Fyzikálny časopis 20 (1970), No. 1

MEASUREMENT OF THERMAL CONDUCTIVITY OF THIN SAMPLES OF A FINITE LENGTH

JURAJ KRIŠTOFIČ, JOZEF LASZ, Bratislava

The paper contains a solution of the problem of measurement of thermal conductivity of a thin plate by the impulse method, taking into account the dimensions of the sample. Conditions are found making it possible to neglect the finite dimensions of the sample. An experimental verification was made on silicon samples of the n-type also for the case when the diemnsions of the sample cannot be neglected.

INTRODUCTION

Paper [1] deals with the problem of measurement of thermal conductivity of a thin plate having a finite thickness, assuming, that its length and width are infinite. When measuring materials having a good thermal conductivity (metals and some semiconductors), it is practically impossible to fulfil these theoretical conditions. This paper deals with the influence of the finite dimensions of the sample on the measurement of thermal conductivity by the impulse method. Numerical computation was made for a simple concrete case. Based on these solutions, the geometry was chosen and measurement were made on the n-type of silicon, confirming theoretical conclusions ove a wide range of temperatures.

THEORY

Assume a line impulse heat source acting in the middle of a plate. The increase of temperature in point x located at a distance r from the line heat source on the surface of the sample (Fig. 1) can be expressed by the following equation

$$T = \frac{Q}{2\pi\lambda bt} \exp\left[-\frac{r^2}{4kt}\right]\left\{2\sum_{n=0}^{\infty} q^n \exp\left[-\frac{(nh)^2}{kt}\right] - 1\right\} + \tag{1}$$

$$+ \frac{Q}{2\pi\lambda bt}\left\{2\sum_{n=0}^{\infty} q^n \exp\left[-\frac{(nh)^2}{kt}\right] - 1\right\}\left[\sum_{m=1}^{\infty} \exp\left(-\frac{(2mH - r)^2}{4kt}\right) + \right.$$
$$\left. + \sum_{m=1}^{\infty} \exp\left(-\frac{(2mH + r)^2}{4kt}\right)\right].$$

in which Q — is the quantity of heat supplied by the impulse, b — the length of the line heat source,

$$q = \frac{1 + R}{1 - R}; \quad R = \left(\frac{\gamma_0 c_0 \lambda_0}{\gamma c \lambda}\right)^{1/2},$$

h — the thickness of the sample, $2H$ — the length of the sample, λ — the coefficient of thermal conductivity of the sample, c — the cpecific heat of the sample, γ — the specific matter of the sample, λ_0 — the coefficient of thermal conductivity of the sole plate, c_0 — the specific heat of the sole plate, γ_0 the specific matter of the sole plate, t — the time.

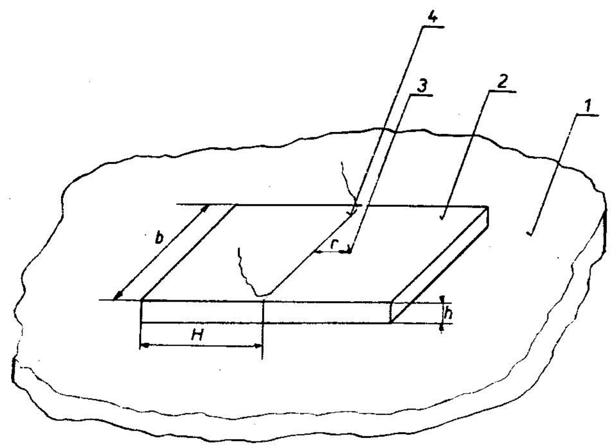


Fig. 1. Scheme of arrangement for measurement of thermal conductivity by the impulse method. 1 — sole plate, 2 — sample, 3 — indication point x, 4 — heating tape.

