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USE OF CORRELATION FUNCTIONS AS PARAMETERS FOR THE ACTIVITY OF COSMIC RADIATION

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The use of correlation functions of the cosmic ray intensities at various stations is suggested as a parameter of the cosmic ray activity. The maximum value of the correlation coefficient depends on the amplitude of the cosmic ray variations both isotropic and anisotropic and, for the given pair of stations, characterizes the degree of the intensity variations. Some examples are given for the period 1963—1964 and May 1966.

In studying various geophysical effects those parameters are widely used which determine the changes or state of various physical quantities more effectively than the absolute values of these quantities do. The indices K_p and A_p marking the changes of the geomagnetic field, ionospheric indices and the like serve as examples. Absolute values of intensity, which in individual cases do not describe the changes themselves are used as a rule directly in cosmic radiation. In paper [1] it was suggested to use correlation functions of intensities of cosmic radiation at two or more stations for determining the parameter which would express the degree of changes in intensity. Statistical error of one hour values of intensity varies between 0·5 % and 0·05 % in most instruments registering the intensity of cosmic radiation at different stations of the world-wide network; some variations have amplitudes comparable with the exactness of measured individual intensity values. Consequently the coefficients of correlation are higher, the more the amplitude of variations will exceed statistical fluctuations.

Let the intensity of cosmic radiation registered in the interval of the length T at two stations be expressed by the functions $F'(t)$ and $G'(t)$.

If we denote

$$F(t_i) = F'(t_i) - \frac{1}{T} \sum_{i=0}^{T} F'(t_i) \qquad (1)$$

$$G(t_i) = G'(t_i) - \frac{1}{T} \sum_{i=0}^{T} G'(t_i)$$

then

$$\sum_{i=0}^{T} F(t_i) = 0 \quad \text{and} \quad \sum_{i=0}^{T} G(t_i) = 0, \tag{2}$$

because the functions $F(t_i)$, $G(t_i)$ are differences between individual values of intensity at certain moments t_i and the average value of the whole interval T. The correlation functions are obtained by the correlation of the values $F(t_i)$ and $G(t_i)$ for various time shifts τ of these functions to one another. We can write

$$r(\tau) = \frac{\sum_{i=0}^{T-\tau} F(t_i)\, G(t_i + \tau)}{\left[\sum_{i=0}^{T-\tau} [F(t_i)]^2 \sum_{i=0}^{T-\tau} [G(t_i + \tau)]^2\right]^{1/2}}. \tag{3}$$

The expression (3) shows that, suppose the equal number of values is retained (i. e. of the length of the inetrval T), 2τ of marginal values disappears at shifting by τ, which may affect the course of the correlation function. Retaining an equal number of correlated values would again cause a shift of the whole interval by τ values in the direction of the shift, which affects the course of the correlation function in the same way. For this reason we should keep to $\tau \ll T$ (especially if there are great variations in marginal fields of the interval T). In this case we may also accept

$$\sum_{i=0}^{T-\tau} F(t_i) \approx 0; \qquad \sum_{i=0}^{T-\tau} G(t_i + \tau) \approx 0, \tag{4}$$

The functions $F(t)$ and $G(t)$ are in a real case composed of real changes in intensity and random changes, which have not the world-wide range such as statistic fluctuations, change in the sensibility and counting level of a measuring instrument, that is

$$F(t_i) = f(t_i) + \varepsilon(t_i), \qquad G(t_i) = g(t_i) + \delta(t_i). \tag{5}$$

With respect to the statistic independence of the fluctuations $\varepsilon(t_i)$ and $\delta(t_i)$ we have for the sufficiently large T

$$\sum_{i=0}^{T} \varepsilon(t_i) = 0; \qquad \sum_{i=0}^{T} \delta(t_i) = 0, \tag{6}$$

