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THE INFLUENCE OF DISPERSION ON MEASUREMENT OF ATTENUATION IN ULTRASONIC WAVEGUIDES

IVAN Ľ. BAJÁK, Žilina

A study is made of the influence of the dispersion on the measurement of the attenuation of longitudinal ultrasonic waves in a cylindrical waveguide, the radius of which is larger than the radius of the transmitting and receiving transducer, when the pulse technique is used. Theoretically and also experimentally it is shown that the deflection of the echoes envelope from exponential shape will be smallest if the ratio of waveguide and transducer radii is near 1.4 (the radii of the transmitter and receiver were considered to be equal).

INTRODUCTION

The measurement of the absorption coefficient of logitudinal ultrasonic waves in a cylindrical waveguide needs an analysis of the influence of dispersion on observed values for its interpretation. This necessity has its origin in the fact that a mechanical signal on the entrance of the guide excites in it a definite number of modes which have various phase velocities. Owing to the difference of phase velocities of modes a complicated interference of the modes is observed and it deforms the measured values. The amplitudes of the modes are determined by boundary conditions at the radial walls of the guide and by pressure distribution across the entrance of the guide.

Redwood [1] dealt with the spreading of longitudinal ultrasonic waves in a cylindrical waveguide of an isotropic solid in which the waves were transmitted by a transducer on the face of which the amplitude of pressure was constant. Redwood assumed that the guide was an ideal liquid cylinder with free walls and chose the boundary conditions analogous to those in a solid, i.e. the pressure was zero at the radial boundary of the guide. Owing to the similarity of the boundary conditions such a type of guide gives us a good picture of the mode structure also in a solid waveguide, but in this latter case the transverse waves arise at its boundary. The loss of energy due to partial conversion of the longitudinal to transverse waves was estimated by McSkimin [2] for the first mode and Redwood [1] extended his analysis to other modes.

Measurements of attenuation in waveguides are frequently performed in samples, the radius of which is larger than the radius of the used transducers. The beam of ultrasonic waves transmitted in such a sample is continually *spreading* and it reaches the radial walls of the guide on a definite length and then will be guided. If we compute the structure of modes and the possible errors (deflection from exponential shape) for this case according to Redwood's assumption that the amplitude of pressure over the face of the transmitter is constant we should get an identity between dispersion and difraction losses for the first group of echos which beahve according to the assumption that the propagation takes place as in an unbounded medium. This is especially in the case of larger radius guides in descrepancy with the experiment.

In the present paper we shall discuss the case when the radius of a waveguide is larger than the radii of transmitting and receiving transducers. We shall limit our analysis to the case when the radius of the transmitter is much greater than the wavelength λ.

THE CYLINDRICAL WAVEGUIDE EXCITED BY THE FAR FIELD OF A PISTON SOURCE

The measurement of the absorption coefficient of longitudinal ultrasonic waves by the impulse-echo method is based on the assumption that the propagation of an ultrasonic pulse containing a large number of cylces is identical with the propagation of a single frequency continuous wave transmitted under the same conditions in an investigated semiinfinite medium. So the continuous wave theory can be used.

We assume the waveguide to be a semiinfinite cylinder of an ideal liquid with free walls. The pressure at the radial walls is zero. The cylindrical coordinate system (r, φ, z) is chosen so as to have its origin in the centre of the cylinder's base and the z axis identical with the axis of the guide. Owing to radial symmetry we can assume that the solution of the wave equation will be independent on φ. Therefore the wave equation for the propagation of waves in an ideal liquid cylinder may be written[3]:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{c_d^2}\frac{\partial^2 p}{\partial t^2} \tag{1}$$

where c_d is the velocity of longitudinal waves in an unbounded medium, $p = p(r, z, t)$ is the pressure at (r, z).

If we take into account only waves spreading in the positive direction of z, the general solution of the equation (1) with the boundary condition $p(b, z, t) = 0$ can be written in the form:

$$p(r, z, t) = \sum_{j=1} G_j \mathscr{I}_0\left(\frac{y_{0j} r}{b}\right) \exp\,[i(\omega t - k_j z)] \tag{2}$$

where b is the radius of the guide and

$$k_j = [(\omega/c_d)^2 - (y_{0j}/b)^2]^{1/2} \tag{3}$$

while y_{0j} are the roots of the Bessel function $\mathscr{I}_0(x)$, G_j are constants and ω is the angular frequency.

