Fyzikálny časopis 20 (1970), No. 2

A CONTRIBUTION TO THE MEASUREMENT OF SMALL ANGLES OF SCATTERED PARTICLES IN A PHOTOGRAPHIC EMULSION

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This paper deals with the exact measurement of the angular distribution of secondary fast particles in nuclear emulsions, especially in the region of small angles. The main consideration is the evaluation of statistical and geometrical errors.

INTRODUCTION

Angular distribution of secondary particles is an important information as regards strong interactions. In case of an elastic scattering of high-energy particles it is especially the region of small angles which is important for the theoretical interpretation of the process [1].

Problems in measuring small angles are caused by the fact that space angles are measured by the coordinate method. The measurement of the coordinates x, y and z is influenced by different errors, on the one hand by the measuring apparatus (Zeiss KSM-2 microscope in our case) and on the other hand by the photochemical processing of the emulsion. The particle track is deformed by a multiple Coulomb scattering and by the distorsions of the emulsion. Moreover, the grains of Ag forming the track of the particle, determine the axis of its path only approximately, statistically. In the region of small angles some specific geometrical corrections take place, too. We tried to consider all the above mentioned factors and to determine their influence on the resultant angular distribution.

MEASURING METHOD AND CALCULATION OF DEFLECTION

In the present paper we shall use the following notations: ϑ — deviation of the particle from the original direction; φ — projection of the angle on the plane of emulsion; Δ_1, Δ_2 — angles of the primary and the secondary tracks with the plane of emulsion (so-called dip angles). For angles defined in this way

$$\cos \vartheta = \cos \varphi \cos \Delta_1 \cos \Delta_2 - \sin \Delta_1 \sin \Delta_2 \quad (1)$$

and for small angles

$$\vartheta^2 = \varphi^2 + (\Delta_1 + \Delta_2)^2 = \varphi^2 + \delta^2 \qquad (2)$$

are valid.

Since the angles in the plane of the emulsion are measured with respect to the fundamental motion of the microscope table, the relation $\varphi = \varphi_1 + \varphi_2$ is valid, where φ_1 is the angle of the primary track and φ_2 the angle of the secondary track with respect to the direction of the table motion. The angles are measured in the direction of the secondary tracks and against the direction of the primary track (related to the point of interaction).

We take variances σ^2, or squared standard deviations or, more exactly, a statistical estimation of these quantities as a measure of errors. With the assumption of the independence of φ and δ according to [2]

$$\sigma_\vartheta^2 = \left(\frac{\partial\vartheta}{\partial\varphi}\right)^2 \sigma_\varphi^2 + \left(\frac{\partial\vartheta}{\partial\delta}\right)^2 \sigma_\delta^2 = \frac{1}{\vartheta^2}\,(\varphi^2\sigma_\varphi^2 + \delta^2\sigma_\delta^2) \qquad (3)$$

is valid.

The angles φ and Δ are measured by the method of the coordinates. The point, the coordinates of which are to be measured, is chosen as a centre of the short track segment Δl (of about 30 μ) in which some grains of developed Ag are placed. The coordinates of the centre of the segment Δl in the direction of the fundamental motion of microscope table (so-called x motion) are measured by the precise turning of the micrometric screw of the table. The direction x is orientated approximately in the direction of the primary track. The coordinate y is measured in the emulsion plane perpendicularly to the direction x, by help of a special micrometer, namely by centering Ag grains lying in the segment Δl. The corrdinate z is measured in the direction perpendicular to the emulsion plane from the boundary of glass plate and the emulsion to the centered upper edges of grains Ag lying in segment Δl.