whence in substituting into (2) there follows

$$\sum_{i=0}^{T} f(t_i) = 0; \qquad \sum_{i=0}^{T} g(t_i) = 0. \tag{7}$$

Now we can express the correlation function to be a function of the shift

$$r(\tau) = \frac{\sum_{i=0}^{T-\tau} f(t_i)\, g(t_i + \tau) + \sum_{i=0}^{T-\tau} \varepsilon(t_i)\, \delta(t_i + \tau)}{\left[\left\{\sum_{i=0}^{T-\tau} [f(t_i)]^2 + \sum_{i=0}^{T-\tau} [\varepsilon(t_i)]^2\right\}\left\{\sum_{i=0}^{T-\tau} [g(t_i + \tau)]^2 + \sum_{i=0}^{T-\tau} [\delta(t_i + \tau)]^2\right\}\right]^{1/2}} \tag{8}$$

if we take into consideration that

$$\sum f(t_i)\, \delta(t_i + \tau) = \sum f(t_i)\, \varepsilon(t_i) = 0$$
$$\sum g(t_i)\, \delta(t_i) = \sum g(t_i)\, \varepsilon(t_i - \tau) = 0 \tag{9}$$

holds with sufficient exactness. It is evident from the expression (8) that the function of correlation will be affected to some extent by the statistic exactness of instruments and that it will often differ substantially from various pairs of stations. Assuming that real variations are much larger than fluctuations, we obtain

$$\sum_{i=0}^{T-\tau} [\varepsilon(t_i)]^2 \ll \sum_{i=0}^{T-\tau} [f(t_i)]^2$$
$$\sum_{i=0}^{T-\tau} [\delta(t_i)]^2 \ll \sum_{i=0}^{T-\tau} [g(t_i)]^2, \tag{10}$$

we get from (8)

$$r(\tau) = \frac{\sum_{i=0}^{T-\tau} f(t_i)\, g(t_i + \tau)}{\left[\sum_{i=0}^{T-\tau} [f(t_i)]^2 \sum_{i=0}^{T-\tau} g(t_i + \tau)]^2\right]^{1/2}}, \tag{11}$$

which is in accordance with the expression (3) if we substitute the functions f, g for the functions F, G. With respect to the independence of fluctuation the other member of the numerator (8) plays a role only in case of autocorrelation for $\tau = 0$, when, however,

$$r(0) = \frac{\sum_{i=0}^{T} [f(t_i)]^2 + \sum_{i=0}^{T} [\varepsilon(t_i)]^2}{\left[\left\{\sum_{i=0}^{T} [f(t_i)]^2 + \sum_{i=0}^{T} [\varepsilon(t_i)]^2\right\}^2\right]^{1/2}} = 1$$

and therefore can also be neglected.

In a common case the detector registers simultaneously both anisotropic and isotropic changes in intensity. With regard to the Earth's rotation aniso-tropy manifests itself as the diurnal variation of intensity, consequently the de-pendence of type $A \cos \omega t_i$ with a 24 hour period may be a simple case of anisotropic function. In a simplified case it may hold for two apparatus located at stations with a difference in longitude between them denoted by λ regard-less of time changes in anisotropy

$$f(t_i) = f'(t_i) + A \cos \omega t_i \tag{13}$$

$$g(t_i + \tau) = g'(t_i + \tau) + B \cos [\omega(t_i + \tau - \lambda)].$$

Analogically with the antecedent we may accept here with sufficient accuracy

$$\sum_{i=0}^{T-\tau} f'(t_i) = \sum_{i=0}^{T-\tau} g'(t_i) = 0. \tag{14}$$

Let us consider that T is much larger than the duration of the isotropic and anisotropic changes of intensity. If we assume the anisotropic variations to predominate during the whole interval, then we may consider the isotropic changes as a part of fluctuations and after a simple computation we obtain

$$r(\tau) \doteq \frac{\cos (\omega\tau - \lambda)}{\left[1 + \left(\frac{\sum \varepsilon'^2}{A^2} + \frac{\sum \delta'^2}{B^2}\right) \frac{1}{\sum \cos^2 \omega t_i}\right]^{1/2}}, \tag{15}$$

where ε', δ' are the new composite fluctuations.

