussed in terms of departures from the scaling law and the dipole formula.

Since the proton form factors are better known let me discuss them first. The most recent data on the scaling law are coming from BONN [14] and SLAC [15]. The measurements of the Bonn group tend to indicate a deviation from the scaling law in the region of q^2 between 1 and 2 $(GeV/c)^2$. Data from SLAC, however, do not confirm this trend for q^2 between 2.5 and 3.75 $(GeV/c)^2$, as can be seen in Figure 4, which shows the data of SLAC together with the BONN data. In Figure 5 a new method of treating the data has been used in the following way [15]. Defining the ratios

$$g_E = \frac{G_E}{G_D}\,, \qquad g_M = \frac{G_M}{\mu G_D}\,,$$

where G_D is the dipole formula, the Rosenbluth formula can be written as

$$R = \frac{(1 + A)\,\sigma(\Theta)}{\sigma_{Dipole}(\Theta)} = g_M^2 + A\,g_E^2.$$

A is a kinematic factor, depending on q^2 and $\mathrm{tg}^2\,\Theta/2$.

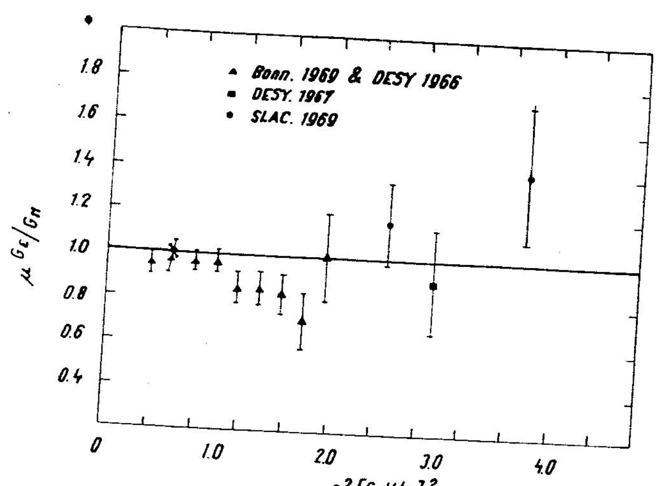


Fig. 4. Values of $\mu G_E^p/G_M^p$. (Compilation by Rutherglen J. R. [1]).

Now, if R is plotted against A the intercept and the slope give g_M^2 and g_E^2 respectively. If the scaling law is true, then the intercept and the slope are both equal to unity.

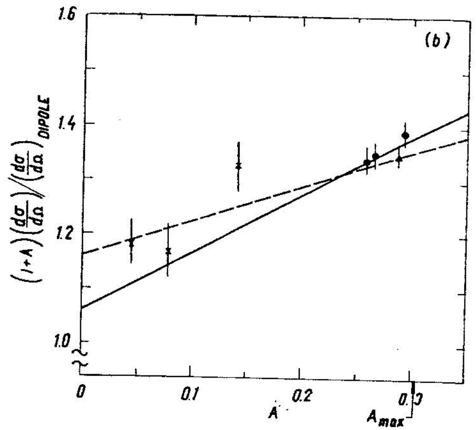


Fig. 5. Elastic e-p scattering data [15]. $q^2 = 1.50\ (GeV/c)^2$; x – BONN (1968); ▲ – DESY (1966); ● – SLAC; --- BONN + DESY: $\mu G_E/G_M = 0.79 \pm 0.09$; —— fit to all data for scaling $G_E = G_M/\mu$.

Now let us consider the absolute values of the form factors. Here the most precise measurements show small but systematic deviations from the dipole-fit. These results are mainly true for the magnetic form factor G_M, because of the suppression of G_E in the cross-section for large q^2. The deviations are graphically shown in Figure 6. The statements about the proton form factors can be summarized as follows:

There is no systematic deviation from the scaling law for an increasing q^2.

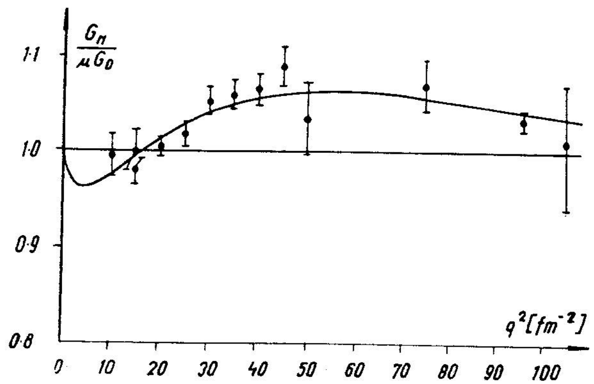


Fig. 6. The magnetic formfactor of the proton normalized to the dipole fit (Compilation by Rutherglen [1]). Curve: four-pole-fit.

