KINETIC ENERGY DISTRIBUTION FOR IONS REACHING THE WALL OF THE DISCHARGE TUBE AT THE LOW-PRESSURE REGIME

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The distribution function for the energy of ions reaching the wall of the discharge tube at the free-fall regime is determined in dependence on the radial course of the potential. The graphical results are presented for a mercury positive column at various Debye lengths. The maximum energy is given by the floating potential. The width of the maximum of the distribution function is approximately equal to the thermal energy of electrons.

INTRODUCTION

The radial potential distribution in the discharge positive column has an influence on the energy distribution function of ions reaching the tube wall. The ions formed at the wall are accelerated by a smaller potential drop than those formed near the axis. In the positive column at low pressures the loss of ion energy caused by collisions with neutral particles can be neglected and therefore the distribution function of ion energy can be calculated directly from the radial course of the potential and the ion density. A stationary and homogeneous column is supposed during the following considerations.

FORMULATION OF THE PROBLEM

Let the function $U(r)$ indicate the potential course in a radial direction (for $r = 0$ is $U = 0$). The electron density distribution in the radial direction can be determined by means of the Boltzmann factor

$$n_{-}(r) = n_0 \exp \left[eU(r)/kT_{-} \right] , \tag{1}$$

where n_0 is the electron density at the tube axis, T_{-} is the electron temperature and e the elementary charge.

For determination of the energy distribution function it is necessary to find out the distribution function for the ion velocity $f(r, v)$ normalized so that we have

$$n_+(r) = \int_0^\infty f(r, v) v \, dv \, .$$

The function $f(r, v)$ does not depend on other coordinates because the column homogeneity in the longitudinal and azimuthal directions is assumed. The function $f(r, v)$ can be determined from the Boltzmann kinetic equation which in the case of cylindrical coordinates and in the stationary case has the form

$$\frac{v}{r} \frac{\partial r f(r, v)}{\partial r} + \frac{a(r)}{v} \frac{\partial v f(r, v)}{\partial v} = N(r, v) \, ,$$

$a(r) = -(e/m_+) \, dU(r)/dr$ is the radial acceleration of the ion and $N(r, v)$ is the collision term representing the number of ions formed in a unit volume of phase space per sec. We assume that the ions are formed exclusively at the electron-neutral molecules collisions and that their initial energy is zero. The collision term then has the form

$$N(r, v) = \alpha n_-(r) \frac{1}{v} \delta(v)$$

where α is the rate of ionization and $\delta(v)$ the Dirac function.

DISTRIBUTION FUNCTION OF THE ION VELOCITY

Under the assumptions mentioned above the solution of the kinetic equation has the form

$$f(r, v) = \frac{1}{rv} \int_0^r \varrho \alpha n_-(\varrho) \frac{\delta[\{v^2 + (2e/m_+) [U(r) - U(\varrho)]\}^{1/2}]}{\{v^2 + (2e/m_+) [U(r) - U(\varrho)]\}^{1/2}} d\varrho$$

By using relation (1) and substituting $r = R$(R is the inner radius of the discharge tube), we get the function for the distribution of the ion velocities at the wall

$$f(R, v) = \frac{\alpha n_0}{Rv} \int_0^R \varrho \exp \left[\frac{eU(\varrho)}{kT_-}\right] \frac{\delta[\{v^2 + (2e/m_+) [U(R) - U(\varrho)]\}^{1/2}]}{\{v^2 + (2e/m_+) [U(R) - U(\varrho)]\}^{1/2}} d\varrho \, .$$

After integration we have

$$f(R, v) = -\frac{\alpha n_0}{Rv} \frac{m_+}{e} \varrho^* \exp \left[\frac{eU(\varrho^*)}{kT_-}\right] \frac{1}{U'(\varrho^*)},$$

where ϱ^* is determined from the condition

$$U(\varrho^*) = \frac{1}{2} \frac{m_+ v^2}{e} + U(R) \, ,$$

$U(R) = U_R$ is the potential of the wall.

DISTRIBUTION FUNCTION FOR ION ENERGY

From an experimental point of view it is advantageous to know the distribution function of the ion-current density j_+ dependent on the kinetic energy by which the ions reach the wall. The ion-current density for ions, having a velocity from the interval $\langle v, v + dv \rangle$, is $dj_+ = vf(R, v) v \, dv$. If the element $v \, dv$ of the velocity space is substituted by the element dW/m_+, where $W = \frac{1}{2} m_+ v^2$ is the kinetic energy of ions, we get

$$dj_+ = -\frac{\alpha n_0}{eR} \varrho^* \exp \left[\frac{eU(\varrho^*)}{kT_-}\right] \frac{dW}{U'(\varrho^*)}$$