Another extreme case may appear in the period when isotropic variations are predominant. We can now put together anisotropic changes with fluctua-tions and obtain

$$r(\tau) \doteq \frac{\sum f'(t_i)\, g'(t_i + \tau)}{\left[\sum f'^2(t_i) \sum g'^2(t_i + \tau)\left\{1 + \frac{\sum \varepsilon'^2}{\sum f'^2} + \frac{\sum \delta'^2}{\sum g'^2}\right\}\right]^{1/2}}. \tag{16}$$

The isotropic and anisotropic parts of the correlation function cannot be separated generally and consequently the purely isotropic or anisotropic correlation coefficient cannot be determined in this way either. Only the

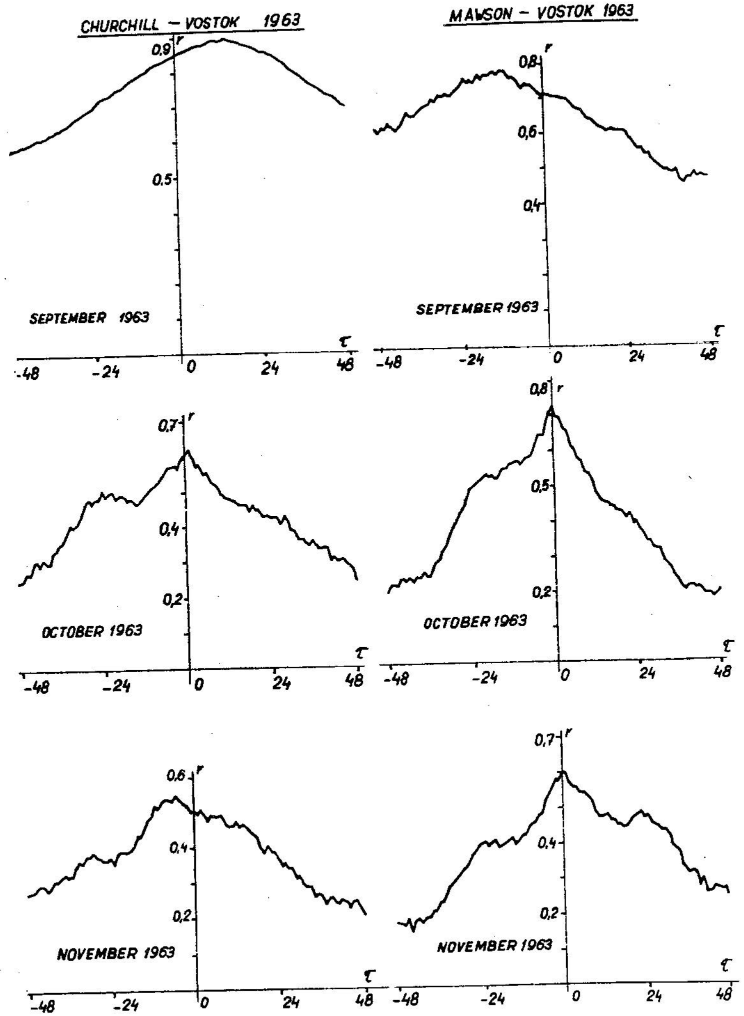

Fig. 1a.

maximal value of the coefficient of correlation for the interval from 0 to $\pm \tau$ can be accepted as a parameter characterizing the activity of cosmic radiation. In the period of a slight activity marginal effects or changes in the counting

