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SOME NONLINEAR FUNCTIONAL MODELS OF FREQUENCY DISCRIMINATION OF THE HUMAN EAR

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On the basis of theoretical treatments of experimental result of the auditory frequency analysis, its non-linear character is shown. To simulate functionally the auditory frequency discrimination ability, the authors describe three models. By all these models the frequency discrimination ability of the human auditory system is arrived at. Which of them is the most adequate as regards the function of the auditory frequency analyser, has to be decided by further experimental investigation of the properties of the human auditory frequency analysis.

INTRODUCTION

The human ear exhibits an example of fine pitch discrimination combined with short transient time. In order to simulate the frequency analysis of the human auditory system, we have to solve the question whether the usual linear analyser will give us adequate help to carry out the functional modelling of the frequency analysis of short signals by hearing.

The linear system consists exclusively of linear elements, the parameters of which are independent of time. For all linear systems the uncertainty relation of communication [1] is valid

$$\Delta f \,.\, \Delta t \geqslant 1, \tag{1}$$

i.e. the product of the tuning width Δf and the transient time Δt is always greater than, or at most equal, to one. Carrying out the frequency analysis by means of the linear analyser we must keep within the limits given by the uncertainty relation.

The experiments of Oetinger [2] and of Liang Chih-an and Chistowitch [3] have shown that in the analysis of short signals the human ear surpasses the limits of uncertainty relation. Thus, as the result of experiments on frequency discrimination of sine-wave pulses, the authors above obtained curves (Fig. 1) representing the frequency difference limen in dependence on the duration of tone pulses, the parameter of the curves being their frequency.

It is quite evident that for short tones of equal duration in the low frequency region the frequency discrimination of the human ear is nearly by an order better than in the linear systems, as the results lie below the line representing the uncertainty relation of communication given by Eq. 1.

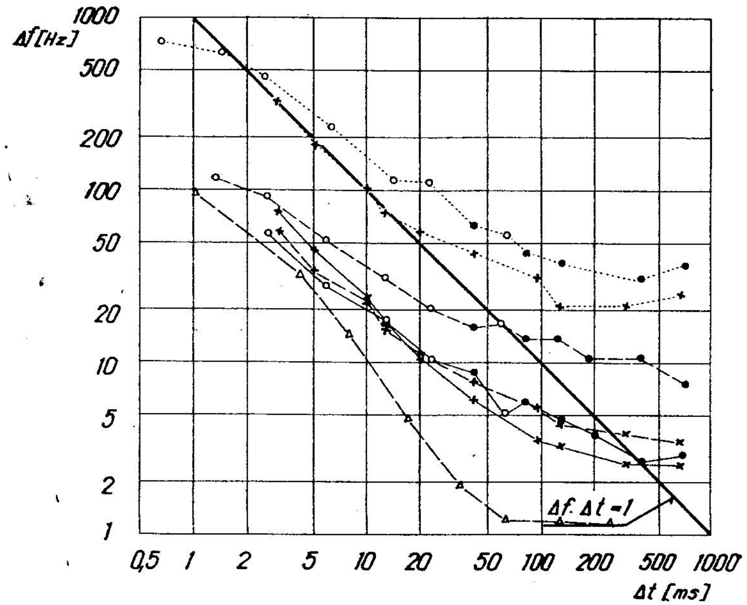

Fig. 1. Just audible change in tone number as a function of the tone duration. ● — Oetinger, Gaussian pulses [2]; ○ — Oetinger, trapeze form pulses [2]; × — Liang- and Chistowitch, rectangular pulses [3]; △ — Cardozo, pulses with null phase and 0.1 sec interstimulation interval [9]; freqvency: ... — 4000 Hz, - - — 1000 Hz, — — 250 Hz.