The wave (2) is given by the superspositión of modes

$$p_j(r, z, t) = G_j \mathscr{I}_0\left(\frac{y_{0j} r}{b}\right) \exp\,[i(\omega t - k_j z)]. \tag{4}$$

The amplitude G_j is given as

$$G_j = \frac{2}{b^2 \mathscr{I}_1^2(y_{0j})} \int_0^b p_0(r)\, \mathscr{I}_0\left(\frac{y_{0j} r}{b}\right) r\, dr \tag{5}$$

where $p_0(r)$ is the distribution of the amplitude of pressure across the entrance of the guide. The relation (5) was deduced by multiplying the equation (4) by $\mathscr{I}_0(y_{0j} r/b) \times r$ and integrating it between the limits 0 and b. Then the ortogonality of the functions $\mathscr{I}_0(y_{0j} x)\sqrt{x}$ on (0, 1) interval was used.

As we can see from (5) the amplitude of the mode depends on $p_0(r)$. We perform the determination of the function $p_0(r)$ in the case $a \gg \lambda$, $a < b$, where b is the radius of the transmitter as follows:

The pressure far from the circular piston source is given (see [4]) as

$$p(r, z, t) \sim \frac{a^2}{R} \frac{\mathscr{I}_1(ka \sin \vartheta)}{ka \sin \vartheta} \exp[i(\omega t - kR,] \tag{6}$$

where R is the distance between (r, z) and the centre of the transmitter, ϑ is the angle between R and the axis of the transmitter, $k = \omega/c_d$, $\mathscr{I}_1(x)$ is the first order Bessel function. The field (6) can be devided to some parts. The first part will be given by the field within the conical surface, the acceptance angle of which is determined by the relation (see Fig. 1)

$$ka \sin \vartheta_1 = y_{12}$$

where y_{12} is the second root of $\mathscr{I}_1(x)$. The n-th part of the field (6) is given

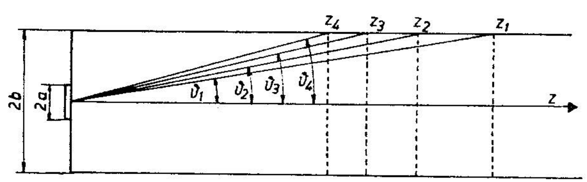

Fig. 1.

by the field between two conical surfaces. The acceptance angle of the externa-surface ϑ_n is given as

$$ka \sin \vartheta_n = y_{1,\,n+1} \tag{7}$$

while the acceptance angle of the internal surface is

$$ka \sin \vartheta_{n-1} = y_{1n} \tag{8}$$

where y_{1n} is the n-th root of $\mathscr{I}_1(x)$. It is evident that the pressure at every conical surface which was chosen in this way is zero.

We shall assume now that each part of the field (6) excites the guide separately and the resulting disturbance that will spread in the guide will be given by a superposition of modes excited by all parts of the field.

The first part of the ultrasonic field reaches the radial wall of the guide at the distance z_1 from the origin, the n-th part at the distance z_n. If $a \gg \lambda$, the first few parts of the field fulfil the relation $\sin \vartheta_n \doteq \operatorname{tg} \vartheta_n$. So we can write for these parts

$$z_n = \frac{kab}{y_{1,\,n+1}} = \frac{2\pi}{y_{1,\,n+1} s_1} \left[\frac{b^2}{\lambda}\right] \tag{9}$$

where $s_1 = b/a$.

The amplitude of pressure of the ultrasonic field in the n-th part will be:

$$p_0(r) = \begin{cases} A \dfrac{a}{k} \dfrac{|\mathscr{I}_1(y_{1,\,n1} r/b|}{r} & \text{for } r \in \left(\dfrac{y_{1n}}{y_{1,\,n+1}} b;\, b\right) \\ 0 & \text{for } r \in \left(0;\, \dfrac{y_{1n}}{y_{1,\,n+1}} b\right) \end{cases}$$

where A is a constant independent from n.