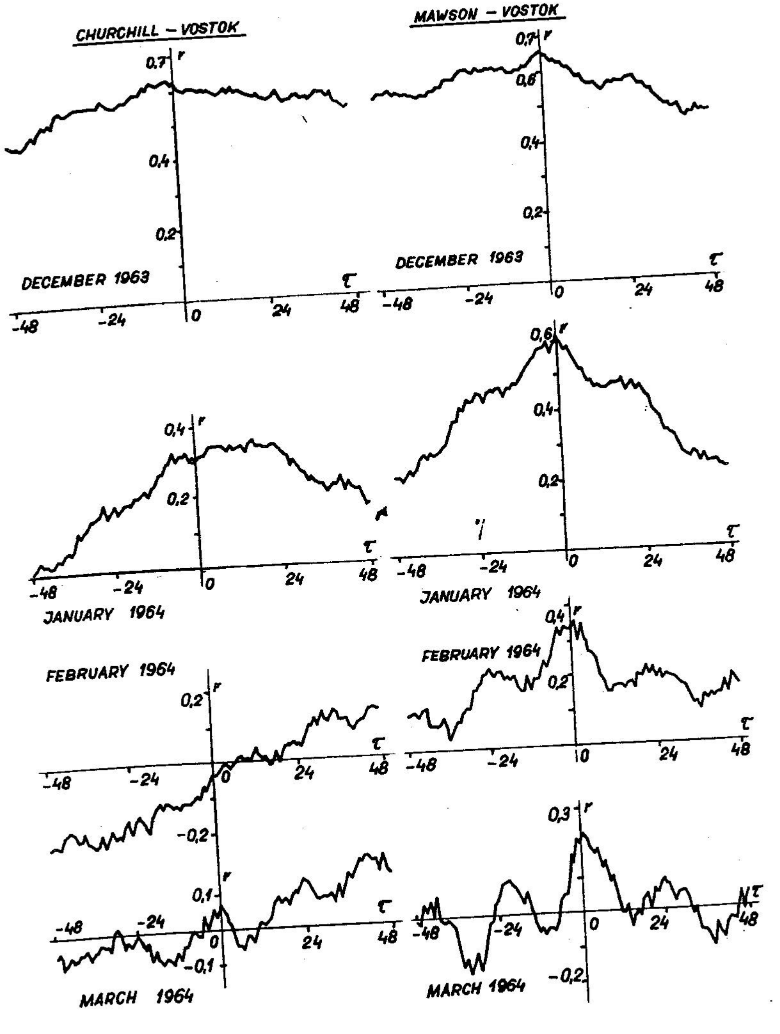

Fig. 1b.

level can cause a relatively steady increase or decrease of the correlation function in a part or the whole interval 2τ. In this case a different pair of stations may be chosen for calculating all the correlation, in the extreme case the local

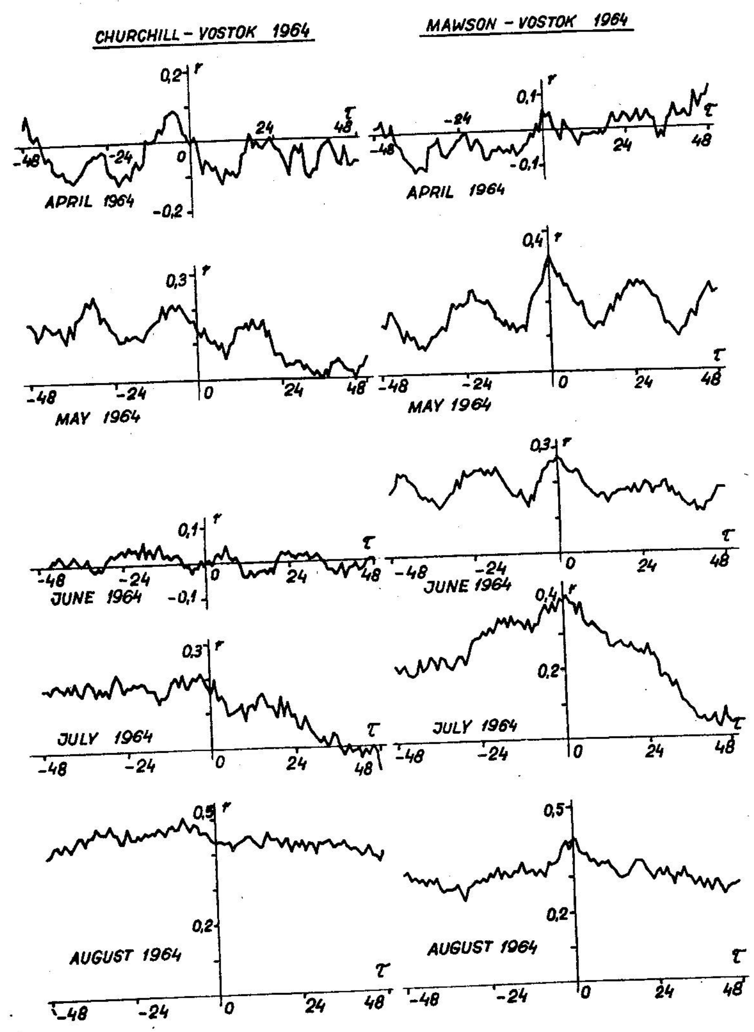

Fig. 1c.