The first member in equation (1) represents the temperature rise when the finite dimensions of the sample are taken into account. The second member represents the contribution to the temperature rise caused by the finite length of the sample, assuming that the heat can dissipate only into the sole plate, computed by the ,,method of fictive heat sources" (see ref. [1]). By arranging equation (1) we obtain the following expression for the temperature rise

$$T = \frac{Q}{2\pi\lambda bt}\left[\sum_{m=0}^{\infty} \exp\left(-\frac{(2mH - r)^2}{4kt}\right) + \sum_{m=1}^{\infty} \exp\left(-\frac{(2mH + r)^2}{4kt}\right) - \right. \tag{2}$$

$$\left. - \exp\left(-\frac{r^2}{4kt}\right)\right]\left[2\sum_{n=0}^{\infty} q^n \exp\left(-\frac{(nh)^2}{kt} - 1\right]\right. .$$

The maximum temperature at the given point is obtained from the condition that $\partial T/\partial t = 0$. It occurs in the time

$$t_m = r^2/4ky, \tag{3}$$

where y is the solution of the equation

$$\{2\sum_{n=0}^{\infty} q^n \exp(-4p^2n^2y - 1)\}\,[\sum_{m=0}^{\infty} \exp - (2ms - 1)^2y\{]\,(2ms - 1)^2y - 1) +$$

$$+ \sum_{m=0}^{\infty} [\exp(-(2ms + 1)^2y)]\,[(2ms + 1)^2y - 1) - [\exp(-y)]\,(y - 1)] +$$

$$+ [\sum_{m=0}^{\infty} \exp(-(2ms - 1)^2y) +$$

$$+ \sum_{m=0}^{\infty} \exp(-(2ms + 1)^2y) - \exp(-y)]\; 8\sum_{n=0}^{\infty} n^2p^2yq^n \exp(-4n^2p^2y) = 0. \tag{4}$$

where $p = h/r$ and $s = H/r$.
If $m = 0$, i. e. if the limited length of the sample is not respected, equation (4) will take the following form

$$y - 1 + \frac{8\sum_{n=0}^{\infty} q^n p^2 n^2 y \exp(-4p^2n^2y)}{2\sum_{n=0}^{\infty} q^n \exp(-4p^2n^2y) - 1} = 0, \tag{5}$$

the solution of which can be found in paper [1].

From equation (3) it is possible to compute the coefficient of thermal conductivity k, provided that the time of the maximum temperature rise and the parameter y are known, while y has to satisfy equation (4) and is a complex function of the parameters p and s. A general solution is too complex and we therefore limit ourselves to the computation of the corrections in the first approximation (taking into account only the first fictive heat source) and shall attempt to find criteria permitting the sample to be assumed to be infinite and to use for measurement expressions derived in paper [1].

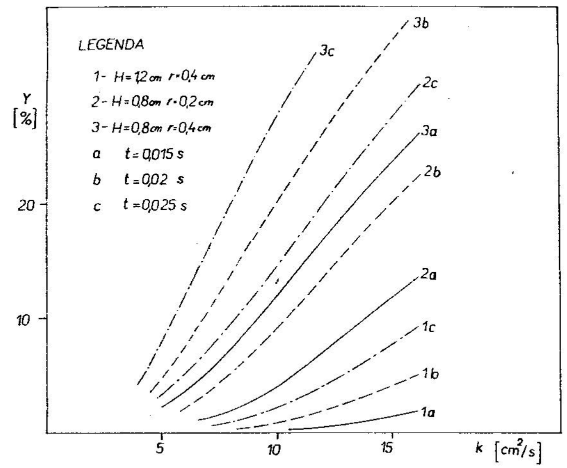
In cases when it is sufficient to consider the effect of the temperature rise due to the first fictive heat source at a distance $2H - r$ from the reference point, the temperature rise due to this fictive source is

$$T_{F_1} = \frac{Q}{2\pi\lambda bt}\left\{2\sum_{n=0}^{\infty} q^n \exp\left(-\frac{(nh)^2}{kt}\right) - 1\right\} \exp\left(-\frac{2H - r)^2}{4kt}\right). \tag{6}$$

A numerical computation was performed for the ratio of $Y = T_{F_1}/T_0$, T_0 being the temperature rise for a sample linearly unlimited, i. e.

$$T_0 = \frac{Q}{2\pi\lambda bt}\left\{2\sum_{n=0}^{\infty} q^n \exp\left(-\frac{(nh)^2}{kt}\right) - 1\right\} \exp\left(-\frac{r^2}{4kt}\right) \tag{7}$$

and the results are plotted in Diagr. 1. With the help of this graph it is possible to find the limits of the measurements of thermal conductivity and of the ratio of H/r, if time is taken as a parameter.



