Fyzikálny časopis 19 (1969), No. 2

THE SU(8) GROUP AND CLASSIFICATION OF ELEMENTARY PARTICLES

ĽUBOMÍR TÓTH, Košice

This paper presents a possibility to extend the SU (4) symmetry of elementary particles to the SU (8) symmetry in analogy with the SU (3) → SU (6) symmetries within the framework of the static quark model. There are obtained wave functions, mass formulae and magnetic moments.

INTRODUCTION

Recently a lot of experimental data on accelerators have been obtained in different countries confirming the existence of new particles and resonant states (resonances); their number now is over 300. To classify them physicists use also parametrical continuous groups — Lie groups. Some successes were obtained in this way in 1961 by Ne'eman and Gell-Mann who suggested, independently, a certain method — *the eightfold way* (because we work therein with eight quantum numbers). The mathematical foundations were found in the Lie groups, particularly in the SU (3) group.

Although *the eightfold way* of Gell-mann—Ne'eman has improved the classification of elementary particles, it is not understandable from the point of view of the unitary model why in nature there are no particles (so-called quarks) corresponding to the minimal dimensional representation — the three-component tensor, which was the fundamental one in the theory of Sakato (sakaton). In 1964 Gell-Mann and (independently) Zweig used a hypothesis according to which all adrons can be constructed within the fremework of SU (3) by means of the mentioned three fundamental particles-quarks or aces with fractional values of quantum numbers.

To avoid difficulties of the SU (3) symmetry (the existence of 9 vector mesons, the relation between masses of scalar and vector mesons etc.) Gürsey, Radicatti and Pais used in 1964 the SU (6) symmetry resulting from an extension of the direct product SU (2) ⊗ S (3). Here SU (2) group describes spin transformations. The principal drawback of the SU (3) and SU (6) theories is that particles with fractional electric charges-quarks have not been observed so far.

To avoid the above mentioned difficulties physicists began to study groups of higher ranks. Several authors [5—7] tried to construct particles by means of four quarks within the framework of the higher SU (4) symmetry in which quarks have integral charges. The necessity of integral charges in strong interactions caused the introducing of a new conserving quantum number, which was called *charm* or *supercharge* and was included in the SU (4) symmetry. The SU (8) group results by an extension of the direct (or Kronecker) product SU (2) ⊗ SU (4), where SU(2) is a spin group.

THE SU (4) GROUP

The SU (4) group is a 15 parameter group of unitary unimodular 4 × 4 matrices, which can be written in the following form

$$U = \exp(iH),$$

where $H = H^{+}$, i. e., the Hermitian conjugate and Sp$H = 0$. The matrix H can be rewritten in the form [15]

$$H = \sum_{k=1}^{15} \omega_k \lambda_k,$$

where ω_k are group parameters and λ_k are generators ($k = 1, 2, \ldots, 15$). Among them we can find three commutating (together) generators, λ_3, λ_8, λ_{15}, so that the rank of the SU (4) group equals three. Generators fulfil the following communication relations

$$[\lambda_i, \lambda_j] = 2i f_{ijk} \lambda_k, \quad \mathrm{Sp}(\lambda_i, \lambda_j) = 2\delta_{ij},$$

where f_{ijk} are structural coefficients.

We shall use one of the models (given in [7]) of the SU (4) group, characterizing the properties of internal symmetries of strong interactions:

	T_3	Y	Z'	Q	N	Z
p	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	1	−1	1
n	$-\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	0	−1	1
λ	0	$-\frac{2}{3}$	$\frac{1}{4}$	0	−1	1
q	0	0	$-\frac{3}{4}$	0	−1	0

i. e. the quartet consists of antibaryons ($N = -1$). This choice within the (extended) SU (8) symmetry is required to obtain multiplets in agreement with the experiments. Here we have introduced a new conserving quantum number

Z' which is called *supercharge* and it is related to the conserving quantity Z (obtain by the author) by the following formula

$$Z = Z' - \tfrac{3}{4}N,$$

thus we have $Q = T_3 + Y/2 + Z'/3 - N/4 = T_3 + Y/2 + Z/3$.

We shall now deal with the occupation of multiplets. In this higher symmetry new particles *appear* comparing to the SU(3) symmetry, and we have 15 vector mesons, 15 pseudoscalar mesons, 20 baryons $\frac{1}{2}^+$and 20 baryons $\frac{3}{2}^+$ [7]. Pseudoscalar and vector mesons are constructed combining quark-antiquark and they belong to the representation 15 because

$$4 \otimes \bar{4} = 15 \oplus 1.$$

A 15-plet of vector mesons also contains a triplet (λ^* — isosinglet and ξ^* — isodoublet) with $Z = 1$ and its antitriplet ($\bar{\lambda}^*$, ξ^*) with $Z = -1$ in addition to the ordinary SU (3) octet (ϱ, K, ω) and the singlet $\mathscr{S}$. 15-plet of pseudoscalar mesons π, K, η, X, λ, ξ has a similar structure.

