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A SURVEY ON WEAK INTERACTIONS[1,2]

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A survey of some parts of the weak interactions theory including the new developments within the last year is presented.

I. INTRODUCTION

The physics of weak interactions of elementary particles has become such a vast subject that it is absolutely impossible to review it within a single talk. I shall there-fore emphasize some parts and neglect others. The guideline in picking out special topics shall be twofold: First, the personal interests of the author and second, the new developments within the last year. No attempt at complete coverage of all data is made; rather, it is hoped that a certain continuity in the presentation has been achieved.

Data with no particular reference attached are taken from reference [1].

II. PHENOMENOLOGY

Within the wide field of weak processes, at least five distinct groups can be separated. They are listed in Table 1 together with their (qualitative) strength.

Table 1

Weak processes

Type	Strength
Leptonic processes	$G \doteq 1.02 \times 10^{-5}\, m_p^{-2}$
Semi-leptonic processes with $\Delta Y = 0$	$G \cos \Theta$, $\Theta \doteq 0.24$
Semi-leptonic processes with $\lvert\Delta Y\rvert = 1$	$G \sin \Theta$
Hadronic processes	$\Delta I = 1/2$ enhanced
CP-violating processes	suppressed

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Θ is, of course, the Cabibbo-angle. A more precise determination distinguishes between Θ_A and Θ_V, i. e. the Cabibbo angles for vector and axial vector matrix elements [2]. For meson decays, the Cabibbo angle tends to be larger by about 10 %.

In connection with Table 1, a few questions immediately come to one's mind. First, what about $\Delta Y = 2$ processes! So far, they have not been observed and present upper limits are

$$\Gamma(\Xi^\circ \to p l^- \nu_l)/\Gamma(\Xi^\circ) < 1.3 \times 10^{-3} \quad l = \mu, e$$

$$\Gamma(\Xi^\circ \to p \pi^-)/\Gamma(\Xi^\circ) < 0.9 \times 10^{-3}$$

$$\Gamma(\Xi^- \to n e^- \nu)/\Gamma(\Xi^-) < 10^{-2}$$

$$\Gamma(\Xi^- \to n \pi^-)/\Gamma(\Xi^-) < 1.1 \times 10^{-3}$$

where $\Gamma(x)$ is the total decay rate of particle x.

Semi-leptonic processes with $|\Delta Y| = 1$ obey the selection rule $\Delta Y = \Delta Q$. If there were a decay mode with $\Delta Y = -\Delta Q$, it would follow that $\Delta I \geqslant 3/2$ in this particular mode. (See for example ref. [3].) This empirical selection rule is well supported by the following experimental limits

$$\frac{\Gamma(\Sigma^+ \to l^+ n \nu)}{\Gamma(\Sigma^- \to l^- n \nu)} < 0.03$$

$$\frac{\Gamma(K^\pm \to \pi^\pm \pi^\pm l^\mp \nu_l)}{\Gamma(K^\pm \to \pi^\pm \pi^\mp l^\pm \nu_l)} \begin{cases} 0.02 & \\ & \text{for} \\ 0.3 & \end{cases} \begin{matrix} l = e \\ \\ l = \mu. \end{matrix}$$

Yet another question concerns neutral lepton currents. There is good evidence that they are totally absent.

$$\Gamma(K^\pm \to \pi^\pm e^+ e^-)/\Gamma(K^\pm) < 4 \times 10^{-7}$$

$$\Gamma(K^\pm \to \pi^\pm \mu^+ \mu^-)/\Gamma(K^\pm) < 2.4 \times 10^{-6}$$

$$\Gamma(K^\pm \to \pi^\pm \nu \bar{\nu})/\Gamma(K^\pm) < 1.2 \times 10^{-6}.$$

The last value [4] is particularly interesting because it concerns a decay which could occur only via second order weak interactions whereas the others are possible by a combination of a weak and an electromagnetic interaction.

III. THE INTERACTION

The above phenomenological constraints and selection rules can be incorporated in an ansatz for the effective weak S-operator in the following way:

$$S = I - \frac{iG}{\sqrt{2}} \int \mathrm{d}^4x \tfrac{1}{2} \{ J_\lambda^+(x) l^\lambda(x) + l_\lambda^+(x) J^\lambda(x) \} + O(G^2). \tag{1}$$

Here, $l^\lambda(x)$ is the lepton current given by

$$l^\lambda(x) = \sum_{l=\mu,e} \overline{\Psi}_l(x) \gamma^\lambda (1 + \gamma_5)\, \Psi_{\nu_l}(x). \tag{2}$$

$J^\lambda(x)$ is the total weak current, i.e. the sum of lepton current l^λ and hadron current h^λ. The hadron current, in turn, consists of a part proportional to $\cos \Theta$ which transforms as a pion under SU_3 and a part proportional to $\sin \Theta$ which transforms as a Kaon under SU_3.