Diagr. 1. Computed values of y for different values of k

In case a certain range of the values of k is assumed and it is intended to measure with a certain chosen accuracy (i. e. the influence of the fictive source is not to exceed a certain percentage), it is possible to read from the graph the dimensions (r and H) of the sample, the time of the maximum temperature increase being limited at the same time.

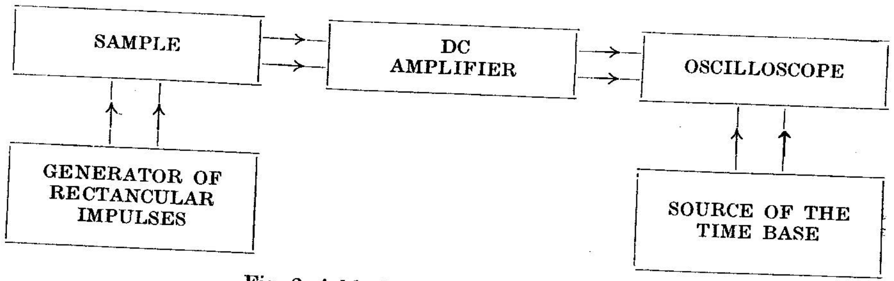


Fig. 2. A block scheme of measurement.

On the basis of the above mentioned analysis an arrangement of the heating and the detecting elements was chosen for $Y < 5\%$. The samples were prepared from a single crystal of silicon of the type n alloyed by phosphorus, cut in a plane [II]. The heating tape, 0.02 centimeter wide, was made by currentless nickeling directly on the ground silicon sample. In the case of the material having a specific resistance 0.001 Ωcm^{-1}, an 800 Å thick oxide film was created under the heating tape because of the great conductivity of the measured material. The natural voltage of the material — Si served for an indication of the temperature rise at the point under consideration. The margines of the sample were fastened in a holder made of teflon. A block scheme of the measuring apparatus is shown in Fig. 2. In the measuring procedure it is important that the source of the heat impulses and the time base of the oscilloscope are synchronized. The results were evaluated from photographs of the oscilloscopic curves. The most important parts of the experimental equipment are the generator of rectangular heat impulses and the dc amplifier. For the evaluation of results equations derived in paper [1] were used, assuming that $q = 1$, so that

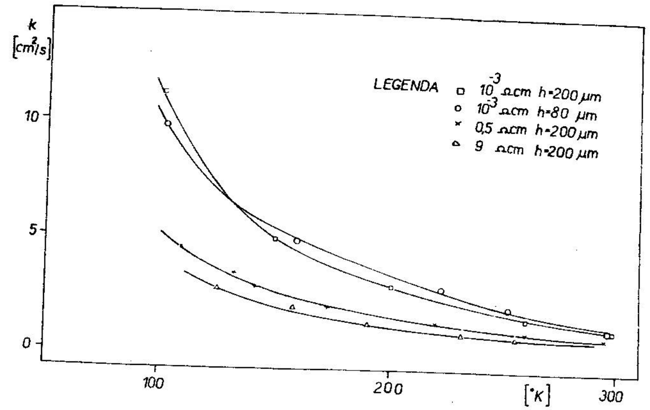
$$k = r^2/2t_m = k_m \tag{8}$$

and

$$k = \frac{r_2^2 - r_1^2}{4t} \cdot \frac{1}{\ln T_1/T_2} = k_p, \tag{9}$$

where T_1 and T_2 are the temperature increases at the points r_1 and r_2 at the time t. The results of measurements are shown in Diagr. 2 and in Table 1. They include results obtained from samples with a different specific resistance and different thickness at temperature ranges between 100—300 °K (0).

The solution of the case presented at the beginning of Chapter Measurements and results makes it possible to find the limits of the use of the results given in paper [1] and thus determine a suitable geometry and the position of the indicator of temperature rise in a certain range of values of k. Even though it is only a first approximation, it is sufficiently characteristic of the practical needs.