Baryons $\frac{1}{2}^+$ belong to the representation 20′ which results as a combination of three quarks, i. e.,

$$4 \otimes \bar{4} \otimes \bar{4} = \overline{36} \oplus 20' \oplus 2 \times \bar{4}.$$

It contains a SU(3) octet (N, Λ, Σ, Ξ) with $Z = 0$, a sextet (σ_1 -isotriplet, $\sigma_{\frac{1}{2}}$ -isodoublet and σ_0 -isosinglet) with $Z = -1$, triplet $-\bar{3}$ ($\tau_{\frac{1}{2}}$ -isodoublet and τ_0 -isosinglet) with $Z = -1$ and a triplet -3 ($t_{\frac{1}{2}}$ isodoublet and t_0 -isosinglet) with $Z = -2$.

Baryon resonances $\frac{3}{2}^+$ belong to the representation 20 which is obtained as a combination of five quarks (two quarks and three antiquarks), for instance

$$15 \otimes 20' = 140'' \oplus \overline{60} \oplus \overline{36} \oplus 20 \oplus 20' \oplus 20' \oplus \bar{4}.$$

This representation contains the SU (3) -decuplet (Ω^-, Ξ^*,Y^*,N^*), a sextet (Ω'^- -isosinglet, Ξ'^* -isodoublet and Y'^* -isotriplet) with $Z = -1$, a triplet (Ω''^- -isosinglet and Ξ''^* -isodoublet) with $Z = -2$, and a singlet (Ω'''^- -isosinglet) with $Z = -3$.

THE SU (8) GROUP

The extension of the algebra of the SU (4) group to the algebra of the SU (8) group (through the direct product SU (4) $\otimes$ SU (2), where SU (2) is a spin group) is obtained by unifying the algebras, where [10]

$$F_\mu \qquad (\mu = 1, 2, \ldots, 15)$$

$$\text{and } J_k \qquad (k = 1, 2, 3)$$

are basical elements of the Lie algebras of SU (4) and SU (2) groups, respectively. We shall consider the direct product

$$J_k \otimes F_\mu. \tag{1}$$

Operators constructed in this way will form the Lie algebra if their commutators are expressed by the same quantities. However, we get:

$$[J_k \times F_\mu, J_l \times F_\nu] = \tfrac{1}{2}[J_k, J_l] \times \{F_\mu, F_\nu\} + \tfrac{1}{2}\{J_k, J_l\} \times [F_\mu, F_\nu],$$

i. e., anticommutators appeare. Therefore we must extend the algebras from which we started. We add to the multiplied algebras the following elements:

$$\sigma_0 = \begin{Vmatrix} 1 & 0 \\ 0 & 1 \end{Vmatrix} = I^{(2)}, \quad \lambda_0 = 2^{-\frac{1}{2}} \begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{Vmatrix} = 2^{-\frac{1}{2}}\, I^{(4)},$$

while $\{\sigma_k, \sigma_1\} = 2\delta_{k1}\sigma_0$, $\{\lambda_\mu, \lambda_\nu\} = 2^{\frac{1}{2}}\, \delta_{\mu\nu}\, \lambda_0 + d_{\mu\nu\varrho}\lambda_\varrho$, where δ_{k1} is the Kronecker symbol and $d_{\mu\nu\varrho}$ the totally symmetric tensor. Thus we must take the U (4) and U (2) groups instead of the SU (4) and SU (2) groups and then get the product of type (1). We may choose the following elements as the basis of that direct product:

$$\Lambda_\mu^{(k)} = \sigma_k \otimes \lambda_\mu \qquad (k = 1, 2, 3), \qquad (\mu = 1, 2, \ldots, 15),$$

where σ_k and λ_μ are generators of the SU (2) and SU (4) groups, respectively. Normalisation conditions of matrices will be written in the form

$$\mathrm{Sp}(\Lambda_\mu^{(k)}, \Lambda_\nu^{(l)}) = 4\delta_{kl}\delta_{\mu\nu}.$$

The elements $\Lambda_\mu^{(k)}$ generate the algebra of the U(8) group and represent its generators. To come back to the special group it is enough to exclude the unit element $\Lambda_0^{(0)}$ and we get the Lie algebra of the SU(8) group. As the SU(8) group is the group of rank 7, so there are seven diagonal generators of $\Lambda_\mu^{(k)}$ among those commuting together. The two last generators (connected with Z) have this physical significance [9]:

$$\Lambda_{15}^{(0)} = \sigma_0 \otimes \lambda_{15} = \tfrac{8^{\frac{1}{2}}}{3}Z, \text{ where } Z = \sigma_0 \otimes \tfrac{3^{\frac{1}{2}}}{8}\lambda_{15}$$

the operator of *supercharge* (*charm*).

$$\Lambda_{15}^{(3)} = \sigma_3 \otimes \lambda_{15} = 2\tfrac{8^{\frac{1}{2}}}{3}\eta_3, \text{ where } \eta_3 = ZS_3\ (S_3 = \tfrac{1}{2}\sigma_3 \otimes I^{(4)}),$$

the operator of the third component of *supermagnetic* moment.