The effective S-operator (1) does not describe hadronic processes. It is not yet clarified, whether our picture of weak interactions can be extended to hadronic processes in a very straightforward manner.

All currents have been taken in the „$V-A$" form. The present upper limits on other terms from neutron and nuclear β-decay have been determined by H. Paul [5]

$$C_S/C_V = -\,0.001 \pm 0.006$$

$$C_T/C_V = -\,0.0004 \pm 0.0003.$$

H. Paul [5] determined the important ratio C_A/C_V also. He obtained

$$C_A/C_V = -\,1.262 \pm 0.008.$$

Among others, the S-operator (1) predicts an electron-neutrino scattering cross-section of the strength G^2. Various experimental limits on this cross-section [6] have been obtained recently. The best is [7]

$$\sigma_{e\nu} < 4\sigma_{theor}.$$

It is interesting that a good estimate for electron-neutrino reactions can also be obtianed from astrophysical considerations [8]:

$$\sigma_{e\nu} = 10^{0\pm2}\sigma_{theor}.$$

IV. MUON DECAY

No exciting new developments took place in the field of muon decay. However, we incorporate a collection of present best data because this is perhaps the standard test of the S-operator (1) and its consequences. The spectrum of electrons with energy E and momentum q stemming from the decay of muons with degree of polarization P is given by

$$N(E, \Omega) = \frac{G^2}{12\pi^4} m_\mu q E E_0 \left\{ 3 - 3x + \tfrac{2}{3}\varrho(4x - 3) + 6\eta\varepsilon\left(\frac{1}{x} - 1\right) - \right.$$

$$\left. - P\frac{q}{E} \cos\Theta\, \xi[1 - x + 2\delta(\tfrac{4}{3}x - 1)] \right\} + O(\varepsilon^2), \qquad (3)$$

where Θ is the angle between the polarization axis and the direction of the electron momentum. Also, we have used

$$\varepsilon = \frac{m_e}{m_\mu} \quad E = \frac{m_\mu^2 + m_e^2}{2m_\mu} \quad x = \frac{E}{E_0} \qquad (4)$$

so that

$$\frac{2\varepsilon}{1 + \varepsilon^2} \leqslant x \leqslant 1. \qquad (5)$$

Predictions of the "V—A" theory (1) as well as experimental data [9] are collected in Table 2.

Table 2

The parameters of muon decay

Parameter		$V-A$ prediction	experiment [9]
Michel	ϱ	$\frac{3}{4}$	0.7518 ± 0.0026
Shape	δ	$\frac{3}{4}$	0.7540 ± 0.0085
Asymetry	ξ	-1	-0.973 ± 0.014
Eta	η	0	-0.12 ± 0.21

The helicity h of the electron has been determined [9] as (-1.0 ± 0.13), in good agreement with the theoretical requirement (-1). The above accuracy allows for the following exclusion of other than $V-A$ terms in muon decay:

$$g_S \leqslant 0.33\, g_V$$

$$g_P \leqslant 0.33\, g_V$$

$$g_T \leqslant 0.28\, g_V.$$

If the ratio of A and V is left open, one obtains the following experimental restriction:

$$0.76\, g_V \leqslant g_A \leqslant 1.20\, g_V.$$

The angle between V and A, restricted to a multiple of π by T-invariance, can be estimated [9] via the relation [10] $\eta^2 \leqslant 1 - h^2$

$$\sphericalangle(V, A) = 180° \pm 15°.$$

V. SEMI-LEPTONIC PROCESSES

The field of semi-leptonic processes is, generally speaking, in very good shape. The $\mu-e$ ratios, the CVC-predictions and many other theoretical values are precisely met by empirical findings. We can and we have learned a lot by neutrino induced processes. New in this area are the following upper limits on neutral currents in neutrino induced reactions [11]

$$\frac{\nu_\mu + p \to \nu_\mu + p}{\nu_\mu + n \to \mu^- + p} = 0.12 \pm 0.06$$

$$\frac{\nu_\mu + p \to \nu_\mu + \pi^+ + n}{\nu_\mu + p \to \mu^- + \pi^+ + p} = 0.08 \pm 0.04.$$