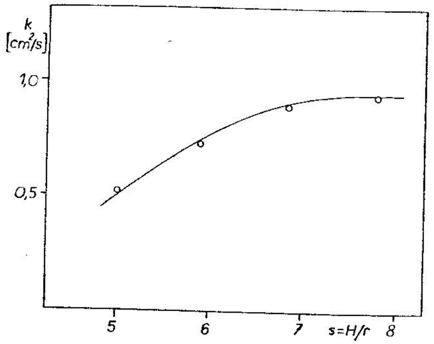
Diagr. 2. Relation between thermal conductivity of silicon and temperature

We shall examine the variations of the values of t_m and y in Eq. (3) in case that the effect of fictive sources is not negligible. In such a case it can be expected that the time of the maximum will increase. It follows from Eq. (3) that y has also to vary since k is a material constant. Such a case was verified by measuring the dependance of y and k on the ratio H/r at a constant temperature. The results are poltted in Diagr. 3A and 3B. The measurements were

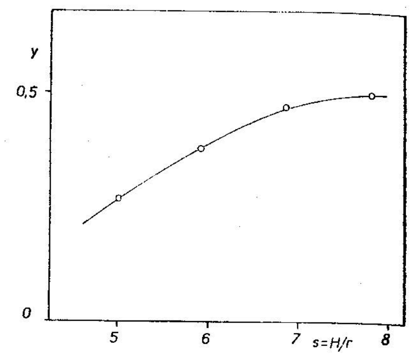
Table 1

A comparison of values k_m and k_p at different temperatures

$\varrho = 0.5\ \Omega cm$							
Θ [°K]	110	153	142	173	220	263	295
k_p [cm²/s]	4.6	4.0	2.8	1.9	1.7	1.4	1.2
k_m [cm²/s]	4.4	3.8	2.8	1.9	1.4	1.0	0.9
$\varrho = 10^{-3}\ \Omega cm$							
Θ [°K]	100	115	150	200	215	260	295
k_p [cm²/s]	10.8	7.3	3.0	2.5	2.2	1.3	1.0
k_m [cm²/s]	11.2	9.5	4.1	3.1	2.5	1.5	1.2



Diagr. 3A. Values of thermal conductivity and parameter y for different arrangements of measurement; $q = 1$.



Diagr. 3B. Values of thermal conductivity and parameter y for different arrangements of measurement; $q = 1$.

based on the assumption that the temperature of the reference point does not change. From diagram 3A we shall determine when it is possible to compute the coefficient of thermal conductivity from equation

$$k = r^2/4t_m y\,, \tag{10}$$

y being the solution of Eq. (5).

For the parameters $q = 1$, $v = 0.8$, the value of s must be $s = H/r > 8$. For the same case the parameter y is plotted as a function of s in Fig. 3B. Knowing its value for the given parameters p and s, it is possible to measure also on smaller samples if the conditions remain the same, i. e. if $p = 0.8$ and $q = 1$.

For different conditions of measurement (different values of p and particularly of q) the criterion $s > 8$ is no longer valid and it is necessary to determine it by measurement for each case. The same holds for the parameter y.

Equation (4) makes it possible to determine y precisely as a function of p, s and q. This leads, however, to complicated computations and to the necessity of using a computor.

A very good agreement was obtained in evaluating the thermal conductivity using equations (8) and (9) — see Table 1. The above agreement of the values of k determined in two different ways confirms the suitability of the geometry used. The experimental results plotted in Diagr. 2 show an agreement with the previous results published. In the range of thicknesses tested (200 to 80 --m), the influence of the thickness on the coefficient of thermal conductivity k does not manifest itself even at the nitrogen temperature. To measure directly the thermal conductivity at temperatures approximately 20 °K or lower,

when an influence of the „size effect“ on the thermal conductivity could be expected, seems to be hard to realize on the given material (large value of k) by the method used. Within the above mentioned temperature renga the influence of the concentration of the admixtures on the thermal conductivity is more pronounced.

The accuracy of measurement depends on the accuracy of time measurement which, with the equipment used, was 2.5 % in the most unfavourable case.

CONCLUSION

The paper presents a solution of the influence of a finite length of the sample on the measurement of the coefficient of thermal conductivity. A numerical computation is presented for a simple case. Conditions are analysed under which the results of paper [1] can be used for thin samples of a finite length. The condition is determined for a concrete case when it is necessary to take into account the finite length of the sample. Theoretical conclusions were verified by mesurements conducted on thin silicon samples.

REFERENCES

[1] Budke O., Krempaský J., Physica status solidi *21* (1967), 175.
[2] Krempaský J., Čs. čas. fyz. *A 16* (1966), 136.

Received May 26, 1969

Katedra fyziky
Elektrotechnickej fakulty SVŠT,
Bratislava