To obtain generators of the SU (8) group in tensorial form it is necessary to perform a similar procedure as in the case of generators, starting from the product of type (1). As the basis of representations of U (4) ⊗ U (2) we shall take directly the elements J_k and F_μ. We get now

$$J_0 \times \sigma_0 + \sum_{k=1}^{3} J_k \times \sigma_k = \left\| \begin{matrix} \frac{1}{2} + J_3 & J_- \\ J_+ & \frac{1}{2} - J_3 \end{matrix} \right\| \quad (2)$$

$$F_0 \times \lambda_0 + \sum_{\mu=1}^{15} F_\mu \times \lambda_\mu = \left\| \begin{matrix} \frac{1}{4} + Q & T_- & K_- & M_- \\ T_+ & \frac{1}{4} - Q + Y + \frac{2}{3}Z & L_- & N_- \\ K_+ & L_+ & \frac{1}{4} - Y + \frac{1}{3}Z & P_- \\ M_+ & N_+ & P_+ & \frac{1}{4} - Z \end{matrix} \right\| \quad (3)$$

From the matrices (2) and (3) we obtain the product of the type (1). By subtracting the trace of the 8 × 8 matrix (which was obtained as the product of (2) and (3)) from it we get the spin-tensor of the group SU (8). This tensor is given in Table 1. For its elements we have

$$(A^\mu_\mu) = 0, \quad (A^\mu_\nu)^+ = A^\nu_\mu,$$

$$[A^\varrho_\sigma, A^\mu_\nu] = \delta^\varrho_\nu A^\mu_\sigma - \delta^\mu_\sigma A^\varrho_\nu, \quad (A^\mu_\nu)^j_i = \delta_{\mu i}\delta_{\nu j} - \tfrac{1}{8}\delta_{\mu\nu}\delta_{ji},$$

where $\mu, \nu, i, j = 1, 2, \ldots, 8$.

The SU (8) group representations will be characterized by the tensor basis of type $\Psi^{j_1 \ldots j_q}_{i_1 \ldots i_p}$, which transforms according to the tensorial character by means of the following formula:

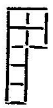
$$(\Psi'^{j'_1 \ldots j'_q}_{i'_1 \ldots i'_p}) = U^{i_1}_{i'_1} \ldots U^{i_p}_{i'_p} (U^+)^{j'_1}_{j_1} \ldots (U^+)^{j'_q}_{j_q} \Psi^{j_1 \ldots j_q}_{i_1 \ldots i_p} \quad (i, j = 1, 2, \ldots, 8).$$

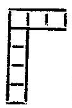
In the group SU (8) there are three invariant tensors [8] δ^i_k, $\varepsilon_{i_1 i_2 \ldots i_8}$ and $\varepsilon^{i_1 i_2 \ldots i_8}$. They can be considered as basic vectors of the one-dimensional subspace of the SU (8) group (its one-dimensional representations).

Further some irreducible tensors (which are obtained ih this model) are written and the corresponding Young diagrams are drawn:

Ψ^A_B  63 [2111111]

$\Psi^{[AB]}_C$  216 [211111]

$\Psi^{[AB]}_{[CDE]}$  1344 [22111]

$\Psi^{[ABC]}_{\{DE\}}$  1800 [31111] .

The dimensionalities of irreducible representations are determined by the formula

$$M[\gamma_1\gamma_2 \ldots \gamma_7] = (1!2! \ldots 7!)^{-1}(\gamma_1 + 7)(\gamma_2 + 6) \ldots (\gamma_7 + 1) \times$$
$$\times (\gamma_1 - \gamma_7 + 6)(\gamma_2 - \gamma_7 + 5) \ldots (\gamma_6 - \gamma_7 + 1) \times \ldots$$
$$\ldots \times (\gamma_1 - \gamma_2 + 2)(\gamma_2 - \gamma_3 + 1) \times (\gamma_1 - \gamma_2 + 1).$$

In the SU (8) symmetry particles are classified according to the subgroups SU(4) ⊗ SU (2). The decomposition of some irreducible representations in these subgroups is given in Table 2.