Having measured the coordinates of two points of the primary track (the distance between them must be long enough in comparison with the segment Δl) and those of two points of the secondary track we have

$$\varphi_1 \doteq \operatorname{tg}\varphi_1 = \frac{y_1 - y_2}{x_1 - x_2}, \qquad \varphi_2 \doteq \operatorname{tg}\varphi_2 = \frac{y_4 - y_3}{x_4 - x_3}.$$

We choose equal lengths of $x_1 - x_2$ and $x_4 - x_3$ and we denote them as usual as the so-called cell length t. Then

$$\varphi = \varphi_1 + \varphi_2 = \frac{y_1 - y_2 - y_3 + y_4}{t}. \qquad (4)$$

Analogously for the dip angles

$$\Delta_1 \doteq \operatorname{tg}\Delta_1 = \frac{z_1 - z_2}{t}\,s, \qquad \Delta_2 \doteq \operatorname{tg}\Delta_2 = \frac{z_4 - z_3}{t}\,s, \qquad (5)$$

$$\delta = \Delta_1 + \Delta_2 = \frac{z_1 - z_2 - z_3 + z_4}{t}\,s$$

is valid; s is the so-called factor of emulsion shrinkage caused by photochemical processing. In our calculations we take the value of $s = 2.3$ [3].

In practice we estimate the track points 1, 2, 3 and 4 in which the coordinates y_i and z_i are measured in such a way that we measure the coordinates y_i, z_i with a relatively short cell length t_0 along the track. According to the size of the so-called second differencies we estimate the region of scattering with the exactness t_0. The edge points of this region are the points 2 and 3. Then we estimate point 1 at the distance $t \gg t_0$ from point 2 against the direction of the primary track and we choose point 4 at the same distance from point 3 in the direction of the primary track (Fig. 1).

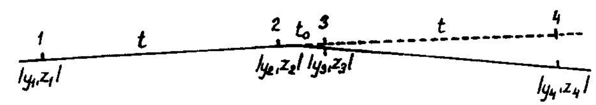

Fig. 1.

The measurement of angles is influenced by the error of the measuring of the coordinates y_i, z_i (we neglect the error of the coordinate x considering the precision of the screw of the microscope table), by a multiple Coulomb scattering and by emulsion distorsions which deflect the particle track from the line in the intervals 1—2 and 3—4. Statistical errors of coordinate measuring were estimated by repeated measuring of these values at different configurations and the density of the track grains. Thus we obtained the values

$$\sigma_y = 0.07\mu, \qquad \sigma_z = 0.19\mu. \qquad (6)$$

Multiple scattering and distorsions were estimated by means of the mean square values of the second differencies $\langle|D_y|\rangle$ and $\langle|D_z|\rangle$ as it is usual in the measuring of multiple scattering [4]. For $t = 500$ μ we have

$$\langle|D_y|\rangle = 0.35\ \mu, \qquad \langle|D_z|\rangle = 0.46\ \mu. \qquad (7)$$

Since the errors of the measuring of coordinates and of the track deformations (multiple scattering and distorsions) are mutually independent, we get

$$\sigma^2_{\varphi_1} = \sigma^2_{\varphi_2} = \frac{k\,\langle|D_y|\rangle^2 + 2\,\sigma^2_y}{t^2} \tag{8}$$

$$\sigma^2_{\Delta_1} = \sigma^2_{\Delta_2} = \frac{k\,\langle|D_z|\rangle^2 + 2\sigma^2_z}{t^2}\, s^2;$$

k depends on the choice of the characteristics of distribution of the random variable. In the given case $k = (\pi/2)^{\frac{1}{2}}$ [5].

Assuming that φ_1 and φ_2 as well as Δ_1 and Δ_2 are independent we get after the substitution of numerical values

$$\sigma_\varphi = 1.15 \text{ m rad} \qquad \sigma_\delta = 3.78 \text{ m rad}, \tag{9}$$

which can be used for the calculation of σ_ϑ from formula (3).

GEOMETRICAL CORRECTIONS OF THE ANGULAR DISTRIBUTION IN THE REGION OF SMALL ANGLES

Fig. 2 shows a plane perpendicular to the direction of the momentum of the primary particle. Onto this plane a unit sphere is projected in the centre of which particle scattering took place. Point A is a point of intersection of the secondary track with the unit sphere. The radius vector of point A is numerically equal to the scattering angle ϑ for small scattering angles. The

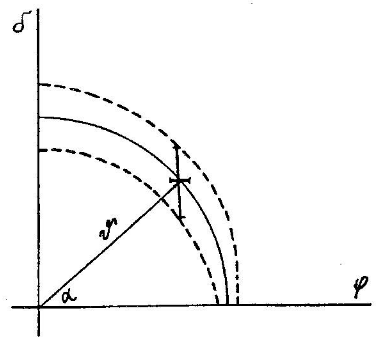

Fig. 2.

