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AREAS OF DEFINITION FOR RELATIVISTIC PARTIAL WAVE AMPLITUDES

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The areas of definition for relativistic two-body into two-body partial wave amplitudes, previously established, are further analysed and some details are added. Several special cases are discussed and the families of the physical cuts are described for reactions involving only stable particles. Additional features relevant to the understanding of the behaviour of the physical cuts are elucidated.

I. INTRODUCTION

Experimental data concerning the behaviour of the strongly interacting particles are analysed often in terms of partial wave amplitudes. If experiments are available both in the low-energy region and at medium energies, the inelastic processes are to be included into the theoretical models which try to explain the data. This fact then leads to the investigation of the basic properties of a typical partial wave amplitude of the form

$$A_l(s) = \int_{-1}^{+1} d(\cos\vartheta) P_l(\cos\vartheta) \int_{t_0}^{\infty} \frac{\varrho(s, t)\mathrm{d}t}{t - t_s}, \tag{1}$$

where a two-body into two-body process with any admissible stable particles is understood,

$$m_1 + m_2 \to m_3 + m_4. \tag{2}$$

The areas of definition for various quantities have been already investigated many times. Let us mention here for instance the investigation of the properties of the Feynman integrals performed by Wu [1], the investigation of the „natural" positions of the branch cuts by Hwa [2] and the more involved treatment by Eden et al [3]. In what follows the areas of definition of a partial wave amplitude (1) are further investigated and some special cases are discussed. The present analysis represents the continuation of our previous paper

[4]. In general, the inelastic case involves a sort of continuation and a more detailed discussion of this case can be found also in refs. [5] and [6].

In relation (1), $s = -(p_1 + p_2)^2$ is the total energy squared in the direct channel, t_s is the (negative) momentum transfer squared $t_s = -(p_2 - p_4)^2$ and in the c. m. system we have

$$t_s = \frac{1}{2s}[-s^2 + s\Sigma - \varkappa + h(s)\cos\vartheta], \tag{3}$$

where

$$h(s) = (s - s_1)^{1/2}(s - s_2)^{1/2}(s - s_3)^{1/2}(s - s_4)^{1/2} = \\ = [(s^2 - s\Sigma + \varkappa)^2 - 4s(s\lambda + \nu)]^{1/2}, \tag{4}$$

and

$$\begin{aligned} \Sigma &= m_1^2 + m_2^2 + m_3^2 + m_4^2, \\ \varkappa &= (m_1^2 - m_2^2)(m_3^2 - m_4^2), \\ \lambda &= (m_1^2 - m_3^2)(m_2^2 - m_4^2), \\ \nu &= (m_1^2 m_4^2 - m_2^2 m_3^2)(m_1^2 - m_2^2 - m_3^2 + m_4^2). \end{aligned} \tag{5}$$

In relation (4) the thresholds are given in the following way

$$s_1 = (m_4 - m_3)^2,\; s_2 = (m_2 - m_1)^2,\; s_3 = (m_1 + m_2)^2,\; s_4 = (m_3 + m_4)^2. \tag{6}$$

The momenta of the external particles are denoted by $p_i (i = 1, 2, 3, 4)$, $\cos\vartheta$ is cosine of the c. m. scattering angle in the direct channel. We consider $z \equiv \mathrm{Re}(\cos\vartheta)$ and t in relation (1) as real quantities which vary in their physical values

$$-1 \leqslant \mathrm{Re}(\cos\vartheta) \leqslant +1, \tag{7}$$

$$t_0 \leqslant t < \infty,$$

t_0 being the threshold value in the t-channel. As far as the inequalities (7) are fulfilled, the partial wave amplitude (1) is not defined in the areas determined by the condition that the denominator in rel. (1) vanishes,

$$t - t_s = 0, \tag{8}$$

t_s given by relation (3). In the following section relation (8) is analysed and for some special cases its mapping in the complex s-plane is described in more details.

In the next section, the procedure for the elastic case is reviewed (compare with MacDowell, ref. [7]). In the third section the basic relations for the inelastic two-body processes are given and the fourth section deals in more detail with the following special cases:

$$\begin{aligned} \gamma + N &\to \pi + N, \\ \pi + N &\to \sigma + N, \\ \gamma + N &\to \varrho + \Delta, \\ \pi + \pi &\to N + \Delta. \end{aligned}$$

All these processes, as they stand, are considered as the s-channel ones and in the two last only the influence of the t-channel forces is included into the discussion; the u-channel forces might be considered after performing the necessary formal rearrangements. The last section summarizes the basic results.

II. THE ELASTIC CASE

We start with the consideration of equation (8) for the elastic scattering case, as e. g. $\pi + N \to \pi + N$. In this case $t_s = -2q^2(1 - \cos\vartheta)$ and with respect to eq. (8) we have

$$t = -2q^2(1 - \cos\vartheta), \tag{9}$$

or

$$\cos\vartheta = 1 + \frac{t}{2q^2}, \tag{10}$$

where q is the momentum in the c. m. system,

$$q^2 = \frac{1}{4s}[s - (M + \mu)^2][s - (M - \mu)^2] \equiv \frac{s^2 - s\Sigma + \varkappa}{4s},$$

$M(\mu)$ is the mass of the nucleon (pion) and, in this case, $\Sigma = 2(M^2 + \mu^2)$, $\varkappa = (M^2 - \mu^2)^2$ and the quantities ν and λ, given by relation (5), vanish.