Table 2

Some irreducible representations of the SU (8) group and their SU (4) ⊗ SU (2) content (m, n).

Dimensions	(m, n) content
63	(15,3), (15,1), (1,3).
216	($\overline{36}$,2), (20′,4), (20′,2), ($\bar{4}$,4), ($\bar{4}$,2).
1344	(140″,2), ($\overline{60}$,4), ($\overline{60}$,2), ($\overline{36}$,4), ($\overline{36}$,2), (20′,6), (20′,4), (20′,4), (20′,2), (20′,2), (20,4), ($\bar{4}$,6), ($\bar{4}$,4), ($\bar{4}$,2).
1800	(140″,4), (140″,2), ($\overline{60}$,2), ($\overline{36}$,4), ($\overline{36}$,2), (20,6), (20,4), (20,2), (20′,6), (20′,4), (20′,4), (20′,2), (20′,2), ($\bar{4}$,4), ($\bar{4}$,2).
$\overline{4032}$	($\overline{160}$,6), ($\overline{160}$,4), ($\overline{160}$,2), (140″,4), (140″,2), ($\overline{84'}$,4), ($\overline{84'}$,4), ($\overline{60}$,4), ($\overline{36}$,6), ($\overline{36}$,2), ($\overline{20'}$,2), ($\bar{4}$,6), ($\bar{4}$,2).

We shall now consider the construction of particles. Mesons are constructed by the combination of quark-antiquark and they are contained in the irreducible representation 63 of the SU (8) group, because

$$8 \otimes \bar{8} = 63 \oplus 1,$$

while 15-plets of vector and pseudoscalar mesons are in the representations (15,3) and (15,1) of the subgroups SU (4) ⊗ SU(2) (see the decomposition in Table 2).

The representation 63 also contains the vector singlet X_1^{*0}, which can be written as (1,3). We can assign to the second irreducible representation and its SU (4) ⊗ SU (2) content (1,1) a pseudoscalar meson X_1^0.

Baryons of the representation 20′are in the representation 216 of the SU (8) group and they result as a combination of one quark-two antiquarks i. e.

$$8 \otimes \bar{8} \otimes \bar{8} = \overline{280} \oplus 216 \oplus 2 \times \bar{8}$$

and the representation (20′,2) of the subgroup SU (4) ⊗ SU (2) corresponds to them.

Table 1

The spin-tensor of the SU (8) group.

The following notations are used:

$T_{\pm} = \frac{\lambda_1 \pm i\lambda_2}{2}$ $K_{\pm} = \frac{\lambda_4 \pm i\lambda_5}{2}$ $L_{\pm} = \frac{\lambda_6 \pm i\lambda_7}{2}$ $M_{\pm} = \frac{\lambda_9 \pm i\lambda_{10}}{2}$ $N_{\pm} = \frac{\lambda_{11} \pm i\lambda_{12}}{2}$ $P_{\pm} = \frac{\lambda_{13} \pm i\lambda_{14}}{2}$

$T_3 = \frac{1}{2}\lambda_3$ $Y = 3^{-\frac{1}{2}}\lambda_8$ $Z = (\frac{3}{8})^{\frac{1}{2}}\lambda_{15}$