maximum closest to $\tau = 0$ or the medium value for the interval $\pm \tau/2$ or the whole interval $\pm \tau$ may be taken.
Fig. 1, e. g. illustrates correlation functions for two pairs of stations: Mawson-
-Vostok, Churchill-Vostok from September 1963 till December 1964. The

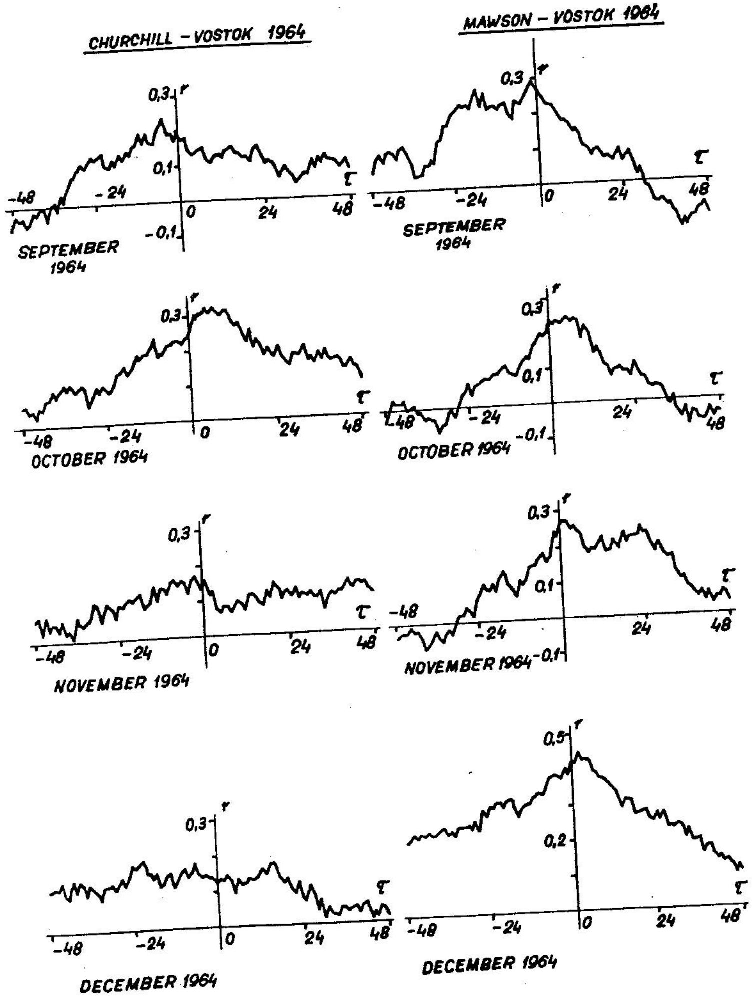

Fig. 1. Correlation functions for the pairs of stations Mawson-Vostok and Churchill-Vostok during the period September 1963—December 1964. One hour values of intensity are correlated, the interval length is always one month, the shift of values at both stations against each other: from 0 to $\pm$ 48 hours.

first pair of stations are Antarctic stations relatively close to each other, the other pair are stations distant from each other and situated in different hemispheres. In most cases the courses of the functions of correlation are similar to each other but the selection of stations plays evidently a substantial role in the maximal values of the coefficients of correlation within the same period.
Fig. 2 shows the courses of various geophysical parameters and maximal