Let us look in the complex s-plane, $s = x + iy$, for the lines where

$$\mathrm{Im}(t) = 0, \tag{11}$$

t given by eq. (9). Since $\cos\vartheta$ is now kept real, the condition $\mathrm{Im}(q^2) = 0$ or

$$y(x^2 + y^2 - \varkappa) = 0, \tag{12}$$

follows from eq. (11). On the other hand, if we are interested in the curves where

$$\mathrm{Im}(\cos\vartheta) = 0, \tag{13}$$

(for t real), the condition $\mathrm{Im}(q^{-2}) = 0$ is obtained using relation (10). Since for any finite and non-vanishing complex number q^2, the condition $\mathrm{Im}(q^2) = 0$ is equivalent to the condition $\mathrm{Im}(q^{-2}) = 0$, it follows that in any two-body elastic scattering case both sets of curves (11) and (13), see Fig. 1., are identical; the common set is given by eq. (12) and it is seen that it consists of the whole real s-axis plus a circle around the origin with the diameter $|s| = \sqrt{\varkappa} = M^2 - \mu^2$. One has then to determine that part of these curves where the inequalities (7) are satisfied.

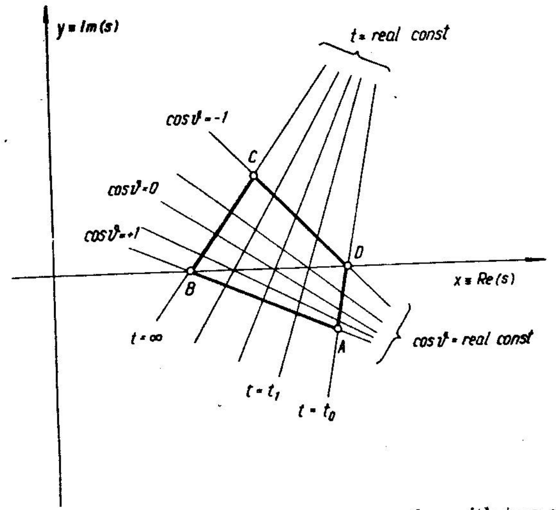

Fig. 1. The area $ABCD$ where $t - t_s = 0$, rel. (8), together with two sets of lines where cos = real const. and t = real const.

III. THE INELASTIC CASE

The process with arbitrary stable particles, relation (2), is now considered. The momentum transfer squared t_s expressed by ralation (3) is used in eq. (8), $t - t_s = 0$. From this relation we have

$$\cos\vartheta = \frac{A + iB}{\sqrt{C + iD}} = \mathrm{Re}(\cos\vartheta) + i\,\mathrm{Im}(\cos\vartheta), \tag{14}$$

where

$$A = x^2 - y^2 + x(2t - \Sigma) + \varkappa, \tag{15}$$
$$B = y(2x + 2t - \Sigma),$$
$$C = (x^2 - y^2)^2 + (x^2 - y^2)(4x^2 - 6x\Sigma + \Sigma^2 - 4\lambda + 2\varkappa) - $$
$$- 4x^3(x - \Sigma) - 2x(2\nu + \varkappa\Sigma) + \varkappa^2,$$

$$D = 2y[(x^2 - y^2)(2x - \Sigma) - 2x^2\Sigma + x(\Sigma^2 - 4\lambda + 2\varkappa) -$$
$$- 2\nu - \varkappa\Sigma].$$

If a given value z is prescribed for $\mathrm{Re}(\cos\vartheta)$ and if $\mathrm{Im}(\cos\vartheta) = 0$, the equations

$$A^2 - B^2 = z^2C, \tag{16}$$
$$2AB = z^2D,$$

follow from relation (14).

Eliminating z^2 from eqs. (16), we obtain the equation $2ABC - D(A^2 - B^2) = 0$, or

$$a(y^2)^3 + b(y^2)^2 + c(y^2) + d = 0, \tag{17}$$

which gives the (off the real s-axis lying part of the) lines where t is real and fixed and z^2 varies through the real values.

The equations (16) and (17) have been discussed with more detail in ref. [4]. For the sake of brevity we give here only the expression for the absolute term. in eq. (17),

$$d = [x^2 + x(2t - \Sigma) + \varkappa]\{x^4t - x^3(t\Sigma - 2\lambda) + 3x^2\nu + \tag{18}$$
$$+ x[t(\varkappa\Sigma + 2\nu) - \nu\Sigma - 2\varkappa\lambda] - \varkappa(\varkappa t + \nu)\}.$$

It is worthwhile to note that if $y = 0$, we have further

$$d = A\left[2\frac{\mathrm{d}A}{\mathrm{d}x}C - A\frac{\mathrm{d}C}{\mathrm{d}x}\right],$$

A, C given by eq. (15), and from the condition d $= 0$ we obtain $\frac{\mathrm{d}}{\mathrm{d}x}(\cos^2\vartheta) = 0$. The last equality implies that the branches $y = y(x)$ of the curve (17) intersect the real s-axis at the points where $\cos^2\vartheta$, varying along the real s-axis, has its extreme values. This fact is demonstrated, e. g., in Figs. 9 and 10. As far as the zero points $z = 0$ lie on the real s-axis, the curve (17) passes throught them, as it is seen from eq. (18).