An unfortunate exception to the optimistic picture of semi-leptonic processes is the K_{l3} decay. Experimental results on this decay are still violently fluctuating. Let us therefore simply quote two of the more recent results. From the measurement of muon polarization in $K_{\mu 3}$ decay one obtains [12] for the ratio ξ of the two form factors [3]

$$\xi = -\,0.9\begin{array}{l}+\,0.5\\-\,0.4\end{array} + i(-0.3 \pm 0.5).$$

If ξ is assumed to be real, one gets [12]

$$\xi = -0.95 \pm 0.3.$$

On the other hand, from measurements of the lepton spectrum, the variation of the form factors with $q^2 = (q_l + q_\nu)^2$ is [13]

$$\lambda^+ = 0{,}045 \pm 0.015,$$

where λ^+ is defined by

$$f_\pm(q^2) = f_\pm(O)\left\{1 + \lambda^\pm \frac{q^2}{m^2} + \ldots\right\}. \qquad (6)$$

Notice in passing that K^* dominance of the form factor yields $\lambda^+ = 0.03$. If λ^- is assumed to be small, ξ is [13]

$$\xi = -0.35 \pm 0.22.$$

The disagreement has, unfortunately, been typical for this field since many years.

VI. HADRONIC DECAYS

The optimistic view we have given about semi-leptonic decays cannot be extended to hadronic decays. Here most details are not understood and only very crude ideas can be modelled into a rough picture of these phenomena. It is generally accepted that $\Delta I = \frac{1}{2}$ transitions are strongly enhanced over $\Delta I = 3/2$ or even higher transitions. Whether this is a good selection rule or just a dynamical mechanism is still under discussion.

It is very hard to do any justice to all the many contributions which try to clarify this field. Let me just mention the Heidelberg group who has recently put some concerted effort into these problems [14]. Maybe the least wrong model is still the naive Quark model [15]. There are 4 different graphs contributing to a hadronic baryon decay (see Fig. 1). It turns out that the first gives Octet enhancement. The others can be made to vanish due to quark statistics [16]. The best one can get is a qualitative picture. Quantitative results cannot even be attempted.

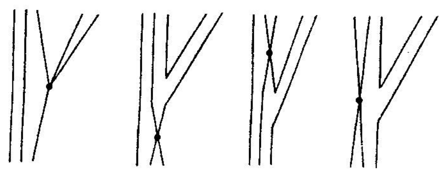

Fig. 1. Hadronic baryon decay in the quark model.

VII. DOUBLE BETA DECAY

Perhaps one of the most exciting events in weak interactions over the last years was the discovery of the double β-decay [17]. So far, three isotopes have been shown to undergo double β-decay. They are listed in Table 3.

The Japanese group [18] finds for the decay of Te^{128} a value of $10^{22\cdot5\pm0\cdot5}$ years. In the last entry of Table 3, we have also shown the estimates for the lepton number violating, neutrino-less decay. Theoretical uncertainties are typical of the order of 2 in the exponent and have only been given in on place. It is interesting to observe that all data lie systematically bellow the lepton number conserving values. Possible consequences have been studied by H. Primakoff and S. P. Rosen [19].

Table 3

Double β-decay

E_0(MeV)	Decay	$T_{1/2}$ (years)	theory *eevv*	theory *ee*
2.5	$Te^{130} \rightarrow Xe^{130}$	$10^{21.3\pm0.1}$	$10^{22.5\pm2}$	$10^{21.3}$
3.0	$Se^{82} \rightarrow Kr^{82}$	$10^{20.2\pm0.2}$	$10^{22.0}$	$10^{21.2}$
0.85	$Te^{128} \rightarrow Xe^{128}$	$>10^{23.3}$	$10^{25.3}$	$10^{23.4}$

VIII. THE STATE OF THE THEORY

As we have already pointed out, the S-operator [1] describes all leptonic phenomena in excellent agreement with experiments. The vector part of semi-leptonic interactions with $\Delta Y = 0$ is on an equally good basis because of the conserved vector current hypothesis. The axial vector part is right now going through a „renaissance“ period since PCAC has made connections with strong interaction models via soft pion techniques. Also $|\Delta Y| = 1$ semi-leptonic processes are satisfactorilly understood by the S-operator (1).

After listing the victories, let us turn to defeats. There is still no consistent picture of hadronic decays and the number of possible models of CP-violation will probably increase instead of decrease in the near future.

A very fundamental question in all particle theory which also touches weak interactions is that of higher orders. In spite of many trials, no progress has been made so far.

In order not to end in a sad mood, let us recall the splendid success of the S-operator (1) in describing leptonic and semi-leptonic processes.

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