From what has been said so far, it follows that frequency discrimination of short signals by the human ear cannot be modelled with the help of linear systems. If we want to find systems attaining the analysing qualities of the human ear, we must necessarily have recourse to nonlinear systems. We shall consider only nonlinear elements in which the ear transfer function, while being nonlinear, is time invariant. Let us mention at least the main characteristics of such systems:

The output signal spectrum may contain other frequencies than those contained in the input signal spectrum, there occurs an energy shift with respect to frequency between the input and output signal spectrum. In a given input signal initial conditions exercise an influence on the steady state. Since

from the steady state of a nonlinear system we can judge its state before letting in the signal, nonlinear systems can act as memory elements. The principle of superposition does not apply to nonlinear systems [4].

Next we shall discuss three possible ways of functional modelling of the frequency analysis of the human ear by means of nonlinear elements.

MODELS BASED ON THE NONLINEAR DISTORTION OF SIGNALS

The first method consists in an intentional distortion of the signal, which is afterwards analysed by means of a linear system.

The principle is as follows: Let us consider two tone signals of the frequency f_0 and $f_0 + \Delta f$. In order to distinguish these signals as to the frequency, it is necessary, according to the uncertainty relation that the analysis time is $\Delta t = 1/\Delta f$. By a nonlinear distortion of the considered signals one obtains higher harmonics $n \cdot f_0$ and $n(f_0 + \Delta f)$ ($n = 2, 3, \ldots$). To distinguish these higher harmonics we need the analysis time

$$\Delta t = \frac{1}{n \cdot \Delta f}.$$

The analysis time of higher harmonics is, therefore, for the factor $1/n$ lower than for signals of ground frequencies. By means of the linear analysis of higher harmonics one can lower the time for frequency discrimination of two tone signals and so perform a simulation of the human ear frequency discrimination ability.

By introducing nonlinearity and by analysing the n-th harmonic component, we have, by means of a linear analyser, with regard to linear analysis, increased the effective frequency discrimination ability n-times; Eq. 1 is transformed into

$$\Delta f_1' \cdot \Delta t = \frac{1}{n}.$$

In this system, it is possible to arrive by an appropriate choice of the number n, representing the order of nonlinearity, at the frequency discrimination ability of the hearing

COINCIDENCE MODELS

Next we shall describe another possibility of modelling the frequency discrimination of the human ear by means of the so-called coincidence filter as Schief has described it in detail [5] In Fig. 2 we see a block diagram of the considered coincidence filter. The analysed signal of the frequency f deviates nito the pulse shaping circuit p in which at a given phase of the input signal an impulse of duration b is always formed. The time interval T between these impulses depends on the frequency f of the analysed signal:

$$T = \frac{1}{f}.$$

In the delay circuit τ the impulses are delayed by the constant time interval of the value τ. For a given frequency band of the analysed signal delayed pulses coincide, as to time, with the nondelayed ones and the coincidence circuit c responds to this by creating the control voltage for the control of the gate circuit g. In the presence of the control voltage the gate is open and allows the input signal to pass to the output of the coincidence filter.

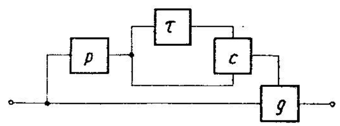

Fig. 2. Block diagram of a coincidence filter.

For the rise time T_n of the coincidence filter we shall take the time interval between the arrival of the signal at the input of the filter and the creation of the first coincidence pulse. This interval depends on the initial phase of the analysed signal and can vary between the values[1])

$$T \leqslant T_n < 2T.$$

The decay time of the coincidence filter is always zero because when the input signal ends, it cannot appear at the output, although the gate circuit g, by virtue of the control voltage created by the coincidence circuit c, remains open for a certain time. The product of the frequency band-width B of the coincidence filter and the so-called rise time T_n is analogous to the product $\Delta f \,.\, \Delta t$ from the uncertainty relation (Eq. 1) valid for linear systems. For narrow band-pass we can delimit the magnitude of the value $B \,.\, T_n$:

$$2\, bf_s < BT_n < 4\, bf_s,$$

where f_s denotes the center frequency of the coincidence filter:

$$f_s \approx \frac{1}{\tau} \quad \text{if} \quad b \ll \tau.$$

Thus the value of the product $B \,.\, T_n$ can theoretically be reduced at will by a suitable choice of the impulse duration b, while in the case of linear filters

[1]) For further details see [5].

the product of band-pass width and decay time is always greater than or equal to one.