With

$$R \equiv x^2 + y^2 - \varkappa, \tag{19}$$

the eq. (17) might be expressed in the form

$$R\{4x^2\varkappa t + 2xt(R + 2\varkappa)(t - \Sigma) - t\Sigma(2t - \Sigma)(R + \varkappa) +$$

$$+ R^2t + \lambda[2x(R + 2\varkappa) + 2(R + \varkappa)(2t - \Sigma)]\} +$$

$$+ \nu(R + \varkappa)[4x^2 + 4x(2t - \Sigma) + 4t(t - \Sigma) + \Sigma^2] - R^2\} = 0. \qquad (20)$$

Now it is seen that if $\nu = 0$ (as, e. g., in the elastic case), the well known circle given by $R = 0$ is to be taken into consideration.

For a given value of the momentum transfer squared, t, the part of the line off the real s-axis, where $\text{Im}(\cos\vartheta) = 0$, might be determined from eq. (17). Only that part is to be taken into account where $z^2 \leqslant 1$. However, it turns out sometimes that as far as the integration in the partial wave amplitude (1) is performed along the physical angles, the integration path $\int_{-1}^{+1} \ldots d(\cos\vartheta)$ is composed of some disconnected parts (c. f. Figs. 5, 6, 9).

If the momentum transfer squared, t, is eliminated from eqs. (16), an equation is obtained for the curves, where $\cos\vartheta$ is real and fixed; we give here only the equation resulting for $z^2 = 1$,

$$\lambda R^2 + \nu(2x - \Sigma)R + \nu^2 = 0. \qquad (21)$$

Eq. (21) gives the dependence $y = y(x)$ for the points off the real s-axis where $z^2 = 1$ and t is real and varies; on the real axis their location $x_{1,2} = x_{1,2}(t)$ is given by

$$tx^2 + x(t^2 - t\Sigma + \lambda) + \varkappa t + \nu = 0. \qquad (22)$$

In the next section, some branches of the aforementioned equations are considered for special processes. For a fixed t, the connections of the branch points or the branch lines are called „physical" if the scattering angle varies through the physical values along them. In the general case ($\Sigma\varkappa\lambda\nu \neq 0$), eqs. (17) and (21) have no common solution, thereby proving essentially the existence of the areas where a partial wave amplitude is not defined. In all figures (except Fig. 5) the singularities arising from the direct s-channel are omitted and the areas under consideration are hatched; only qualitative schemes are drawn. In the computations the following numerical values have been used: the mass of the nucleon squared, $M^2 = 45{\cdot}16$; the mass of the $\Delta(1236)$ squared, $\Delta^2 = 77{\cdot}97$; the mass of the ϱ-meson squared, $m_\varrho^2 = 30{\cdot}25$; all quantities are expressed in units where the pion mass μ is unity and also $\hbar = c = 1$. Δ means the N^*_{33} resonance of the πN-system.

Before entering into details we recall the main results of ref. [8]: The sign of $\cos\vartheta$ is the same at the complex conjugated points on the same sheet of the s-plane; on the real s-axis, along the curve $x = x(t)$, given by eq. (22), $\cos\vartheta$ changes sign at the thresholds $s_i (i = 1, 2, 3, 4)$ given by eq. (6); at these thresholds, the curve $x = x(t)$ has its extremes. By means of the function $h(s)$, eq. (4), a two-sheeted Riemann surface can be introduced, the two sheets being

connected along the real s-axis between the points s_1 and s_2 and between s_3 and s_4. Then the sign of $\cos\vartheta$ at a point on the first sheet is opposite to that on the second sheet. The introduction of this Riemann surface is understood also in the next section and as far as it is not stated otherwise, the values of $\cos\vartheta$ are understood to be those on the first (physical) sheet.

IV. EXAMPLES AND SOME DETAILS

1. The first two examples represent a special case of the reaction (2) where $m_2 = m_4$ and therefore $\lambda = 0$.

(i) The photoproduction of pions on nucleons, $\gamma + N \to \pi + N$.

In addition to the set of points arising from the t-channel continuum, mentioned in ref. [5], one finds that the area under investigation lies between the curves given by eqs. (21) and (17) for $t = 4\mu^2$, and to the left from their points of intersection $P^{(\pm)}$ $[x = M^2 - \frac{3}{2}\mu^2,\ y = \pm \frac{3}{2}\mu (M^2 - \mu^2)^{1/2}]$. This area is very narrow.