We assume now that only the n-th part exists. The modes excited by this part will have the amplitudes

$$G_{jn} = \frac{2A\, A_{jn}}{k\, s_1\, \mathscr{I}_1^2(y_{jc})} \tag{10}$$

where

$$A_{jn} = \int\limits_{y_{1n}/y_{1,\,n+1}}^{1} \mathscr{I}_0(y_{0j}x)\, |\mathscr{I}_1(y_{1,\,n+1}x)|\, \mathrm{d}x \tag{11}$$

and the disturbance $p_n(r, z, t)$, $z \geqslant z_n$ excited in the waveguide will be described by the equation

$$p_n(r, z, t = \sum_{j=1} G_{jn} \mathscr{I}_0 \left(\frac{y_{0j} r}{b}\right) \exp{(\mathrm{i}\, \Psi_{jn})} \tag{12}$$

where the phase

$$\Psi_{jn} = [\omega t - k_j(z - z_n) - kz_n + (n - 1\, \pi]$$

is given by the requirement that the modes (12) must be in phase or in phase opposition with the disturbance (6) at the distance $z = z_n$. (The phase opposition of the mode will be expressed by the minus sign.) The term $(n-1)\pi$ in Ψ- is a consequence of the change of the sign of $\mathscr{I}_1(x)$ with the change of n. As $a \gg \lambda$ and $\sin \vartheta_n \doteq \operatorname{tg} \vartheta_n$ we can put $R \doteq z_n$ and so we can neglect the change of phase within each part of the field.

The resulting disturbance that will spread in the guide in the distance $z \geqslant z_1$ is

$$\underset{z \geq z_1}{p(r, z, t)} = \sum_{j=1} [\sum_{n=1} G_{jn} \exp{\{\mathrm{i}[(k_j - k)z_n + (n-1)\pi]\}}] \exp{[\mathrm{i}(\omega t - k_j z)]} \tag{13}$$

If we compare (13) with (2) the G_j can be written

$$G_j = \sum_{n=1} G_{jn} \exp\{\mathrm{i}[(k_j - k)z_n + (n-1)\pi]\} = |G_j| \exp{(\mathrm{i}\Psi_j)}. \tag{14}$$

Let the receiving transducer be placed in the waveguide at the distance $z \geqslant z_1$. The electrical response of the receiver on the wave $p(r, z, t)$ is

$$U(z) \sim \underset{\text{(during 1 periode)}}{\text{maximum}} \int_0^{2\pi} \int_0^{C} p(r, z, t)\, r\, \mathrm{d}r\, \mathrm{d}\varphi \tag{15}$$

where C is the radius of the receiver.

From (10)--(15) we get

$$U(z) \sim |\sum_{j=1} U_j \exp{[\mathrm{i}\,(\omega t - k_j z + \Psi_j)]}| \tag{16}$$

where

$$\frac{U_j}{U_1} = \frac{y_{01} \mathscr{I}_1^2(y_{01}) \mathscr{I}_1 \left(\dfrac{y_{0j}}{s_2}\right) |\sum\limits_{n=1} A_{jn} \exp{(\mathrm{i}\, \varphi_{jn})}|}{y_{0n} \mathscr{I}_1^2(y_{0n}) \mathscr{I}_1 \left(\dfrac{y_{01}}{s_2}\right) |\sum\limits_{n=1} A_{1n} \exp{(\mathrm{i}\, \varphi_{1n})}|} \tag{17}$$

while $s_2 = b/c$ and φ_{jn} are given by the relation

$$\varphi_{jn} = (k_j - k)z_n + (n-1)\, \pi. \tag{18}$$

If $c a y_{0j}/\omega b \ll 1$, the relation (18) can be written as

$$\varphi_{jn} = -\frac{1}{s_1} \left(\frac{y_{0j}^2}{2y_{1,\,n+1}}\right) + (n-1)\, \pi. \tag{19}$$

The electrical output voltage (16) can be graphically computed as the absolute value of the sum of vectors. The phase angle between the vector representing mode 1 and the vector representing mode j at any z is given by

$$\gamma_{1j} = (k_1 - k_j)z + (\Psi_j - \Psi_1). \tag{20}$$

We can see from (16) and (17) that the envelope of echoes causes loss or gain also in an ideal liquid cylinder. If the interference of modes is constructive, a gain, but with a destructive interference of modes, a loss is found.

The integrals A_{jn} can be numerically computed and the curve

$$\underset{z \geq z_1}{\alpha(z)} = 20 \log \frac{U(z_1)}{U(z)} \text{ [dB]} \tag{21}$$

gives the correction of errors that arise as the consequence of dispersion.