We should realize that we cannot get the Fourier components of the frequency spectrum by means of the coincindence filter in the form as described, because the output signal has the same shape as the corresponding section of the input signal. The coincidence filter lets the integral multiples of the basic band pass, too, in so far as the harmonic frequencies fulfill the condition of coincidence of a higher order. This undesirable quality of the filter can be removed by a preliminary filtering of the signal by the help of a linear filter which, however, must not have too high a selectivity in order that its transient time may not reduce the advantages of the short rise time of the coincidence filter.

Coincidence analysis of neural discharges carried out as mentioned above makes it possible to explain the fine frequency discrimination of the ear for short tone signals.

MODELS SIMULATING THE NEURON NETWORK OF THE AUDITORY SYSTEM

As a last method we shall mention the functional simulation of the pitch discrimination of the human ear by means of a neuron interaction in neuron networks. As a special case of the neuron interaction we shall deal with lateral inhibition [7], the most investigated neuron interaction found in eye and skin [8]. By its means the resonant curve of the linear system can be greatly contracted so that the whole frequency discrimination ability is improved. Let us consider by means of a mathematical description of the mentioned neuron interaction a simplified plane neuron network in which the neurons are ordered in n rows and m columns (Fig. 3). Each column represents a neural path leading the excitation from a reception element in the first row through the neurons connected in series to the perception center represented by the final row. We suppose that the pulse rate produced in the neurons of the first row simulating the hair cells disposed along the basilar membrane, is proportional to the displacement of the basilar membrane. The neurons of the $(j - 1)$-th row interact (excitating or inhibiting) with the neurons of the j-th row. Neuron network with certain neuron interaction may serve as a functional model of pitch discrimination in the auditory system, since the input pulse rate curve $y_1(x)$ (Fig. 3) could be transformed in a more sharp output curve $y_n(x)$ enabling better pitch resolution. The transformation between individual rows in the considered neuron networks can be described by means of equations in a linear space

$$Y^{(j)} = \hat{O}^{(j)}\, Y^{(j-1)}, \tag{2}$$

where in the discrete case $Y^{(j)}$ and $Y^{(j-1)}$ are vectors, the components $Y_1^{(j)}$, $Y_2^{(j)}$, ... $Y_m^{(j)}$ and $Y_1^{(j-1)}$, $Y_2^{(j-2)}$, ... $Y_m^{(j-1)}$ of which are equal to the pulse rate of the corresponding neurons of the j-th and $(j-1)$th row respectively.

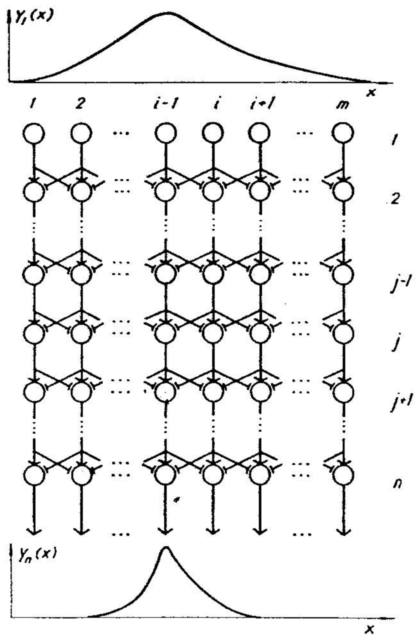

Fig. 3. Simplified neuron network.