From the pion pole term $t = \mu^2$, in addition to the point $s = M^2$ (compare with Fig. 9 of ref. [5]), one obtains also a branch cut which lies partially off the real s-axis (for a qualitative picture see here Fig. 2), Namely, the whole curve arising from the integration $\int_{-1}^{0} \ldots dz$ for this pole term $t = \mu^2$ is contracted into the point $s = M^2$, on the second sheet; at the point $s = M^2$ on the first sheet, the whole integration path $\int_{0}^{1} \ldots dz$ is contracted which emerges through

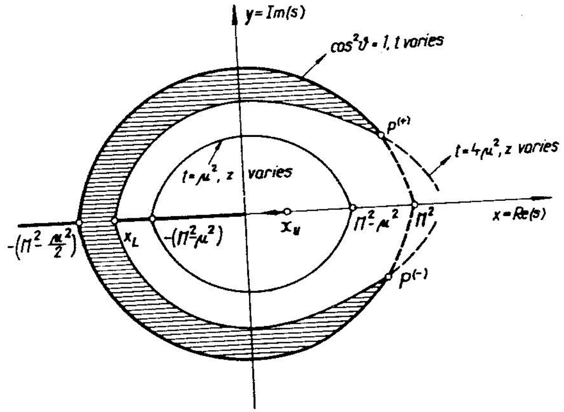

Fig. 2. The points where $t - t_s = 0$ for the process $\gamma + N \to \pi + N$.

this threshold from the second sheet. The other part $\int_0^1 \ldots dz$, on the first
sheet, begins at $s = M^2 - \mu^2$ (where for $t = \mu^2$ we have $\cos\vartheta = 0$), conti-
nues above and below the real s-axis up to the point $x = -(M^2 - \mu^2)$, $y = 0$
and then it contains the whole negative real s-axis. All three curves, shown in
Fig. 2 are close to the circle $R = 0$.

The areas of definition arising from the u-channel are identical with the
location of singularities of ref. [5]; namely, from the continuum $u \geqslant (M + \mu)^2$
a cut arises for $-\infty < x \leqslant x_u$; $x_u = M^3/(M + \mu) - M\mu$. The crossed
nucleon pole term $u = M^2$ has two contributions in the s-plane: The integration
path of the integral $\int_{-1}^{+1} \ldots dz$ of the first (righthand) contribution is all contracted
at the point $s = M^2$; however, the integration path of the second (left-hand)
contribution of the integral $\int_{-1}^{+1} \ldots dz$ is extended along the whole negative
x-axis.

Using the expression for the coefficient d, eq. (18), the point x_L, Fig. 2,
(the intersection of the curve where $t = 4\mu^2$ and z varies with the negative
part of the s-axis) can be determined. We have obtained

$$x_L = -M^2 + \frac{5}{8}\mu^2 - \frac{\mu^2}{2}\frac{\mu^2}{M^2}\frac{3}{8}\left\{\frac{3}{8} + \frac{7}{64}\frac{\mu^2}{M^2} + 0\left(\frac{\mu^4}{M^4}\right)\right\}. \tag{23}$$

The value (23) is different from $-\left(M^2 - \frac{\mu^2}{2}\right)$, which gives the intersection
of the curve where $\cos^2\vartheta = 1$ and t varies with the negative x-axis (cf. Fig. 2):
this fact demonstrates that the area where a partial wave amplitude is not
defined, is present also in the photoproduction process under investigation.

(ii) We consider the process $\pi + N \to \sigma + N$, where σ represents a strongly
interacting pion pair with quantum numbers $I^G J^P = 0^+0^+$ and with the mass,
say, about 400 MeV. For simplicity we treat σ as a single, stable particle; its
mass is denoted by m_σ.

As to the zero points of the Mandelstam, denominators arising in the s-plane
from the t-channel, we have the following picture:

The curve where $\cos^2\vartheta = 1$ and t varies along the real values has the form
as it is qualitatively seen in Fig. 3.

The integration path $\int_{-1}^{+1} \ldots dz$ corresponding to the beginning of the t-channel
continuum, $t = \text{const.} = 9\mu^2 = t_0$ (see Figs. 4a, b; the process shown in Fig.
4a does not give rise to an anomalous threshold in the t-channel) starts at the

points T_1, T_2 where $\cos\vartheta = -1$ (Figs. 3 and 5); it passes along the real axis
from T_1 and from T_2 to the point T'. From T' it lies off the real s-axis and
itersects with it again at the point T'' (Fig. 5) and then it contains the whole
negative part of the real x-axis. For $t = t_0 + \varepsilon$ (ε small, real, positive), the
corresponding point T' is shifted somewhat to the right from the position

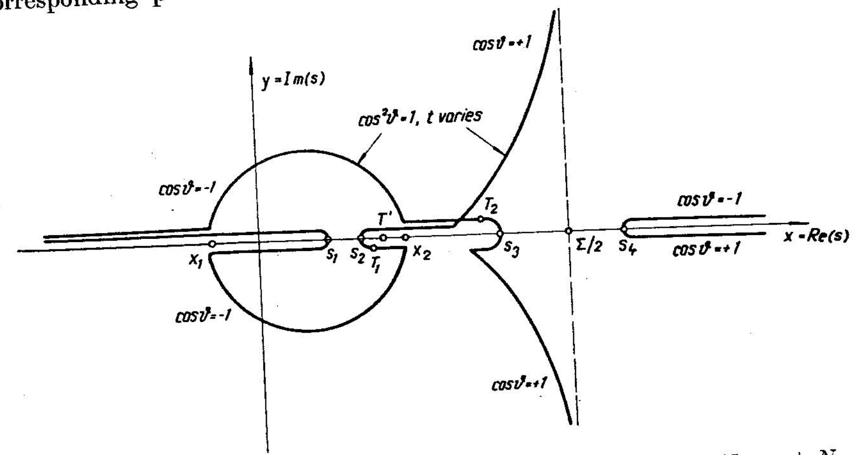

Fig. 3. The curve where $\cos^2 = 1$ and t varies for the process $\pi + N \to \sigma + N$.

