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# THE PRESET-COUNT AND PRESET-TIME IN THE MEASUREMENT OF RADIOACTIVITY

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The present paper describes a way of determining preset count and preset-time by a choiced standard deviation.

In cases where it is not possible to adjust precisely an evaluated prechoice on the apparatus, relations are derived which enable to express numerically the change of the beforehand selected relative standard deviation in dependence on the modified preset.

## INTRODUCTION

The activity of a radioactive source is expressed by the ratio of the measured counts to the counting time as well as to the efficiency of the detecting counter. If the radioactivity is of the same order as the background, a correction for the background is needed. We discuss the problem of the length of the counting time so as to get the chosen accuracy of measurement. The accuracy of measurement is quantified by means of the standard deviation or by the coefficient of variation, that means the standard deviation divided by the mean value.

Earlier investigators have dealt with the problem of the length of measuring the sample with the background as well the background alone [1—7]. We will give a survey of this problem especially as regards the practical use of the relations derived. Finally we shall complete the solution by the derived relations for correcting the chosen coefficient of variation for cases where the calculated value for the time- and count-preset cannot be precisely adjusted on the apparatus.

## 1. THE THEORETICAL VALUE OF THE COEFFICIENT OF VARIATION

The letter $n$ will denote the mean count rate $n_i$ of the sample. The standard deviation will be $\sigma$ and $D$ the dispersion. The coefficient of variation is



$$c = \sigma/\bar{n}; \qquad c^2 = D/\bar{n}^2. \tag{1}$$

If we measure the background alone, the symbol will be denoted by $P$, and for the sample with background by $V + P$. For values regarding the sample itself we use $V$. Therefore:

$$n_V = n_{V+P} - n_P, \tag{2}$$

where $n_V$ is the count rate of the sample only.

### a. Preset time

Consider a sequence of particles emitted according to a Poisson process with the theoretical count rate $n$. The probability that $m$ numbers of particles emitted in the time $T$ will be equal to a whole number $i$ is given by

$$\frac{(nT)^i}{i!}\,e^{-nT}, \qquad i = 0, 1, 2, \ldots \tag{3}$$

where the mean value of the count rate in the time $T$ equals $\bar{m} = nT$ and the dispersion $D(m) = nT$.

The mean value and the dispersion of the measured count rate $n = m/T$ are

$$\bar{n} = \frac{\bar{m}}{T} = \frac{nT}{T} = n, \tag{4}$$

$$D(n) = \frac{D(m)}{T^2} = \frac{nT}{T^2} = \frac{n}{T},$$

the same values o the sample itself

$$n_V = n_{V+P} - n_P \tag{5}$$

$$\bar{n}_V = \bar{n}_{V+P} - \bar{n}_P = n_{V+P} - n_P = n_V$$

$$D(n_V) = D(n_{V+P}) + D(n_P) = \frac{n_{V+P}}{T_{V+P}} + \frac{n_P}{T_P},$$

where $T_{V+P}$ is the counting time for the sample with background and $T_P$ the counting time for the background only.

The square of the coefficient of the variation $c$ for the chosen time $T_{V+P}$ and $T_P$ is

$$c^2 = \frac{1}{n_V^2}\left(\frac{n_{V+P}}{T_{V+P}} + \frac{n_P}{T_P}\right) \tag{6}$$

### b. Preset count

If we assume that the emission of particles is in agreement with the Poisson process and proceeds according to the theoretical count rate $n$, in the time $t$ there will be emitted $N$ particles as a random amount, with a gamma distribution. Therefore the probability that the number of $N$ particles will be detected in the interval of time $\tau$ to $\tau + \Delta\tau$ is

$$\frac{n^N}{(N-1)!}\,\tau^{N-1}\,e^{-n\tau}\,\Delta\tau. \qquad (7)$$

The mean value $1/t$ is

$$\frac{n}{N-1} = \left(\overline{\frac{1}{t}}\right)$$

and the dispersion

$$D\left(\frac{1}{t}\right) = \frac{n^2}{(N-1)^2\,(N-2)}. \qquad (8)$$

The mean value and the dispersion of the detected count rate $n = N/t$

$$\bar{n} = N\left(\overline{\frac{1}{t}}\right) = \left(\frac{N}{N-1}\right)n \qquad (9)$$

$$D(n) = N^2 D\left(\frac{1}{t}\right) = \left(\frac{N}{N-1}\right)^2 \frac{n^2}{N-2}$$

It may be noted that the mean value and dispersion differ mathematically from relations (4), although we take into consideration that $N = nT$. For a long time $T$ and a great number of pulses $N$ this type of difference will not be too marked [8].