If $\hat{O}^{(j)}$, $\hat{O}^{(j-1)}$, ... $\hat{O}^{(2)}$ are bounded linear operators, the transformation between the j-th and 1-st rows can be written in the form

$$Y^{(j)} = \hat{O}_f \, Y^{(1)},$$

where

$$\hat{O}_f = \hat{O}^{(j)} \, . \, \hat{O}^{(j-1)} \, . \dots \, \hat{O}^{(2)}.$$

It is possible to determine the vectors $Y^{(1)}$ and $Y^{(n)}$ within the framework of psychoacoustics at least in the first approximation and thus the searched operator $\hat{O}_f$ characterizing the auditory system as well. In the case of a continuous distribution of the pulse rate on the neuron rows the pulse rate is represented by the continuous function $y_j(x_j)$ and the operator equation (2) takes the form of an integral transformation with the kernel $K(x_j, x_{j-1})$

$$y_j(x_j) = \int K(x_{j-1}, x_j) \, . \, y_{j-1}(x_{j-1}) \, \mathrm{d}x_{j-1}.$$

In order to illustrate the function of a concrete neuron system we take as an example the neuron network with the interaction given by the transformation formula of pulse rate vector components

$$Y_i^{(j)} = \alpha\{Y_i^{(j-1)} - k[Y_{i+1}^{(j-1)} + Y_{i-1}^{(j-1)}]\}. \qquad (3)$$

The relation (3) leads to the simple neuron network (Fig. 3), in which the pulse rate $Y_i^{(j)}$ produced in the i-th neuron disposed along the j-th row is proportional to the difference of the pulse rate $Y_i^{(j-1)}$ of the i-th neuron of the previous $(j-1)$-th row of the direct path and the k-multiple of the sum of neurons pulse rates $Y_{i+1}^{(j-1)}$ and $Y_{i-1}^{(j-1)}$ of the $(j-1)$-th row of the neighbouring paths. In this way, at every neuron three synapses are situated. One of the synapses represents the excitation input of the neuron and the other two synapses coming from the neighbouring neurons, represent inhibition inputs. By means of the coefficient α the resulting curve is normalized.

Let $y_j(x_j)$ be the pulse rate of neurons in the j-th row placed in the point with the coordinate x_j. Taking into account that the function $y_j(x_j)$ is assumed to be continuous, beginning with the 4-th order we may neglect all the terms in the Taylor expansion of this function supposing that the distance between neighbouring neurons in each row denoted by Δx_{j-1} is small. The resulcing pulse rate of the neurons in the j-th row will be

$$y_j(x_j) = \alpha \left\{ (1 - 2k) y_{j-1}(x_j) - k(\Delta x_{j-1})^2 \left. \frac{\mathrm{d}^2 y_{j-1}(x_{j-1})}{\mathrm{d}x_{j-1}^2} \right|_{x_{j-1}=x_j} \right\}. \qquad (4)$$

From the relation (4) it follows that there is a sharpening of the peak in the curves of the resonance type depending on the choice of the coefficient of the cross coupling k and the distance of neighbouring neurons. The neural discharges produced in the basilar membrane according to the place theory of hearing, are processed in the logical mesh circuits of the neuron network. Here the first step, therefore, is the linear preliminar analysis by the basilar membrane and the conversion of the displacement of the basilar membrane into pulse rate in the corresponding neural path. Only, then the nonlinear process is set in motion by the mutual influence of the adjacent nerve fibres. This is the most likely explanation of the high frequency discrimination of the human ear in spite of the rather flat resonant curves of the basilar membrane.

CONCLUSIONS

On principle it is possible to find such nonlinear systems in which the frequency analysis of short signals arrives at values experimentally found in hearing. It is difficult to assert, for the time being, which of these methods of nonlinear analysis most approaches the real function of the human ear in the role of the frequency analyser. Only further work in this field will give us an answer to this question.[2])

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