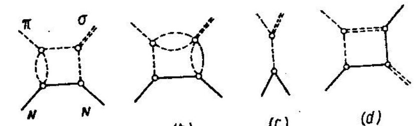

Fig. 4. Some basic diagrams for the process $\pi + N \to \sigma + N$.

given in Figs. 3 and 5 and the corresponding point T'' lies a little more to the
left from the position given in Fig. 5.

We describe now the integration path $\int_{-1}^{+1} \ldots dz$ for the pion pole term, $t = \mu^2$,
fig. 4c, on both sheets; this result has been obtained after varying slowly the
masses involved and taking into account the changes of sign of $\cos\vartheta$ at the
thresholds.

On both sheets (see Fig. 6) this integration path is composed of two parts;
the first one passes from the points A, A^* (with the subscripts corresponding
to the sheets), where $\cos^2\vartheta = 1$, to the point B. (In our example (i) this part
is contracted into the threshold point $s = M^2$). The second part passes from
B' (at B and B' we have $\cos\vartheta = 0$) off the real s-axis to the point C and then
it contains the whole negative part of the real s-axis. The physical connection

of the branch points, as far as $t = \mu^2$ and $\cos\vartheta$ is real and $-1 \leqslant \cos\vartheta \leqslant +1$,
are shown in Fig. 6. We emphasize that on the first sheet only the positive
values of $\cos\vartheta$ are met and therefore *it is erroneous* to perform the integration

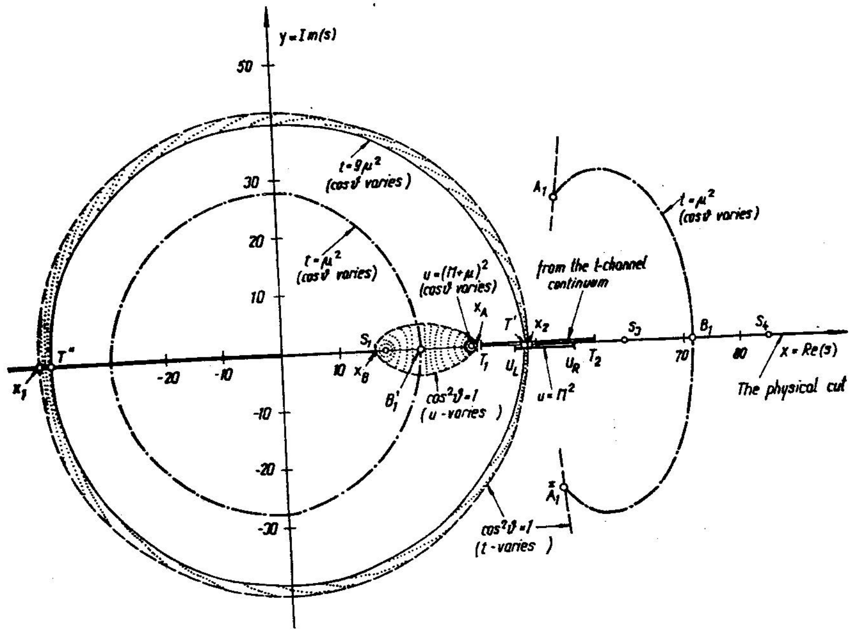

Fig. 5. The points where a partial wave amplitude of the process $\pi + N \to \sigma + N$ is not defined; cf. rel. (8).

$\int_{-1}^{+1} \ldots dz$ for the pole term $t = \mu^2$ on the first sheet. This point might be con-
fronted with several references, where its importance was not recognized.
A similar result is valid also for the pion pole term $t = \mu^2$ in $\pi + N \to \varrho + N$
(and in other cases). Thus, in Fig. 6 the lines $O_2C_2B_2'B_1A_1$, $(-\infty_2)$ $C_2B_2'B_1A_1^*$,
$A_2^*B_2B_1'C_1$ $(-\infty_1)$, and $A_2B_2B_1'C_1O_1$ correspond to the integration $\int_{-1}^{+1} \ldots dz$
($t = \mu^2$) along the physical angles.

For the zero points of the Mandelstam denominators arising from the u-channel we have the following results:

From the continuum beginning at $u_0 = (M + \mu^2)$, Fig. 4d, an area arises, where a partial wave amplitude is not defined. This area lies between a part of the curve α) where $\cos^2\vartheta = 1$ and u varies and where β) $u = (M + \mu)^2$