The mean value and the dispersion for the sample alone

$$n_V = n_{V+P} - n_P$$

will be:

$$\bar{n}_V = \bar{n}_{V+P} - \bar{n}_P = \frac{N_{V+P}}{N_{V+P}-1}\,n_{V+P} - \frac{N_P}{N_P-1}\,n_P \qquad (10)$$

$$D(n_V) = D(n_{V+P}) + D(n_P) = \left(\frac{N_{V+P}}{N_{V+P}-1}\right)^2 \frac{n_{V+P}^2}{N_{V+P}-2} + \left(\frac{N_P}{N_P-1}\right)^2 \frac{n_P^2}{N_P-2},$$



if $N_{V+P}$ is the count of pulses for the sample and background. The square of the coefficient of variation for the selected count of pulses $N_{V+P}$ and $N_P$ is:

$$c^2 = \frac{\left(\frac{N_{V+P}}{N_{V+P}-1}\right)^2 \frac{n_{V+P}^2}{N_{V+P}-2} + \left(\frac{N_P}{N_P-1}\right)^2 \frac{n_P^2}{N_P-2}}{\left[\frac{N_{V+P}}{N_{V+P}-1}\,n_{V+P} - \frac{N_P}{N_P-1}\,n_P\right]^2}. \qquad (11)$$

## 2. APPROXIMATE COUNTING TIME

Let us consider the following examples:

a. The counting time for the sample with background and for the background alone is the same. We choose such a counting time that

$$T_{V+P} = T_P = T.$$

The count rates $n_{V+P}$ and $n_P$ are arbitrary. If the coefficient of variation for $n_V$ is small enough, for instance 0·05, we can express $T$ and get

$$c^2 = \frac{1}{n_V^2}\,\frac{n_{V+P} + n_P}{T}$$

$$T = \frac{1}{c^2 n_V^2}\,(n_{V+P} + n_P)$$

$$T = \frac{1}{c^2 n_V}\,\frac{n_V + 2n_P}{n_V},$$

where

$$\xi = \frac{n_V}{n_P},$$

then

$$T = \frac{1}{c^2 n_V}\left(1 + \frac{2}{\xi}\right). \qquad (12)$$

If we choose the counting time $T$ we must first know the approximate value of $\xi$. It may be more convenient to estimate $\xi$ smaller, so as to get the counting time rather longer than shorter. In cases with a sample less active than the background we shall have $\xi < 1$. It is less probable that we will have to measure samples with an activity $\xi < 0{\cdot}01$. In Fig. 1 there is the graphic dependence of the product $Tn_V$ on the value $\xi$ for different coefficients of variation.

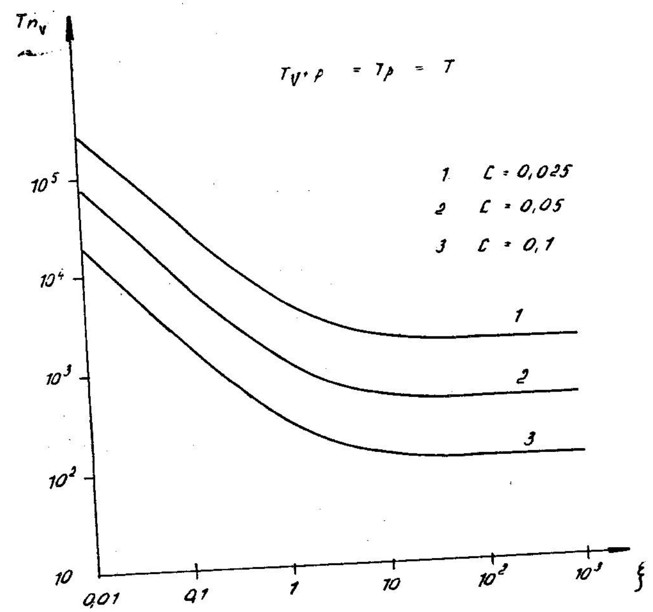


Fig. 1. Preset time.