COMPUTATION OF THE DISPERSION LOSSES

The computation of $|\sum_{n=1}^{5} A_{jn} \exp(\mathrm{i}\, \varphi_{jn})|/|\sum_{n=1}^{5} A_{1n} \exp(\mathrm{i}\, \varphi_{1n})|$, Ψ_j and of U_j/U_1 was performed for various values of b/a in cases when the radii of transmitting and receiving transducers are equal ($s_1 = s_2 = s$). We take only the first five

Table 1

	$\lvert\sum_{n=1}^{5} A_{jn} \exp(\mathrm{i}\varphi_{jn})\rvert / \lvert\sum_{n=1}^{5} A_{1n} \exp(\mathrm{i}\varphi_{1n})\rvert$			
s \ j	2	3	4	5
1.1	.1369	.0822	.0439	.0168
1.2	.1383	.0774	.0406	.0210
1.4	.1429	.0709	.0443	.0177
1.6	.1433	.0673	.0285	.0154
1.8	.1479	.0654	.0262	.0285
2.0	.1480	.0626	.0238	.0280
2.3	.1491	.0597	.0242	.0214
2.5	.1492	.0490	.0252	.0135

Table 2

s \ j	$\Psi_j - \Psi_1$				Ψ_1
	2	3	4	5	
1.1	— 3.308	— 1.443	— 2.721	— 3.308	— 0.721
1.2	— 1.760	— 1.144	— 2.029	— 3.870	— .664
1.4	— 1.538	— .686	— 1.281	— 2.284	— .569
1.6	— 1.341	— .339	— .204	— 2.873	— .498
1.8	— 1.211	— .031	.239	— 1.742	— .440
2.0	— 1.085	.188	.557	— .647	— .396
2.3	— .946	.527	.887	.656	— .346
2.5	— .881	.931	1.047	1.506	— .319

parts of the ultrasonic field into account as the prevalent quantity of ultrasonic energy is carried in them and the amplitude of the pressure rapidly decreases with the increasing angle ϑ. The accuracy of the computed values is about 2 %. The results of the computation are in Table 1, Table 2 and Table 3.

Table 3

	U_j/U_1	($s_1 = s_2$		
s \ j	2	3	4	5
1.1	— .082	.032	— .010	.002
1.2	— .065	.008	.064	— .006
1.4	— .011	— .030	.021	— .004
1.6	.042	— .042	.005	.006
1.8	.089	— .038	— .007	.015
2.0	.127	— .023	— .014	.007
2.3	.173	.004	— .018	— .008
2.5	.202	.014	— .017	— .009

The minus sign at the value U_j/U_1 in Table 3 means that the phase angle between the vector representing mode 1 and the vector representing mode j is $\gamma_{1j} + \pi$ (see also relation (20)).

We can see from Table 3 that the modulation of the echo envelope will be smallest if $s \sim 1.4$.

The correction curve (21) can be constructed for every size of s. The echoes that correspond to the distance $z < z_1$ may be corrected according to Seki et al. [5] while the echoes for $z \geqslant z_1$ are corrected by (21).

THE EXPERIMENTAL STUDY OF THE DISPERSION EFFECT IN AN ULTRASONIC WAVEGUIDE

As we had not a suitable sample of an isotropic solid, the experimental investigation of the dispersion effect was performed on two samples of a single crystal of Si. The sample number 1 (radius 10 mm, length 30 mm) and also the sample number 2 (radius 7 mm, length 22 mm) were optically polished (± 1 μm) and the axis of the cylinder was indentical with the ⟨1, 1, 1⟩ direction of Si. Redwood [1] has made the measurement of dispersion effects in a single crystal of Ge ⟨1, 0, 0⟩ and he showed that the results were similar to those in fused silica. Besides, the single crystal of Si is aelotropic in the ⟨1, 1, 1⟩ direction, and we can assume that the deviations caused by anisotropy will be small.

We realized two cases: $s = 2$ and $s = 1.4$. The correction curve (21) was graphically constructed for $s = 2$, while the phase angle between the vector representing mode 1 and mode j was for every z computed from relation (20) which can be for $c_d y_{0j}/\omega b \ll 1$ rewritten in the form

$$\gamma_{1j} = \frac{y_{0j}^2 - y_{01}^2}{4\pi} z' + (\Psi_j - \Psi_1)$$

where z' is the distance z in $[b^2/\lambda]$ units. The distance of neighbouring maxima or minima can be determined from the length L_{1j} on which the phase between the first and the j-th mode changes from 0 to π. We get from the relation (20):

$$L_{1j} = \frac{4\,\pi^2}{y_{0j}^2 - y_{01}^2} \left[\frac{b^2}{\lambda}\right].$$

The values L_{1j} ($j = 1, 2, 3, 4$ and 5) are in Table 4.