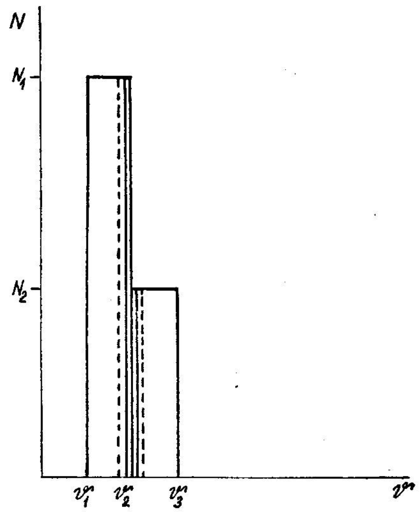

Fig. 3.

corresponding statistical errors σ_φ and σ_δ are determined by ellipses with the half-axes $\vartheta \pm \sigma_\varphi$ and $\vartheta \pm \sigma_\delta$. In polar coordinate $\varphi = \vartheta \cos\alpha$, $\delta = \vartheta \sin\alpha$ and $\sigma^2_\vartheta = \cos^2\alpha(\sigma^2_\varphi - \sigma^2_\delta) + \sigma^2_\delta$. Assuming that the scattering is symmetric about the axes, the mean value of the error of the scattering angle $\langle\sigma_\vartheta\rangle$ is equal to

$$\langle\sigma_\vartheta\rangle = \frac{2}{\pi}\int\limits_0^{\pi/2} \sqrt{(\sigma^2_\varphi - \sigma^2_\delta)\cos^2\alpha + \sigma^2_\delta}\, d\alpha. \tag{10}$$

By inserting $\varepsilon^2 = (\sigma^2_\delta - \sigma^2_\varphi)/\sigma^2_\delta = 0.91$ we can simplify expression (10) and express the corresponding elliptical integral [6] as

$$\langle\sigma_\vartheta\rangle = \frac{2\sigma_\delta}{\pi}\left\{\frac{\pi}{2}\left(1 - \frac{\varepsilon^2}{4} - \frac{\varepsilon^4}{64} - \ldots - \left[\frac{(2n-1)!!}{2^n n!}\right]^2 \frac{\varepsilon^{2n}}{2n-1} - \ldots\right)\right\} \doteq$$

$$\doteq \sigma_\delta - \frac{1}{4}\,\frac{\sigma^2_\delta = \sigma^2_\varphi}{\sigma_\delta} \doteq 2.92. \tag{11}$$

The angles are measured in miliradions (m rad). The advantage is that with the given assumption the value $\langle\sigma_\vartheta\rangle$ does not depend on the scattering angle ϑ.
Let us have a certain distribution of the scattering angles measured by the described method. We demonstrate this angular distribution by a histogram. $\vartheta_1, \vartheta_2, \vartheta_3, \ldots$ are the dividing values of the chosen intervals of the histogram $\Delta\vartheta = \vartheta_2 - \vartheta_1 = \vartheta_3 - \vartheta_2$ (Fig. 3). We choose $\Delta\vartheta \geqslant 2 \ll \sigma_\vartheta\rangle$.
We separate a subinterval of the width $\langle\sigma_\vartheta\rangle$ for each of the considered intervals $\Delta\vartheta$. For the sake of simplicity, the distribution of particles in each of these subintervals will be substituted by „a line“ in its center. We assume that for all $N'_1 = N_1\langle\sigma_\vartheta\rangle/\Delta\vartheta$ particles of the first subinterval we have measured the scattering exactly for the angle $\vartheta_2 - \frac{1}{2}\langle\sigma_\vartheta\rangle$. For N'_2 we assume similarly that for all $N'_2 = N_2\langle\sigma_\vartheta\rangle/\Delta\vartheta$ particles of the second subinterval the scattering exactly will be for the angle $\vartheta_2 + \frac{1}{2}\langle\sigma_\vartheta\rangle$. In fact, owing to the scattering of the measured values, a part of the particles of the first subinterval can be among those of the second interval and conversely. The probability of these „passes“ can be estimated by means of areas, see Fig. 2.