b. Present counts for samples with background and for background are equal. In the above case we choose $N_{V+P} = N_P = N$, while the counting times $t_{V+P}$ and $t_P$ are different. From equation (11) is derived

$$c^2 = \frac{\left(\dfrac{N}{N-1}\right)^2 \dfrac{n_{V+P}^2}{N-2} + \left(\dfrac{N}{N-1}\right)^2 \dfrac{n_P^2}{N-2}}{\left[\left(\dfrac{N}{N-1}\right) n_{V+P} - \left(\dfrac{N}{N-1}\right) n_P\right]^2} \tag{13}$$

$$c^2 = \frac{1}{N-2} \frac{n_{V+P}^2 + n_P^2}{n_V^2}$$



$$N = 2 + \frac{1}{c^2}\left(1 + 2\,\frac{\xi + 1}{\xi^2}\right),$$

where again we use the term

$$\frac{n_V}{n_P} = \xi.$$

It is necessary to know at least the approximate value of $\xi$. From Fig. 2 we can read the number of pulses $N$, which we have to choose for a preset count according to different values of the coefficient of variation.

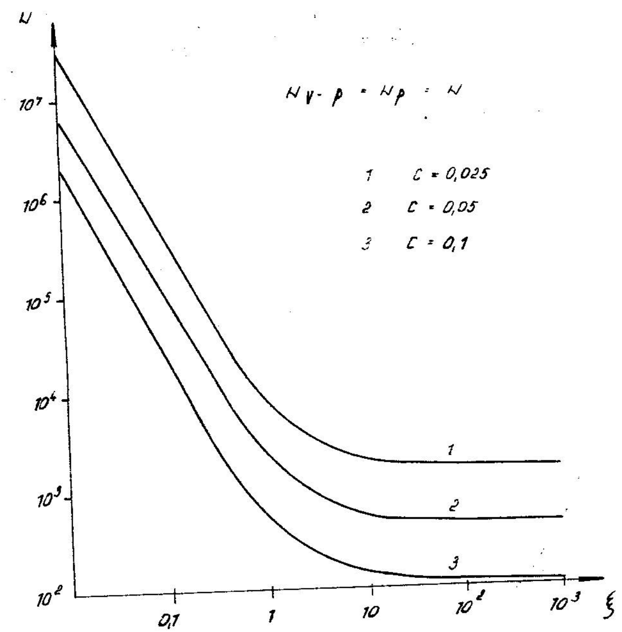


Fig. 2. Preset count.

## 3. CORRECTION FOR INEXACT TIME OF MEASUREMENTS

### a. Preset time

In practice it may occur that it is impossible to set precisely the calculated preset time $T$, but a time $T'$ can be calculated with the validity of

$$T' = \eta T. \tag{14}$$

For practical application it may be more suitable to put $\eta < 1$.

In such a case the formerly chosen coefficient of variation will also change from $c$ to $c'$. Our task here is to determine the size of the change of the coefficient of variation.

From equation (12) we can express the square of the coefficient of variation and get

$$c^2 = \frac{1}{Tn_V}\left(1 + \frac{2}{\xi}\right). \tag{15}$$

Substituting (14) into (15) we obtain

$$c'^2 = \frac{1}{\eta T n_V}\left(1 + \frac{2}{\xi}\right). \tag{16}$$

Comparing the last relations we obtain

$$c^2 = \eta c'^2$$

and from it

$$c' = \eta^{-1/2}\, c \tag{17}$$

In Fig. 3 we see the course $\eta^{-1/2}$ in dependence on $\eta$ for the values $0{\cdot}1 \leq \eta \leq 1$.