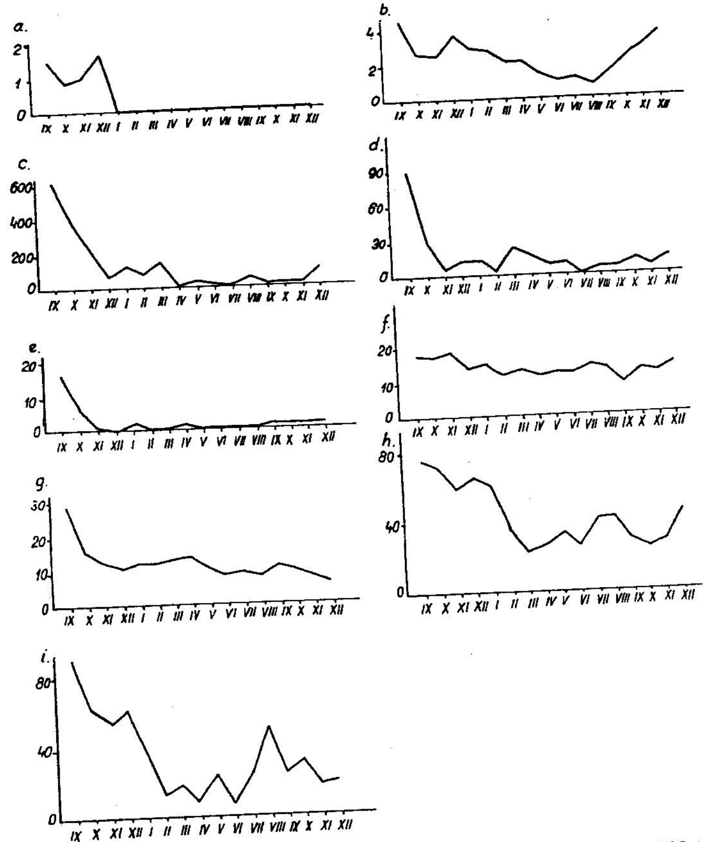

Fig. 2. The course of geophysical and solar parameters from September 1963 to December 1964: Time variations of the parameters a—g are plotted using the data published in [2]. a-ionospheric polar blackout index; b-ionospheric auroral blackout index; c-medium diurnal areas of solar spots in millionths of solar hemisphere; d-total number of all eruptions during one month; e-total number of chromospheric eruptions with the magnitudes 2 and more; f-index of the green coronal line ($\lambda = 5203$ Å) — daily average in a month; g-average daily value of the geomagnetic index A_p; h-maxima of the correlation coefficients for Mawson-Vostok; i-maxima of the correlation coefficients for Churchill-Vostok.

values of correlation coefficients in each month of the period between September 1963 and December 1964. Correlations among these parameters are evident, a more detailed study of these correlations, however, requires a fairly large number of correlation coefficients as to the length of periods as well as to the