$\frac{1}{20} + \frac{1}{4}J_3 + J_3Q$	$(\frac{1}{2} + J_3)T_-$	$(\frac{1}{2} + J_3)K_-$	$(\frac{1}{2}Q + J_3)M_-$	$J_-(\frac{1}{4} + Q)$	J_-T_-	J_-K_-	J_-M_-
$(\frac{1}{2} + J_3)T_+$	$-\frac{1}{2}Q + \frac{1}{2}Y + \frac{1}{3}Z + \frac{1}{4}J_3 - J_3Q + J_3Y + \frac{2}{3}J_3Z$	$(\frac{1}{2} + J_3)L_-$	$(\frac{1}{2} + J_3)N_-$	J_-T_+	$J_-(\frac{1}{4} - Q + Y) + \frac{2}{3}J_-Z$	J_-L_-	J_-N_-
$(\frac{1}{2} + J_3)K_+$	$(\frac{1}{2} + J_3)L_+$	$-\frac{1}{2}Y + \frac{1}{6}Z + \frac{1}{4}J_3 - J_3Y + \frac{1}{3}J_3Z$	$(\frac{1}{2} + J_3)P_-$	J_-K_+	J_-L_+	$J_-(\frac{1}{4} - Y + \frac{1}{3}Z)$	J_-P_-
$(\frac{1}{2} + J_3)M_+$	$(\frac{1}{2} + J_3)N_+$	$(\frac{1}{2} + J_3)P_+$	$-\frac{1}{2}Z + \frac{1}{4}J_3 - J_3 - J_3Z$	J_-M_+	J_-N_+	J_-P_+	$J_-(\frac{1}{4} - Z)$
$J_+(\frac{1}{4} + Q)$	J_+T_-	J_+K_-	J_+M_-	$\frac{1}{2}Q - \frac{1}{4}J_3 - J_3Q$	$(\frac{1}{2} - J_3)T_-$	$(\frac{1}{2} - J_3)K_-$	$(\frac{1}{2} - J_3)M_-$
J_+T_+	$J_+(\frac{1}{4} - Q + Y) + \frac{2}{3}J_+Z$	J_+L_-	J_+N_-	$(\frac{1}{2} - J_3)T_+$	$-\frac{1}{2}Q + \frac{1}{2}Y + \frac{1}{3}Z - \frac{1}{4}J_3 + J_3Q - J_3Y - \frac{2}{3}J_3Z$	$(\frac{1}{2} - J_3)L_-$	$(\frac{1}{2} - J_3)N_-$
J_+K_+	J_+L_+	$J_+(\frac{1}{4} - Y + \frac{1}{3}Z)$	J_+P_-	$(\frac{1}{2} - J_3)K_+$	$(\frac{1}{2} - J_3)L_+$	$-\frac{1}{2}Y + \frac{1}{6}Z - \frac{1}{4}J_3 + J_3Y - \frac{1}{3}J_3Z$	$(\frac{1}{2} - J_3)P_-$
J_+M_+	J_+N_+	J_+P_+	$J_+(\frac{1}{4} - Z)$	$(\frac{1}{2} - J_3)M_+$	$(\frac{1}{2} - J_3)N_+$	$(\frac{1}{2} - J_3)P_+$	$-\frac{1}{2}Z - \frac{1}{4}J_3 + J_3Z$

Baryons $\frac{3}{2}^+$ are in the representation (20,4) of the subgroup SU (4) ⊗ SU (2), which appears at the decomposition of the representation 1344 of the group SU (8). The representation 1344 arises from the combination

$$8 \otimes 8 \otimes \bar{8} \otimes \bar{8} \otimes \bar{8} = \overline{4032} \oplus \overline{3080} \oplus 2 \times$$
$$\times\ 5544 \oplus 2 \times \overline{4200} \oplus 1344 \oplus 1800 \oplus 6 \times \overline{280} \oplus 6 \times 216 \oplus 6 \times \bar{8}.$$

As to the other registered resonances we can choose a subgroup of some irreducible representation for any resonance (obviously the same holds for the SU (6) group). Few resonances observed so far are in individual subgroups and the fundamental characteristics of a number of them have not been studied sufficiently, so that it is difficult to classify them now.

PHYSICAL CONSEQUENCES OF THE SU (8) SYMMETRY

At first we shall construct wave functions of particles. For the representation (20′,2) of baryons $\frac{1}{2}^+$ the spin part of the wave function must be the eigenfunction of the operator of the whole spin S^2 and S_z with the eigenvalue $\frac{3}{4}\hbar^2$ and $\frac{1}{2}\hbar$, respecively. This sλin part of the wave function may be taken in the form

$$6^{-\frac{1}{2}}(\underset{abc}{\uparrow\uparrow\downarrow} + \underset{abc}{\uparrow\downarrow\uparrow} - 2\underset{abc}{\downarrow\uparrow\uparrow})$$

and it is symmetrical in bc. The internal (*supercharge*) part of the baryons wave functions is the antisymmetric one in the interchange of quarks in this representation. Therefore the notation we shall use has to be understood as follows

$$a[bc] = a(bc - cb)(2)^{-\frac{1}{2}}.$$

We give the wave functions of some particles [11]:

$$P \to p[\lambda\bar{q}]6^{-\frac{1}{2}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow - 2\downarrow\uparrow\uparrow)$$
$$N \to n[\lambda\bar{q}]6^{-\frac{1}{2}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow - 2\downarrow\uparrow\uparrow)$$
$$\Lambda^0 \to 6^{-\frac{1}{2}}(p[\overline{pq}] + n[\overline{nq}] - 2\lambda[\lambda\bar{q}])6^{-\frac{1}{2}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + 2\downarrow\uparrow\uparrow)$$
$$\sigma_1^- \to n[\bar{p}\lambda]6^{-\frac{1}{2}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow - 2\downarrow\uparrow\uparrow)$$
$$\tau_{\frac{1}{2}}^0 \to 2^{-\frac{1}{2}}(\lambda[\bar{p}\lambda] - q[\overline{pq}]6^{-\frac{1}{2}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow - 1\downarrow\uparrow\uparrow).$$