If we know the probability P_1 according to which a particle measured in the first subinterval belongs to the adjacent interval and if we know the number of particles in the first subinterval N'_1, then the number of patricles subtracted from the first interval and added to the second will be equal to

$$\Delta N_1 = \frac{\langle\sigma_\vartheta\rangle}{\Delta\vartheta}\, N_1 P_1. \tag{12}$$

The probability P_1 is a ratio of areas S_1/S_2, where S_1 is an area limited by a circle with the radius ϑ_2 and by an outer ellipse which is limiting the errors of the angular value $\vartheta_2 - \frac{1}{2}\langle\sigma_\vartheta\rangle$; S_2 is the total area of errors belonging to the angle $\vartheta_2 - \frac{1}{2}\langle\sigma_\vartheta\rangle$.

$$P_1 = \frac{S_1}{S_2} =$$

$$= \frac{(\vartheta_2 - \frac{1}{2}\langle\sigma_\vartheta\rangle + \sigma_\varphi)(\vartheta_2 - \frac{1}{2}\langle\sigma_\vartheta\rangle - \sigma_\delta) - \vartheta_2^2}{(\vartheta_2 - \frac{1}{2}\langle\sigma_\vartheta\rangle + \sigma_\varphi)(\vartheta_2 - \frac{1}{2}\langle\sigma_\vartheta\rangle + \sigma_\delta) - (\vartheta_2 - \frac{1}{2}\langle\sigma_\vartheta\rangle + \sigma_\varphi)(\vartheta_2 - \frac{1}{2}\langle\sigma_\vartheta\rangle - \sigma_\delta)} \doteq$$

$$\doteq \frac{2.01\vartheta_2 - 7.92}{9.86\vartheta_2 - 14.40}. \qquad (13)$$

From the second interval we shall subtract

$$\Delta N_2 = \frac{\langle\sigma_\vartheta\rangle}{\Delta\vartheta} N_2 P_2, \qquad (14)$$

where P_2 is again a ratio of areas S_1'/S_2' from Fig. 2. The area S_1' is limited by a circle with the radius ϑ_2 and by an ellipse limiting the errors of the value $\vartheta_2 + \frac{1}{2}\langle\sigma_\vartheta\rangle$ and S_2' is the total area of errors of the value $\vartheta_2 + \frac{1}{2}\langle\sigma_\vartheta\rangle$.

$$P_2 = \frac{S_1'}{S_2'} =$$

$$= \frac{\vartheta_2^2 - (\vartheta_2 + \frac{1}{2}\langle\sigma_\vartheta\rangle - \sigma_\varphi)(\vartheta_2 + \frac{1}{2}\langle\sigma_\vartheta\rangle - \sigma_\delta)}{(\vartheta_2 + \frac{1}{2}\langle\sigma_\vartheta\rangle + \sigma_\varphi)(\vartheta_2 + \frac{1}{2}\langle\sigma_\vartheta\rangle + \sigma_\delta) - (\vartheta_2 + \frac{1}{2}\langle\sigma_\vartheta\rangle - \sigma_\varphi)(\vartheta_2 + \frac{1}{2}\langle\sigma_\vartheta\rangle - \sigma_\delta)} \doteq$$

$$\doteq \frac{2.01\vartheta_2 + 7.92}{9.86\vartheta_2 + 14.40}. \qquad (15)$$

THE APPLICATION OF THE METHOD TO THE MEASUREMENT OF THE SCATTERING ANGLE OF DEUTERONS

The above described method has been used for measuring the scattering angle of deuterons on the atom nuclei of the photographic emulsion NIKFI BR-2. The nuclear photographic emulsion of the type NIKFI BR-2 has been irradiated by a deuteron beam with the mobility 2.43 GeV/c in the High Energies Laboratory of the Joint Institute for Nuclear Research in Dubna. By means of measuring the coordinates y and z with the cell $t = 500\ \mu$ along the primary track we have detected the cases of deuteron scattering to very small angles practically without a subjective error. We have found 47 cases of deuteron

angular scattering in the interval 6—100 m rad on the total length of 10.2 m.