while the condition for ϱ^* acquires the form $U(\varrho^*) = W/e + U_R$. If we denote the inverse function to $U(r)$ as $X(U)$ then $1/U'(\varrho^*) = X'[U(\varrho^*)]$. Let $\Phi(W) dW = dj_+/j_0$ be the distribution function of the ion-current density, then we have

$$\Phi(W) \, dW = -\frac{\alpha n_0}{eR} \frac{1}{j_0} X(W/e + U_R) \, X'(W/e + U_R) \exp \left(\frac{W + eU_R}{kT_-}\right) dW \, ,$$

where j_0 is the total ion-current density at the wall, while $j_0 = -\alpha n_0 A/eR$, where

$$A = \int_0^{-eU_R} X(W/e + U_R) \, X'(W/e + U_R) \exp \left(\frac{W + eU_R}{kT_-}\right) dW \, .$$

The final form for $\Phi(W)$ then is

$$\Phi(W) \, dW = \frac{1}{A} X(W/e + U_R) \, X'(W/e + U_R) \exp \left(\frac{W + eU_R}{kT_-}\right) dW \, . \qquad (2)$$

DISCUSSION AND GRAPHIC RESULTS

For the determination of the distribution function it is necessary to know the radial course of the potential. In the region of the quasineutrality the potential course is given by the Langmuir — Tonks theory [1] in the form

$$s = \sqrt{\eta}\,(1 - 0{\cdot}2\eta - 0{\cdot}026\eta^2 - 0{\cdot}0065\eta^3 - \ldots)\,,$$

where $s = r\alpha(m_+/2kT_-)^{1/2}$ and $\eta = -eU/kT_-$. This theory is applicable for the inner part of the positive column only, because near the wall a plasma-sheath is formed. The boundary, where the Langmuir-Tonks theory holds no more, is given by the values $s_0 = 0.7722$ and $\eta_0 = 1.155$. If the Debye length is much smaller than the inner radius of the tube, it is possible to neglect the ionization in the sheath as well as the sheath curvature. Consequently, for the potential course in the sheath the Ott results [2] can be used. The resulting potential course was obtained by a continuous connection of both courses by a graphic method.

The maximum width W_0 of the distribution function $\Phi(W)$ is given by the potential difference U_R between the axis and the discharge tube wall. This difference is estimated precisely enough by means of the floating potential U_f [2]

$$W_0 = -eU_R = -eU_f = \frac{1}{2}kT_- \ln\frac{\varepsilon m_+}{2\pi m_-}; \quad \varepsilon = 2{\cdot}718\ldots$$

As the exponential function and the product XX' decrease with decreasing W in the relation (2), the function $\Phi(W)$ reaches its maximum for $W = W_0$. In the direction of smaller energies Φ decreases approximately as $x \exp x$, so that the width ΔW of the maximum of the function $\Phi(W)$ is given approximately by the radial potential drop across the quasineutrality region, where the greatest probability of ionization is. Thus

$$\Delta W \approx \eta_0 kT_-.$$

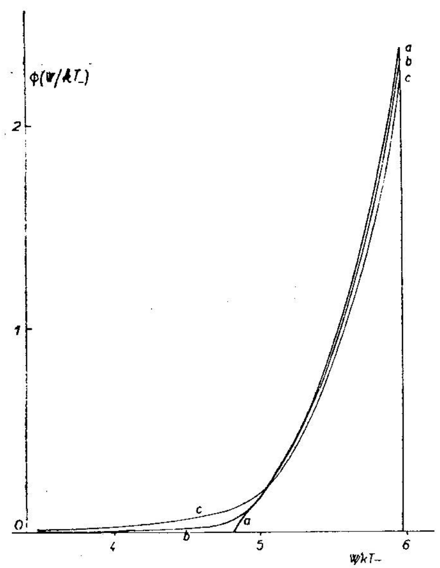

Fig. 1. The normalized functions $\Phi(W/kT_-)$ for low pressure discharge in mercury vapours: a) $h/R = 0$, $(d/R = 0)$; b) $h/R = 0{\cdot}01$, $(d/R = 0{\cdot}068)$; c) $h/R = 0{\cdot}0315$, $(d/R = 0{\cdot}215)$. d is the thickness of the plasma-sheath.

Because of very little probability of ionization in the plasma-sheath, Φ obtains here only slight values.

The normalized functions $\Phi(W)$ are plotted in Fig. 1 for mercury vapours. The ratio h/R (h is the Debye length) was chosen as a parameter for Φ. For $h/R > 3 \times 10^{-2}$ the Debye length is already comparable with the tube radius. In such a case the theory for the radial potential calculation according to [1] and [2] cannot be employed.

REFERENCES

[1] Langmuir I., Tonks L., Phys. Rev. *34* (1929), 876.
[2] Ott W., Z. Naturforschung *17a* (1962), 962.

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