Baryon resonances of the subgroup (20,4) are constructed by means of the combination of two quarks-three antiquarks. The spin part of the wave function must be the eigenfunction of the operators of the whole spin S^2 and S_z with the eigenvalue $\frac{15}{4}\hbar^2$ and $\frac{3}{2}\hbar$. There was found the combination fulfilling

$$2^{-\frac{1}{2}}(\underset{abcde}{\uparrow\uparrow\uparrow\uparrow\downarrow} - \underset{abcde}{\uparrow\uparrow\uparrow\downarrow\uparrow}),$$

the antisymetric one in de. The internal part of the wave function is the antisymmetric function in the interchange of antiquarks and the symmetrical function in the interchange of quarks. Therefore the notations we shall use must be understood as

$$[abc]\{de\} = 6^{-\frac{1}{2}}(abc - bac + bca - cba + cab - acb)2^{-\frac{1}{2}}(de + ed).$$

The internal parts of the wave functions of some baryon resonances will be:

$$\Xi^{*-} \to [\overline{pnq}]\{n\lambda\} \qquad \Xi^{*0} \to [\overline{pnq}]\{p\lambda\}$$

$$Y^{*+} \to [\overline{pnq}]\{pp\} \qquad Y^{*0} \to 2^{-\frac{1}{2}}([\overline{p\lambda q}]\{p\lambda\} - [\overline{n\lambda q}]\{n\lambda\})$$

$$N^{*++} \to [\overline{n\lambda q}]\{pp\} \qquad N^{*0} \to 2^{-\frac{1}{2}}([\overline{p\lambda q}]\{pn\} - [\overline{n\lambda q}]\{nn\})$$

$$Y^{*+\prime} \to [\overline{pn\lambda}]\{pp\} \qquad \Xi^{\prime\prime 0} \to [\overline{pn\lambda}]\{pq\}$$

$$\Omega^{-} \to [\overline{pnq}]\{\lambda\lambda\} \qquad \Omega^{\prime\prime -} \to [\overline{pn\lambda}]\{qq\}.$$

Here we have used the graphical (model) way of the expression of wave functions for a better physical understanding. Using the above *graphs* it is not difficult to express the wave function in the explicit (analytic) form.

We shall deal with the mass formulae of particles now. To calculate elementary particle masses we shall use ordinary methods for deriving mass formulae [12]. A 15-plet of pseudoscalar mesons of the representation (15,1) has within the SU (4) the following mass formulae:

$$m_\eta^2 - m_\pi^2 = \tfrac{4}{3}(m_K^2 - m_\pi^2) \tag{4a}$$

$$m_\lambda^2 - m_K^2 = m_\xi^2 - m_\pi^2 \tag{4b}$$

$$m_{X_{15}^0}^2 - m_\pi^2 = \tfrac{1}{6}(m_K^2 - m_\pi^2) + \tfrac{3}{2}(m_\xi^2 - m_\pi^2). \tag{4c}$$

Here (4a) is the well known formula of Gell-mann—Ocubo and the formulae (4b), (4c) give us relations for *new* pseudoscalar mesons. The latest Rosenfeld Tables [13] contain only two new convenient particles: The meson resonance $0(0^-)$ — — X^0 (or η') with the mass $m = 958{,}3$ MeV and the square mass $m^2 =$ $= 0{,}918$ Gev^2 and the second resonance $0(0^-)$ — $E(1420)$ with $m = 1424$ MeV and $m^2 = 2{,}03$ Gev^2. In our scheme there are also two convenient (in quantum numbers) particles X_{15}^0 and λ^0. Out of the four possible ways of making the theory agree with the experiment only two are suitable. First our X_{15}^0 meson can be assigned to the particle η' Then from the mass formulae we obtain masses of the other two meson resonances:

$$m_\xi^2 = 0{,}595 \text{ Gev}^2 \quad \text{and} \quad m_\lambda^2 = 0{,}822 \text{ Gev}^2.$$

The second particle $E(1420)$, because of the great difference of its m^2 compared to the others, can be assigned to the pseudoscalar singlet (1,1) of the repre-

sentation 1. Secondly the particle $E(1420)$ can be ascribed to our X_{15}^0 meson. Then for the other particles we find

$$m_\xi^2 = 1{,}32 \text{ Gev}^2 \quad \text{and} \quad m_\lambda^2 = 1{,}54 \text{ Gev}^2$$

where η' must be assigned to λ^0. Their difference is for $m^2 \sim 65$ %, that is for $m \sim 8$ %, which is a rather good agreement.