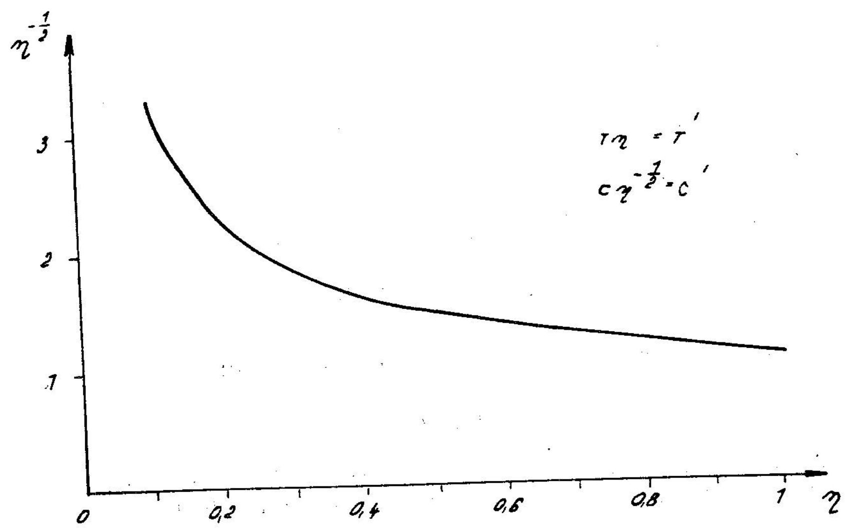


Fig. 3. Correction for inexact preset time.



b. Preset count

In cases of measuring certain samples often a case may occur where it is impossible to set precisely the determined value of count preset, but only a value close to the determined one. From the apparatus we can expect a recording of $N$ counts, but in fact we can set only values of $N'$ counts:

$$N' = \eta N, \tag{18}$$

where $\eta < 1$.

In dependence on the preset count there changes also the original value of the coefficient of variation from $c$ to $c'$

$$c'^2 = \varrho c^2. \tag{19}$$

Next, we determine $\varrho$. From (13) we express the coefficient of variation $c$. Using (18) the following relation is valid for the square of the new coefficient of variation:

$$c'^2 = \frac{1}{\eta N - 2}\left(1 + 2\,\frac{\xi + 1}{\xi^2}\right), \tag{20}$$

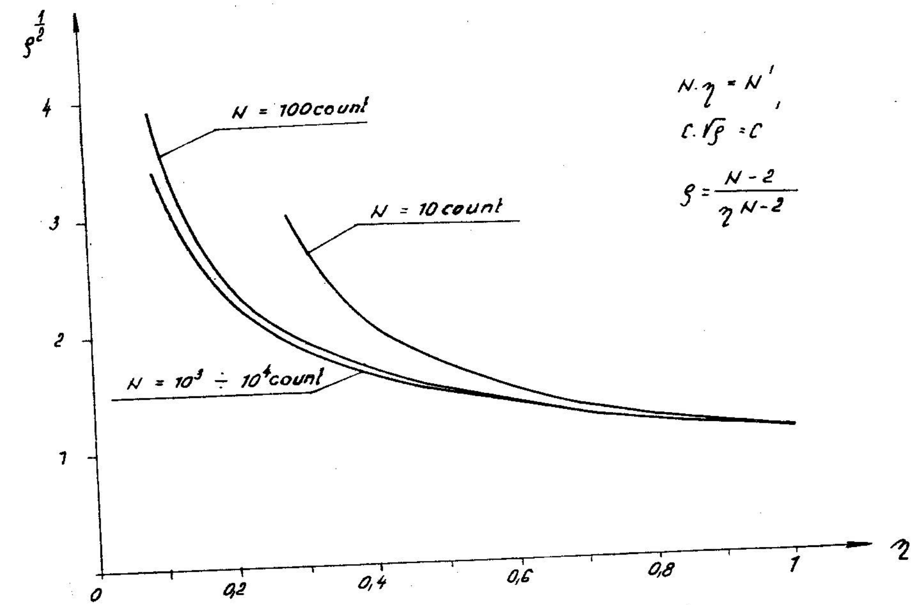


Fig. 4. Correction for inexact preset count.

where we substituted according to (18). The determined values for the two coefficients of variations will be substituted into (19) and we shall obtain

$$\left[\frac{1}{N-2}\left(1+2\frac{\xi+1}{\xi^2}\right)\right]\varrho = \frac{1}{\eta N-2}\left(1+2\frac{\xi+1}{\xi^2}\right)$$

$$\varrho = \frac{N-2}{\eta N-2}\,. \tag{21}$$

In Fig. 4 the course of $\sqrt{\varrho}$ is shown in dependence on the value $\eta$ for different values of the calculated preset count $N$.

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