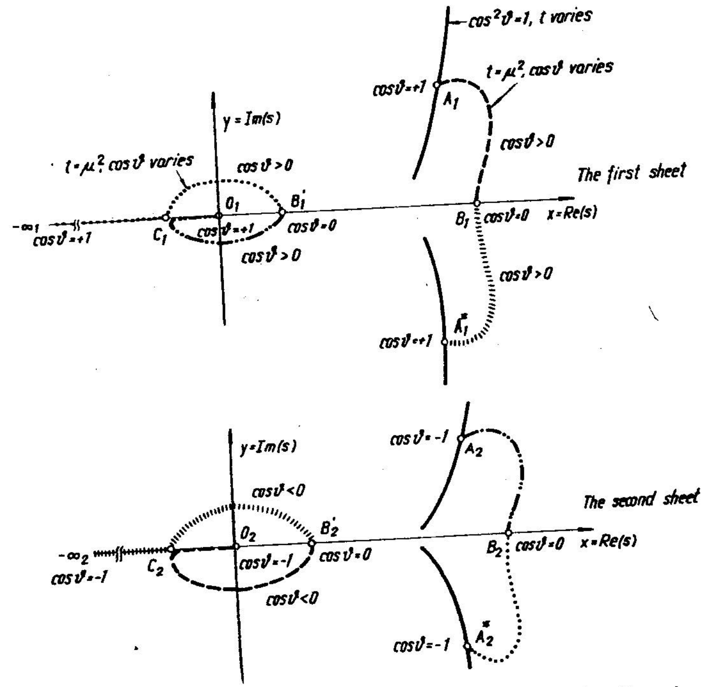

Fig. 6. The physical branch cuts of a partial wave amplitude for the pion pole term $t = \mu^2$ in the process $\pi N \to \sigma N$ (or $\sigma N \to \varrho N$, etc.). The lines with the same sign represent the physical continuation of the cuts. On the second sheet the sign of $\cos\vartheta$ is opposite to that on the first sheet.

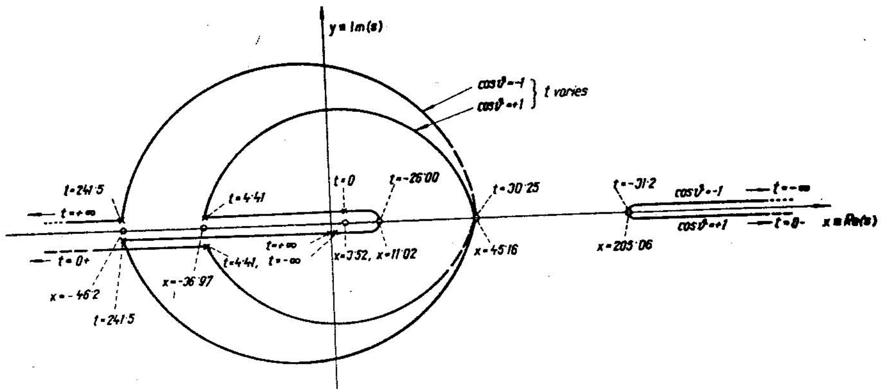

Fig. 7. The curve where $\cos^2\vartheta = 1$ and t varies for the process $\gamma N \to \varrho\Delta$.

and $\cos^2 \vartheta$ varies, compare with. Fig. 5. One „corner" of this area is at the point $x_A = (M^3 + \mu m_\sigma^2)/(M + \mu) - M\mu$, $y = 0$, and it contains also a part of the real s-axis, $-\infty < x \leqslant x_B \equiv (M^3 + \mu^2 m_\sigma)/(M + m_\sigma) - M m_\sigma$.

The exchanged nucleon pole term $u = M^2$ contributes by a cut between the point U_L and U_R and between $s = 0$ and $s = -\infty$.

2) The following examples represent two special cases, when, off the real s-axis, the curve $\cos^2 \vartheta = 1$ and t varies consists of two parts, one part lying inside the other and the t-dependence on them is opposite. This fact then implies that the physical lines for the higher values of the momentum transfer, t, tend in the first case to the outher part from outside while in the second case to the inner part from inside (if all real values of t would be allowed, then the partial wave amplitude would be defined only in a narrow gap between the two aforementioned parts). In what follows only the influence of the t-channel forces is analysed.

(iii) The photoproduction of the vector mesons, $\gamma + N \to \varrho + \Delta$. The

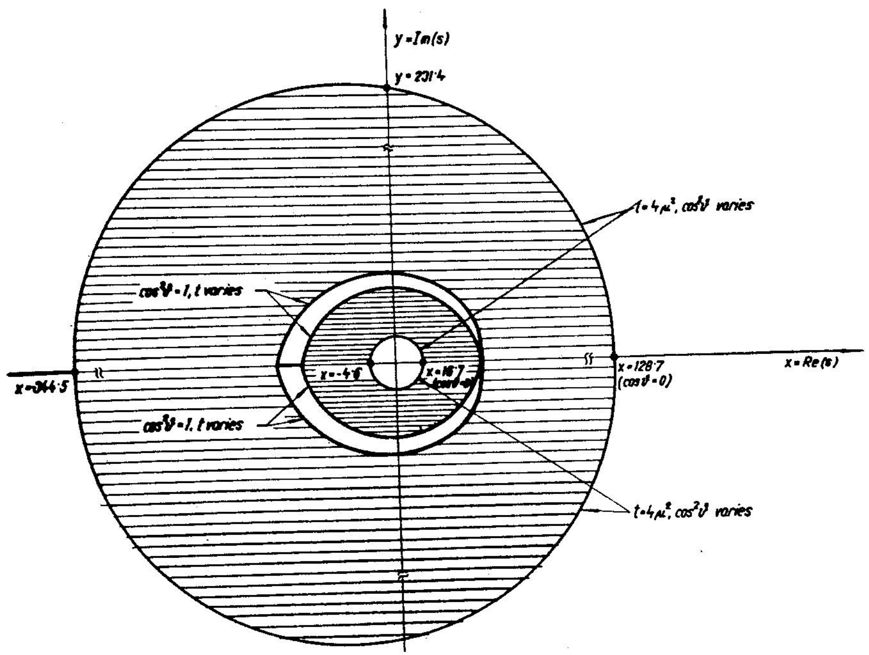

Fig. 8. The area where $t - t_s = 0$ with $4\mu^2 \leqslant t < \infty$, $0 \leqslant \cos^2 \vartheta \leqslant 1$ for the process $\gamma N \to \varrho\Delta$.