A 15-plet of vector mesons of the representation (15,3) has within SU (4) the following mass formulae:

$$m_\varrho^2 + 3m_\omega^2 = 4m_{K^*}^2.$$

$$m_{\lambda^*}^2 + m_\varrho^2 = m_{\xi^*}^2 + m_{K^*}^2.$$

$$2(m_\varphi^2 + m_\omega^2 + m_\varrho^2) = 3(m_{\xi^*}^2 + m_{K^*}^2).$$

The first formula (Gell-Mann's relation) is not fulfilled (for the ω^0 meson). A hypothesis is suggested about $\omega^0 - \varphi^0 - X_1^0$ mixing. In the result we get

$$(m_\omega^2 - m_\varrho^2)(m_\varphi^2 - m_\varrho^2)(m_{X_1^*}^2 - m_\varrho^2) =$$

$$= -2(m_\omega^2 + m_\varphi^2 + m_{X_1^*}^2 - 2m_{K^*}^2 - 2m_{\xi^*}^2 + m_\varrho^2)(m_{K^*}^2 - m_\varrho^2)(m_{\xi^*}^2 - m_\varrho^2).$$

Unfortunately this formula (looking like Schwinger's formula in SU (3) cannot be tested because of the lack of experimental data. Combining the representations (15,1) and (15,3) into the representation 63 of the group SU (8), we have the following general mass formula

$$m^2 = m_J^2 \sum_{(A,B)} \Psi_B^{A*}\Psi_B^A + \Delta_1 \sum_{(A,B)} \{\delta(A - \bar\lambda) + \delta(B - \lambda)\} + \Delta_2 \sum_{(A,B)} \{\delta(A - \bar q) +$$
$$+ \delta(B - q)\} + h\Big(\sum_{(A)} \Psi_A^{*A}\Big)\Big(\sum_{(A)} \Psi_A^A\Big)$$

where the first term is invariant under SU (4) and the other terms are invariant under SU(2). In case of pseudoscalar mesons ($J = 0$) Sp$\Psi = 0$ and the last term disappears.

Mass formulae for baryons of the representation (20,2) are:

$$m_\Xi + m_N = \tfrac{1}{2}(3m_\Lambda + m_\Sigma)$$

$$m_{\sigma_0} - m_{\sigma_{\frac{1}{2}}} = m_\Xi - m_\Sigma$$

$$m_{t_0} - m_{t_{\frac{1}{2}}} = m_\Sigma - m_N \tag{5}$$

$$m_N + m_{t_{\frac{1}{2}}} = \tfrac{1}{2}(3m_{\tau_0} + m_{\sigma_1})$$

$$m_{\tau_{\frac{1}{2}}} = \tfrac{1}{2}(m_\Xi + m_{t_{\frac{1}{2}}}).$$

We can find only one baryon $\frac{1}{2}^+$ in the Rosenfeld Tables, namely $N'(1470)$ with the mass $m = 1470$ Mev. There are three possibilities how to fit this particle into our scheme, from which we prefer the following one: We take $m_{t_{\frac{1}{2}}} = 1470$ Mev and from the mass relations we find

$$m_{t_0} = 1724 \text{ Mev} \quad \text{and} \quad m_{\tau_{\frac{1}{2}}} = 1394 \text{ Mev}.$$

The mass formula for baryon resonances of the subgroup (20,4) has this form:

$$m = m_0 + h_1\{\tfrac{5}{3} - Y) - \tfrac{1}{3}(\tfrac{1}{2} - Z)\} + h_2(\tfrac{1}{2} - Z), \tag{6}$$

i. e., we have equidistances not only for the hypercharge Y (as in the unitary symmetry) but for the supercharge Z, too. Unfortunately, in the Rosenfeld Tables there are no *new* convenient particles (except the well-known SU (3) decuplet), so that we can determine only

$$h_1 = 146 \text{ Mev} \quad \text{and} \quad 2m_0 + h_2 = 2326 \text{ Mev}.$$

Mass formulae (5) and (6) of the representations (20′,2) and (20,4) can be combined within the representation 1344 of the SU (8) group by the formula

$$m = m_J + k_1 Y + k_2 T(T + 1) + k_3 Y^2 + k_4 Z + k_5 Z^2, \tag{7}$$

where the coefficients $m_{\frac{1}{2}}$, $m_{\frac{3}{2}}$, k_i are expressed through the constants α_1, α_2, β_1, β_2, m_0, h_1 and h_2, where:

$$\alpha_1 = m_N - m_\Sigma, \qquad \alpha_2 = m_\Xi - m_\Sigma,$$

$$\beta_1 = m_N - m_{\sigma_1}, \qquad \beta_2 = m_{t_{\frac{1}{2}}} - m_{\sigma_1}.$$

The first term in (7) is invariant under SU (4) and the other terms are invariant under SU (2).