Table 4

j	2	3	4	5
L_{1j} $[b^2/\lambda]$	1.5991	.57129	.29630	.1818

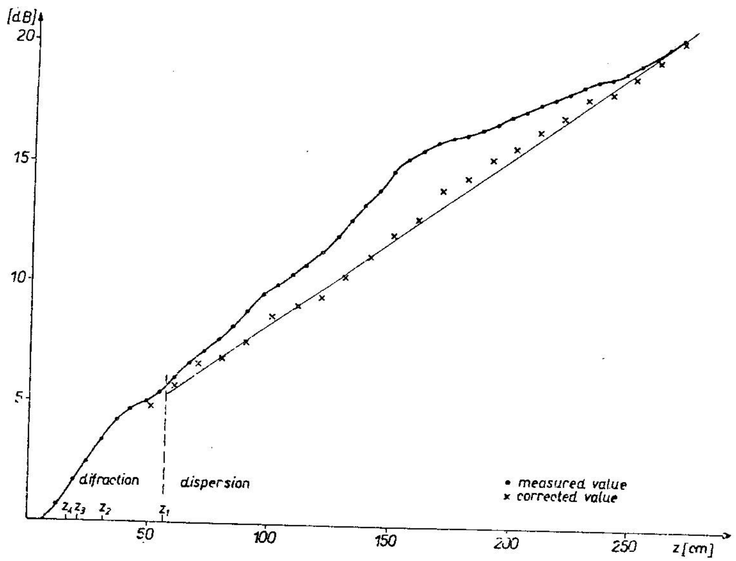

Fig. 2. The plot of attenuation against distance at 66 MHz in sample number 1 if the ratio of specimen and transducer radius equals 2. The measured values are corrected as to the influence of the dispersion for the distance $z \geq z_2$.

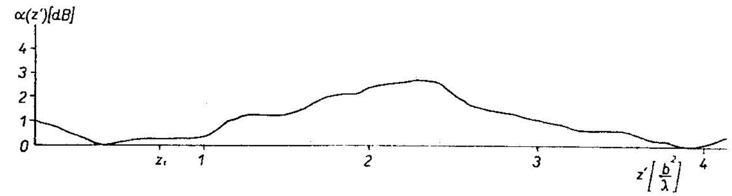

Fig. 3. The apparent dispersion loss in output voltage [dB] plotted against the distance from the transmitter z' $[b^2/\lambda]$ in the cylindrical waveguide, the radius of which is twice as large as the radii of the transmitting and receiving transducers.

With regard to the predominant role of the first two modes on the modulation of the echo envelope, the distance between the two main maxima (minima) will be about 3.198 $[b^2/\lambda]$. As the unit $[b^2/\lambda]$ is relatively big, a large number of echoes is necessary if several main maxima (minima) are to be observed. The correction (21) is important especially if there are not even two main maxima (minima) on the envelope of echoes. When there is a large number of maxima (minima) on the echo envelope we can make the correction according to Redwood [1] so that the amplitudes of echos are plotted in logarithmic scale