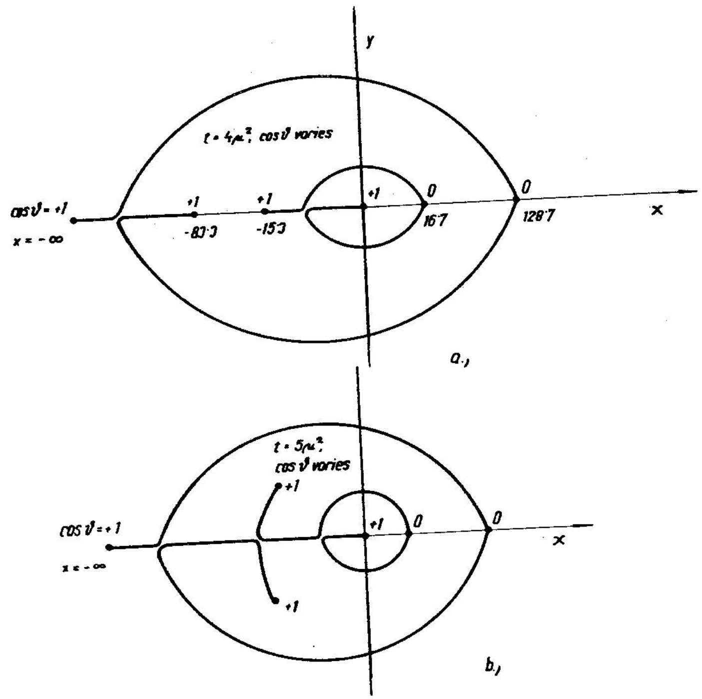

Fig. 9. The physical branch lines ($0 \leqslant \cos^2 \vartheta \leqslant 1$) for $\gamma + N \to \varrho + \Delta$ if: a) $t = 4\mu^2$, b) $t = 5\mu^2$ (the first sheet is understood). Another possibility is obtained by $y \to -y$.

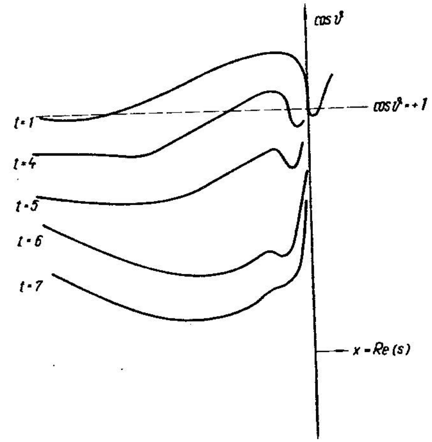

Fig. 10. The dependence $\cos \vartheta = f(x)$ on the real s-axis for $t = 1, 4, 5, 6$ and $7\mu^2$ of the process $\gamma + N \to \varrho + \Delta$. The integration is to be performed only for $\cos^2 \vartheta \leqslant 1$. The physical branch lines intersect the real x-axis (compare with Fig. 9) at the extreme points of the aforementioned dependence.

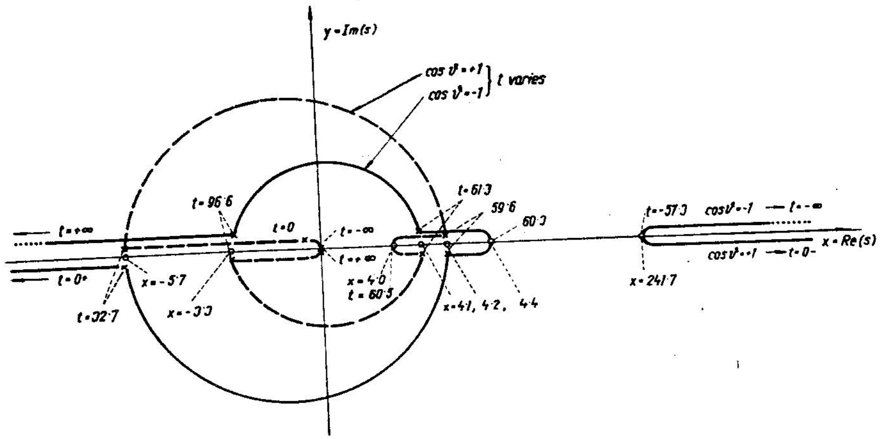

Fig. 11. The curve where $\cos^2 \vartheta = 1$ and t varies for the process $\pi\pi \to N\Delta$.