As three out of four quarks have a zero charge, the calculation of the magnetic moments of mesons and baryons is not done by the ordinary procedure, which was used in the SU (3) → SU (6) theories. We suggest a hypothesis about anomalous quark magnetic moments:

	Q	T_3	Z	μ/μ_0
p	1	$\frac{1}{2}$	1	$1 + \delta_1$
n	0	$-\frac{1}{2}$	1	δ_2
λ	0	0	1	δ_3
q	0	0	0	δ_4

We have here introduced anomalous magnetic moments (in the Bohr magnetons) as small corrections to the Dirac magnetic moments. Then, for instance, we get for some baryons the magnetic moments:

$$P \to 1 + \delta_1 - \delta_3 - \delta_4, \qquad N \to \delta_2 - \delta_3 - \delta_4,$$

$$\Lambda^0 \to - \delta_4, \qquad \Sigma^+ \to 1 + \delta_1 - \delta_2 - \delta_4.$$

If we assume for the doublet quarks (p, n) the same origin of the anomality (peripheral charges) then $\delta_1 = - \delta_2$. If we determine the other parameters δ_i from experimental data (according to the Rosenfeld Tables)

$$\mu_P = 2.79 \text{ M}_B, \quad \mu_N = -1.91 \text{ M}_B, \quad \mu_{\Lambda^0} = (-0.73 \pm 0.16)\text{M}_B,$$

we can estimate the magnetic moment μ_{Σ^+}. Its latest experimental value is $\mu_{\Sigma^+} = (2.5 \pm 0.7)\text{M}_B$. Taking into account the accuracy of the experiment (for μ_{Λ^0}) we obtain within the framework of SU (4) the value $\mu_{\Sigma^+} = 3.81$ M_B (the difference is 19 %) and within that of SU (8) we get the value $\mu_{\Sigma^+} = 3.62$ M_B (the difference is 13 %).

CONCLUSION

In the present paper we have obtained within the framework of the theory SU(4) → SU (8) some new results in comparison to the usually used SU (3) → SU (6) symmetries. There are mainly the mass formulae for the new meson and baryon resonances. To explain the deviation of masses ω and φ from those obtained from mass formulae a new hypothesis is assumed of the $\omega^0 - \varphi^0 - X_1^{*0}$ mixing of these vector mesons. However, the obtained formula for masses cannot be used because of the lack of experimental data. In this paper the hypothesis of anomalous quark magnetic moments is also given to explain the magnetic moments μ_P, μ_N, μ_{Λ^0} and a fairly good accordance is obtained for μ_{Σ^+}(in spite of the breaking up of the SU (4) symmetry). The drawback of this theory is that it is a static and approximate model like all the other preceding theories of this kind. The test of this theory depends largely on new experimental data.

REFERENCES

[1] Ne'eman J., Nucl. Phys. 26 (1961), 222.
[2] Gell-Mann M., Phys. Rev. 125 (1962), 1067.
[3] Zweig G., CERN Reports 8182/TH. (1964), 401, CERN Reports 8419/TH. (1964), 412.
[4] Gürsey F., Pais A., Radicati L. A., Phys. Rev. Lett. 13 (1964), 299.

[5] Tartjane P., Teplitz V. A., Phys. Rev. Lett. 11 (1963), 447.
[6] Glashow S. L., Bjørken O., Phys. Lett. 11 (1964), 255.
[7] Amati D., Bacry H., Nuyts J., Prentki J., Nuovo Cimento 34 (1964), 1732.
[8] Нгуен Ван Хьеу, *Лекции по теории симметрии элементарных частиц*. Издательство ОИЯИ, Дубна 1966.
[9] Кадышевский В. Г., Мурадян Р. М., Тавхелидзе А. Н., Тодоров И. Т., Д — 1929. Издательство ОИЯИ, Дубна 1964.
[10] Кадышевский В. Г., Мурадян Р. М., Смородинский Я. А., Р — 2061. Издательство ОИЯИ, Дубна 1965.
[11] Weiskopf V. F., SU (2) → SU(3) → SU (6). CERN 66-19, Geneva 1966.
[12] Боголюбов Н. Н., *Физика высоких энергий и теория элементарных частиц* (сборник). Издательство Наукова думка, Киев 1967.
[13] Rosenfeld A. H., Barash-Schmidt N., Barbaro-Galtieri A., Price L. R., Söding P., Wohl Ch. G., Roos M., Willis W. J., *Data on Particles and Resonant States*. CERN UCRL 8030 – Pt. 1(Rev.), Geneva 1968.
[14] Хамермеш М., *Теория групп*. Издательство МИР, Москва 1966.
[15] Behrends R. E., Dreitlein J., Fronsdal C., Lee W., Rev. Mod. Phys. 34 (1962), 1.

Received October 17th, 1968

Katedra teoretickej fyziky a geofyziky
Prírodovedeckej fakulty UPJŠ,
Košice