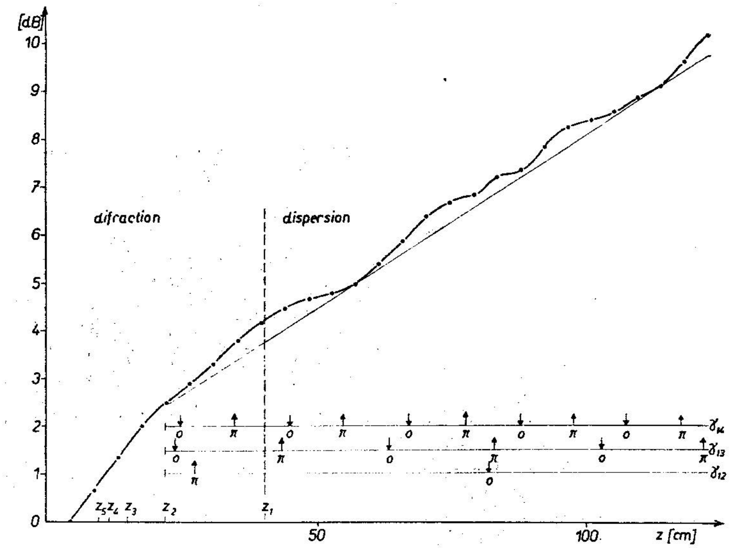

Fig. 4. The plot of attenuation against the distance from the transmitter at 66 MHz in sample number 2 if the ratio of the specimen and transducer radii equals 1.4 (one probe). The places where the 2nd, 3rd and 4th modes are in phase or in phase opposition to mode 1 are indicated in the scales by γ_{12}, γ_{13}, γ_{14}. The arrows in the scales show whether the competent mode increases or decreases the attenuation.

and the attenuation is read from the slope of the straight line linking the maxima (minima) of the envelope. This value of attenuation must be corrected also with regard to conversion losses and the coefficient of reflection.

In our case the transducers with a 22 MHz fundamental frequency were used. The radius of the transducer was 5 mm. The glycerine oil was taken for the bond transducer-sample.

The ratio $s = 2$ was realized at the sample number 1. The HF generator operated at 66 MHz and fed the transmitting transducer with 2.5–4 μs pulses. The one probe method was used. The echoes were amplified, detected, and showed on the oscilloscope as shown in Fig. 5. The comparative waveguide attenuator operated on intermediate frequency was used for the measurement of the decrease of echoes. The results of the measurement are in Fig. 2. The $b^2/\lambda = 70.4$ cm, $\lambda = 14.19 \times 10^{-3}$ cm in this case.

The echoes were divided into two groups. The first group contained the echoes that correspond to $z < z_1$ and diffraction losses are prevalent on their envelope. (This group of echos can be corrected according to Papadakis [6].) On the envelope of all others echoes the dispersion effect was observed. The correction curve (21) was plotted (see Fig. 3) and it was subtracted from the curve of echo envelope in Fig. 2 for $z \geqslant z_1$ (in Fig. 2 and also in Fig. 4 it is shown that correction (21) can be applied already for $z \geqslant z_2$). The straight line was obtained from the slope from which the attenuation can be read.

The ratio $s = 1.4$ was brought about in sample 2. The modulation of the echo envelope was less than .5 dB in this case. The results of the measurement are in Fig. 4 and the photography of echos is in Fig. 5.

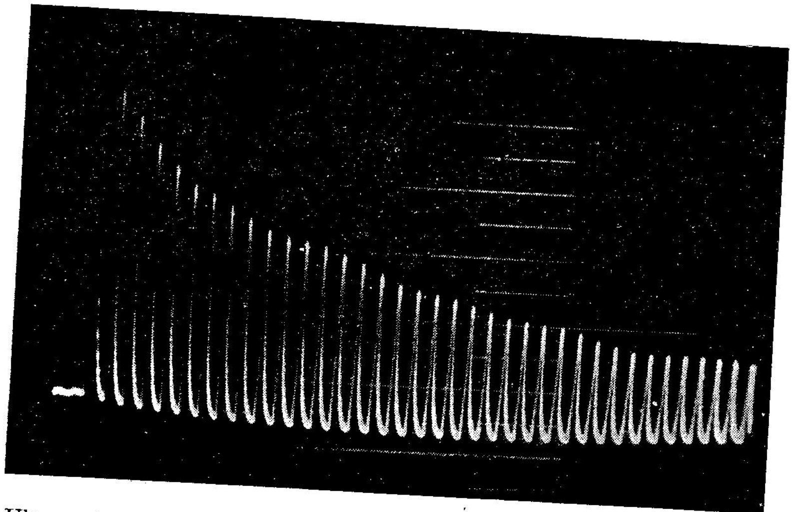

Fig. 5. Ultrasonic echo patterns at 66 MHz in sample number 2. The ratio of the sample's radius and the radius of the transmitter (one probe method) is 1.4.

As the deflection of echo envelope from the exponential shape was small we did not construct the correction curve (21) for this case and the comparison of the computation with experimental data was made by the determination of the positions of maxima and minima on the envelope of echoes. There are scales in Fig. 4 which show where the second, third and fourth mode are in phase or in phase opposition to mode 1. The arrows in the scales show whether the particular mode causes an increase or decrease of the attenuation.

We can say that in both cases the computations of the dispersion losses are in good agreement with experimentally found values.*)

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Katedra fyziky
Vysokej školy dopravnej,
Žilina

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