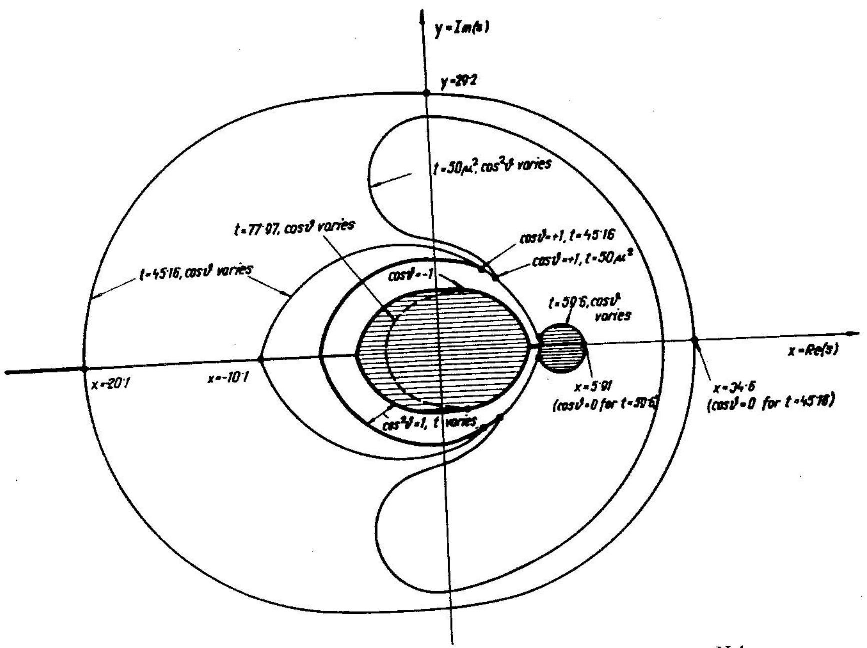

Fig. 12. The points where $t - t_s = 0$ for the process $\pi\pi \to N\Delta$.

curve $\cos^2 \vartheta = 1$ and t varies has the form of Fig. 7. In Fig. 8 the area is seen where the partial wave amplitude (1) is not defined, $t - t_s = 0$, using in relation (7) the threshold value $t_0 = 4\mu^2$. The physical branch lines for $t = 4\mu^2$ and $t = 5\mu^2$ are shown in Fig. 9. This tape of connections may be easier understood if the dependence $\cos \vartheta = f(x)$, Fig. 10, is taken into account. As to Fig. 9, the continuation of the physical connections with $-1 \leqslant \cos \vartheta \leqslant 0$ can be found on the second sheet.

(iv) The process $\pi + \pi \to N + \Delta$. The curve $\cos^2 \vartheta = 1$ and t varies is shown in Fig. 11 and the area where $t - t_s = 0$, using in rel. (7) $t_0 = (M + \mu)^2 = 59{\cdot}6$, is seen in Fig. 12. together with the physical connections for some constant values of t.

(v) In addition we give Fig. 13, where the qualitative picture of the area can be seen where $t - t_s = 0$ for the reaction $\pi + \Delta \to \varrho + N$.

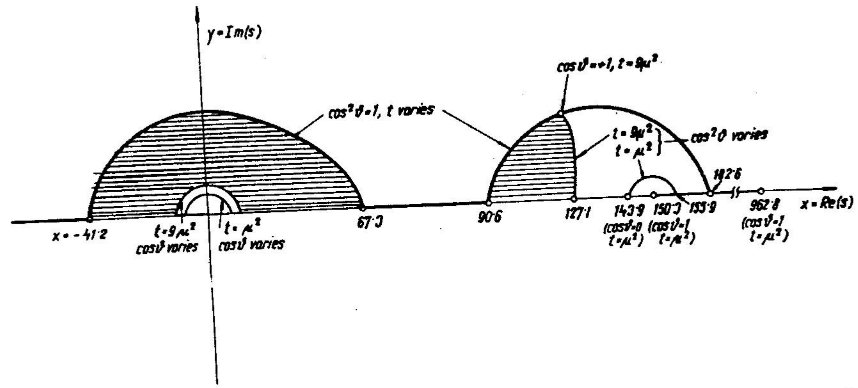

Fig. 13. The qualitative picture of the area where $t - t_s = 0$ for the reaction $\pi + \Delta \to \varrho + N$. The area arises for $9\mu^2 \leqslant t < \infty$, $0 \leqslant \cos^2 \leqslant 1$. Also the physical branch line for the pion pole term $t = \mu^2$ is seen (the other part covers $-\infty < x \leqslant 0$). The mirror image is omitted.

V. SUMMARY

For some relativistic two-stable-body inelastic processes areas have been described where the Mandelstam denominators of the partial wave amplitudes vanish as far as the momentum transfer squared and the scattering angle have physical values. If the procedure is used that the continua in the crossed channels are replaced by (stable or also unstable) objects (the so-called particle approximation), instead of the aforementioned areas only some isolated physical branch cuts are met in a partial wave amplitude [9]. For a given value of the momentum transfer squared, the location of the *physical* branch lines can be considered as one indicator about the importance of the corresponding exchange. On the other hand, if the physical branch lines are deformed (thereby involving a continuation, e. g., in the complex cos ϑ-plane), the location of the branch lines with respect to one another, can be exchanged and the considerations concerning the relative importance of the corresponding exchange might lead to quite different conclusions. However, if any of the objects involved becomes unstable, the physical cuts are deformed and they are asymmetrically located with respect to the real s-axis. A forth coming paper will be devoted to such a case. If in the complex s-plane the integration along the physical branch lines is performed, no kind of continuation in the complex cos ϑ-plane is necessary; this fact might be suitably used if the particle approximation were applied in a dynamical model of strong interactions.

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