SOME REMARKS ON GENERALIZED CURRENT ALGEBRAS[1]

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Some results of the current algebras extended to non-equal times are surveyed and new sum rules are presented for vector and axial vector currents.

In order to have a complete picture of processes with many particles, we ought to know the space-time dependence of the retarded commutators Instead, some equal-time current algebras and divergence conditions are at our disposal. The first step to generalize the current algebra to non-equal times was made by Okubo [1]. He postulated a set of commutators on the light cone[2] for the source of the meson octet, through a general algebraic structure containing f-, d-type and singlet terms. It was possible to get many new relations, among others the Barger-Rubin relation.

Another motivation for the current algebra of non-equal time was the need of clarifying the infinite momentum method. In this connection [2, 3] first the equal-time commutation relations are formally extended to a spacelike hyperplane $nx = 0$, $n^2 > 0$, $n_0 > 0$ then $n^2 \to 0$, the spacelike hyperplane approaches the light cone. In this manner a light cone algebra is postulated from which one can reproduce the Fubini sum rule.

Starting from [2, 3], Brandt [4] was able to show the Bjorken scale law.

We begin here with the remark that the divergence conditions bearing the influence of the symmetry breaking express a change in time, therefore, they represent an important restriction for the current commutators of non-equal time. This idea has been investigated in [5]. Without going into details, we present a simple example for conserved vector currents.

From equal-time commutators and current conservation we get for a small x_0

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[2] Light cone singularities give important contributions to the high energy behaviour of the scattering amplitude.

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$$\left[V_0^a\left(\frac{x}{2}\right), V_l^b\left(-\frac{x}{2}\right)\right] = if_{abc}\delta(\underline{x})\, V_l^c\left(-\frac{x}{2}\right) + S_{V_0V_l} + \alpha x_0 \partial_k \delta(\underline{x}) \times$$

$$\times\left[\delta_{kl} f_{abc} V_4^c\left(-\frac{x}{2}\right) - i\varepsilon_{klm}\left(\sqrt{\frac{2}{3}}\,\delta_{ab}\, A_m^0\left(-\frac{x}{2}\right) + d_{abc} A_m^c\left(-\frac{x}{2}\right)\right)\right],$$

where $S_{V_0V_l}$ is the vacuum expectation value of (1) for $x_0 = 0$ (the so-called Schwinger term) and $\alpha = 1(0)$ for field (quark) algebra.

Taking (1) between spinless single particle states of the same mass, and denoting the Fourier-transform of this matrix element by $t_{0l}^{ab}(p_1, p_2, Q)$, we can easily arrive at the following sum rule

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}Q_0 t_{0l}^{ab}(p_1, p_2, Q)\, g(k_0 - Q_0) = if_{abc} P_l F^c(t)\, g\left(k_0 - \frac{\Delta_0}{2}\right) -$$

$$- i\alpha f_{abc} F^c(t) P_0 \left(Q_l - \frac{\Delta_l}{2}\right) \frac{\mathrm{d}g(z)}{\mathrm{d}z}\bigg|_{z = k_0 - \frac{\Delta_0}{2}} \tag{2}$$

Here we used

$$P_\mu = \frac{1}{2}(p_{1\mu} + p_{2\mu}),$$

$$\Delta_\mu = p_{2\mu} - p_{1\mu}, \quad t = \Delta^2 \tag{3}$$

$$g(z) = (\exp iz\delta - 1)/iz$$

with a small δ. $F^c(t)$ means the usual vector form factor. With the exception of the terms of order δ and δ^2 everything else has to be neglected in (2). One can discuss (2) most easily in the limit $p_0 \to \infty$. In such a way some new information [5] can be obtained beside the Fubini sum rule. In particular, (2) predicts the field algebra as the right choice.

In the usual Regge-asymptotics the integral (2) does not converge if we expand the function $g(k_0 - Q_0)$ and interchange the summation and integration, that is, in this case the n' th term equals the integral coming from the n'th time derivative of the current. Nevertheless, if we allow certain deviations from the simplest Regge-behaviour at high energy, new sum rules can be tested. In particular the Serpukhov experiments show a complicated asymptotics [6], which can be probably explained by Regge cuts [7] and (or) by oscillating cross-sections [8].

Consider an example for the axial-vector current in the above sense and examine the relation

$$\langle p \left| \left[\frac{\mathrm{d}^2\chi^+(t)}{\mathrm{d}t^2}, \quad \chi^-(t)\right] \right| p\rangle = 0, \tag{4}$$

$\chi^\pm$ mean chirality operators, $|p\rangle$ is a proton state. (4) follows from PCAC and the pion commutator. Inserting a complete system of states, we get [9] for the soft-pion-proton total cross-sections

$$\int_{m_\pi}^{\infty} \mathrm{d}\omega\, \omega(\sigma_0^{\pi^- p} - \sigma_0^{\pi^+ p}) = 0. \tag{5}$$

Hence we conclude that the difference of the total cross sections has the least one sign change at high energies. At very high energies only small contributions may come to (5). A possible shape satisfying this conditon is

$$\sigma_0^{\pi^- p} - \sigma_0^{\pi^+ p} = c\,\frac{\sin b\omega^l}{\omega^k}, \tag{6}$$

where $l > 0$ and $k + l > 2$; b, c are constants. Indeed, the asymptotic form of (5) corresponding to (6) is equal to

$$\frac{cb^{(k-2)/l}}{l}\,\frac{\cos ba^l}{(ba^l)^{k+l-2/l}} \qquad (a \text{ very large}) \tag{7}$$

We see that not only decreasing but also increasing cross-sections of the oscillating type are allowed.

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