Fig. 4 shows the measured angular distribution. The maximum dispersion of the scattering angle is $(\sigma_\vartheta)_{max} = 3.78$ m rad. For angles from the interval 6—10 m rad σ_ϑ can get larger than 50 % ϑ, according to the values φ and δ. Therefore the cases from this interval are considered only for the geometrical correction for the minimal angle $\vartheta = 10$ m rad.

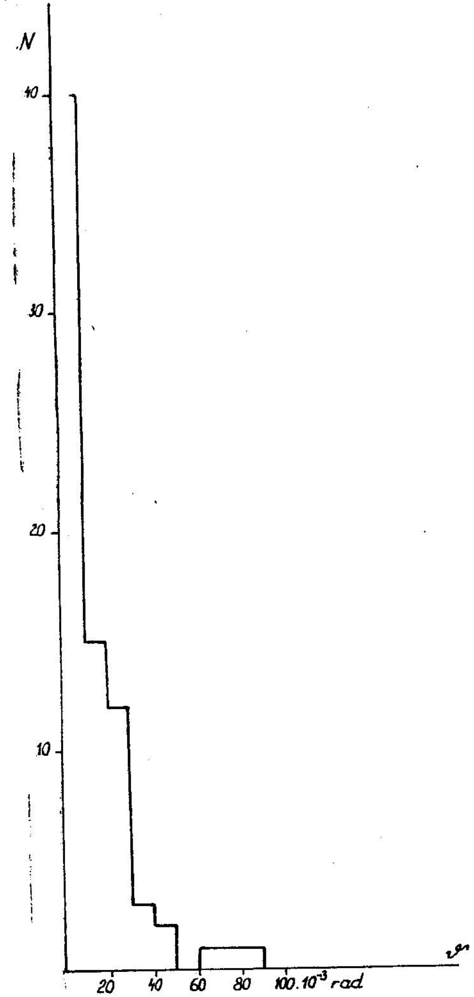

Fig. 4.

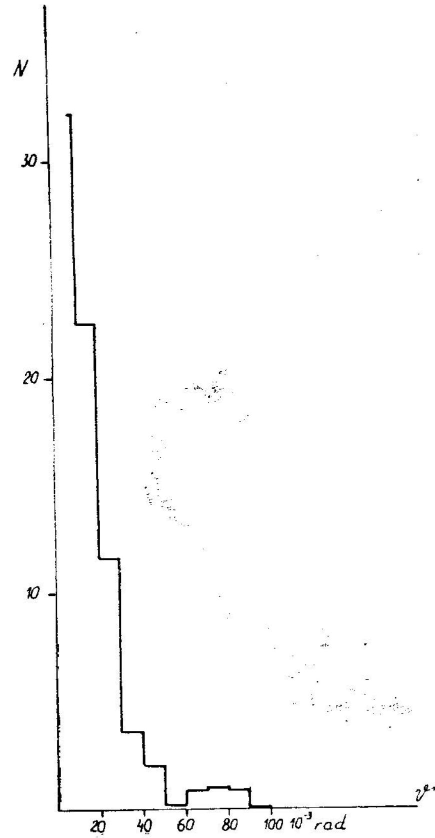

Fig. 5.

Fig. 5 shows the same angular distribution as Fig. 4 after the above mentioned geometrical corrections have been made. It can be seen that there are considerable corrections especially in the region of very small angles, where the dispersion of measured values is relatively the largest one. The suggested method of geometrical corrections defines more precisely the angular distribution in the region of the especially important extrapolation of differential effective cross-sections with respect to the zero angle.

This method can be easily applied to other particles, eventually to a different material.

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Received January 6th, 1970

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