There may be deviations of 10% between 1 and 2 $(GeV/c)^2$. The dipole-fit is not perfect, but provides a good representation.

From the electron-deuteron scattering the neutron form factors are less accurately determined, because of necessary extraction mechanisms.

The data on the magnetic form factor G_M^n of the neutron are not in disagreement with the dipole formula. For the electric one, G_E^n, the only precise measurement was done by the scattering of thermal neutrons by atoms [16], giving only the slope by

$$\left.\frac{d\, G_E^n}{d\, q^2}\right|_{q^2=0} = 0.50 \pm 0.01\ (GeV/c)^{-2}.$$

Data from elastic electron-deuteron scattering at $q^2 < 0.15\ (GeV/c)^2$ are in reasonable agreement with this slope.

New data on the formfactors of the neutron have been obtained by measuring the ratio [17]

$$R = \frac{d^3\sigma/d\Omega_e\, dE_e\, d\Omega_n}{d^3\sigma/d\Omega_e\, dE_e\, d\Omega_p} = \frac{a_n + b_n\, tg^2\, \Theta/2}{a_p + b_p\, tg^2\, \Theta/2},$$

from quasi-elastic electron-deuteron scattering, where electron-neutron and electron-proton coincidences are simultaneously measured. This ratio has the advantage of being relatively insensitive to assumptions about the deuteron wave function.

The combined information on the electric form-factor of the neutron obtained so far is shown in Figure 7. The data are consistent with $G_E^n = 0$.

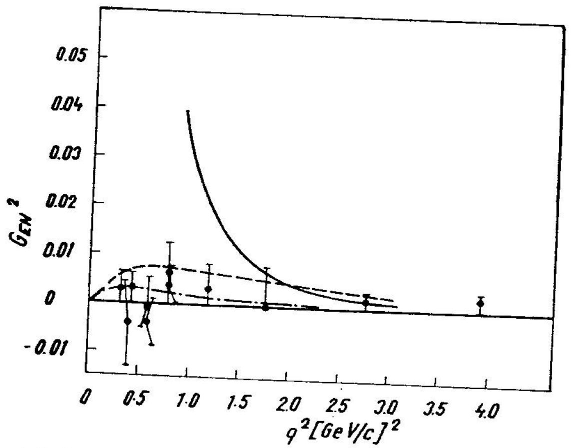
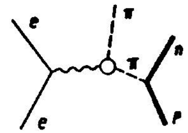


Fig. 7. Experimental data on the electric form-factor of the neutron G_E^n (Compilation by Rutherglen J. R. [1]). ---- $G_{EN} = -\tau G_{MN}$; -.-. $G_{EN} = \tau/1 + 4\tau\, G_{MN}$; —— $G_{EN} = G_D$.

Let us finally come to the pion formfactor. Information on this formfactor can be obtained by inelastic electron-proton scattering under special kinematic conditions, where the following diagram is the dominant one



Since this diagram cannot be separated from others in a gauge invariant manner, isolation of the pion-form factor for space-like momenta is difficult. The experiments can be fitted with a pion formfactor equal to that of the proton, but the data are also compatible with a simple ϱ-meson dominance model [18].

Independent from this experiment Chou and Yang [19] recently proposed a method of evaluating the pion form factor by extrapolating elastic $\pi^- p$ scattering at high energies. They get a pion form factor which is falling off less rapidly than that of the proton (Fig. 8).

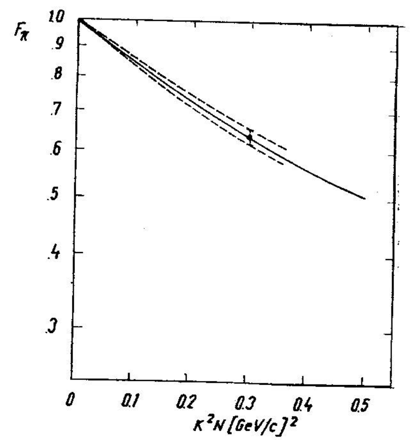


Fig. 8. The pion formfactor for space-like momentum transfer as calculated from $\pi^- p$ scattering by Chou and Yyng [19].

In addition to the topics I discussed in this seminar many exciting questions concerning electromagnetic interactions, as for instance inelastic electron-nucleon scattering, photoproduction and vectormeson dominance, have cropped up and are waiting to be solved.

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