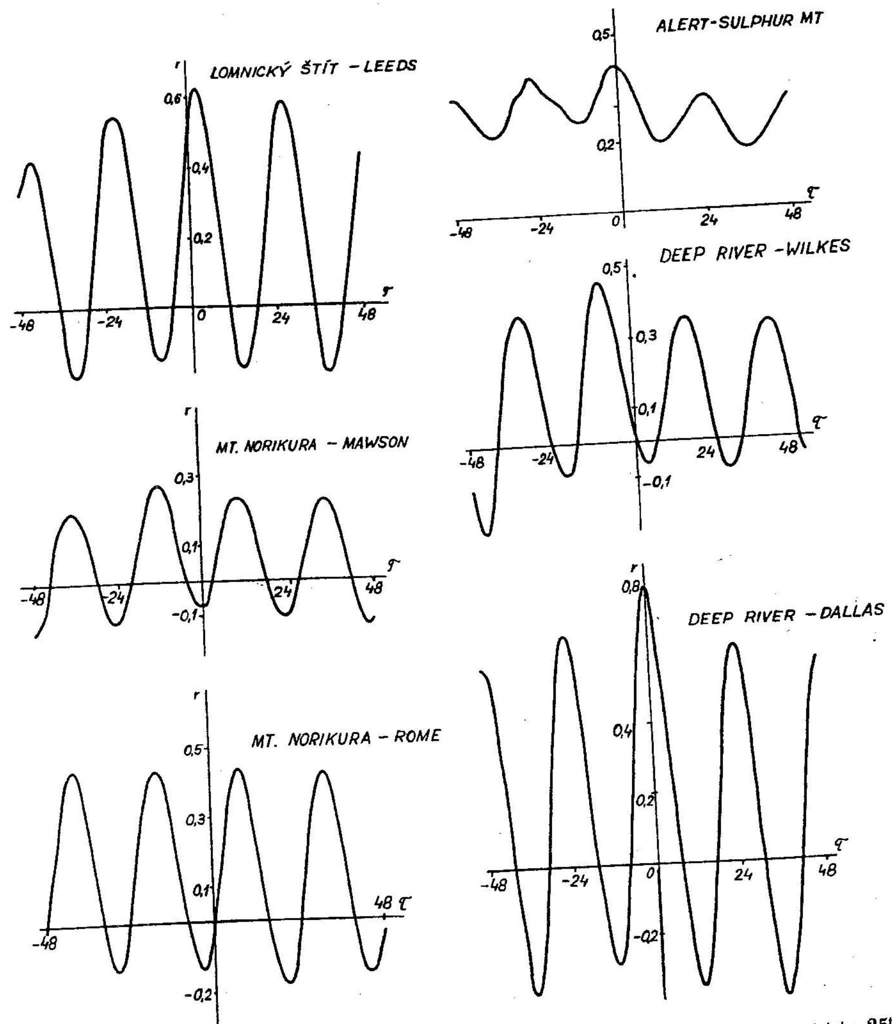

Fig. 3. Correlation functions for some pairs of stations during the period from 1st to 25th May 1966. 1. Alert-Sulphur Mt., 2. Deep River-Wilkes, 3. Deep River-Dallas, 4. Lomnický štít-Leeds, 5. Norikura-Mawson, 6. Norikura-Rome.

number of stations. The effect of the choice of stations is evident from Fig. 3 where functions of correlation for the period from 1st to 25th May 1966 from several pairs of stations are shown. Contrary to most curves illustrated in Fig. 1 an event is demonstrated here in which anisotropic variations played a decisive role. The influence of the choice of a station for a given period and changes for the same station in time are illustrated also in Fig. 4, where are given autocorrelation functions for some stations and periods.

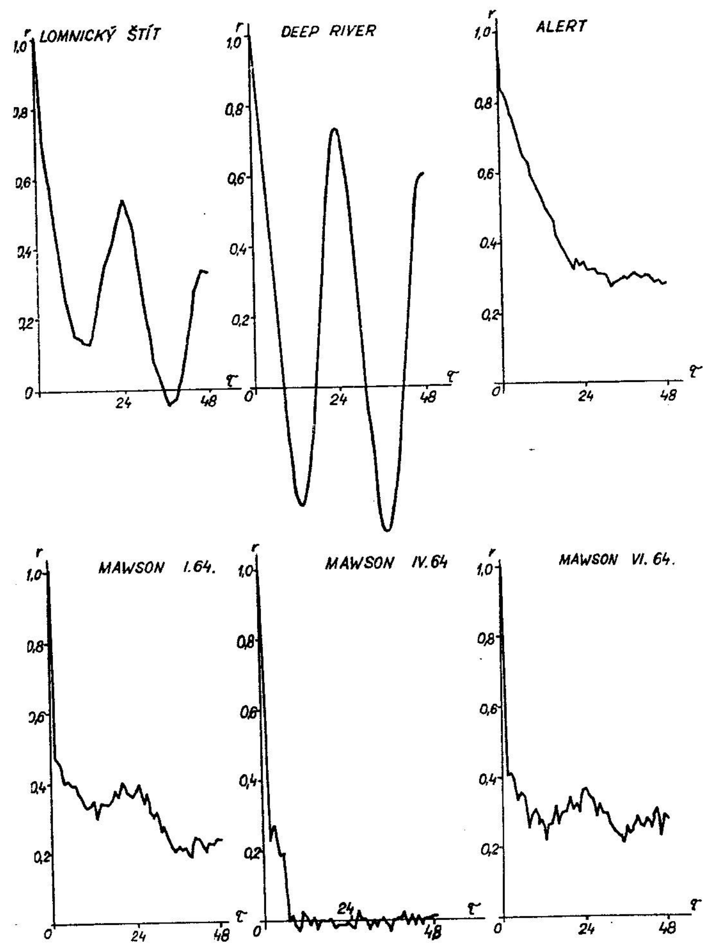

Fig. 4. Autocorrelation curves for the period from 1st to 25th May 1966 for the stations: 1. Lomnický štít, 2. Deep River, 3. Alert and for the period covering January, April and June 1964 for the station Mawson (curves 4, 5 and 6 respectively).

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