Fyzikálny časopis 18 (1968), No. 1

THE SU(6) BOUND-STATE MODEL OF BARYONS AND SUM RULES FOR ELECTROMAGNETIC MASS DIFFERENCES

PETER LICHARD, Bratislava

Sum rules for electromagnetic mass differences are reviewed on the basis of a simple SU(6) self-consistent model. Mass formulae of Coleman—Glashow, Weinberg—Treiman and Katsumori—Faustov are obtained together with some new ones. The devations from SU(3), SU(6) and quark model sum rules are also estimated.

I. INTRODUCTION

Electromagnetic mass differences can give us valuable information on the group or internal structure of hadrons, and they can serve as a first test of various dynamical models. In the present paper we shall calculate the electromagnetic mass differences of baryons and baryon resonances starting from a simple bound-state model within the framework of the SU(6) symmetry. In order to enable the comparison of our results with the preceding ones, we shall make a brief review of previous works.

Weinberg and Treiman [1] dealt with the electromagnetic corrections to isotopic spin conservation. They have shown that masses of particles belonging to the same isotopic multiplet have to fulfil the relation

$$M(T_3) = \alpha T_3^2 + \beta T_3 + \gamma.$$

This condition is identically satisfied in every iso-multiplet with $T \leqq 1$ but in the case of the $3/2^+$ Δ quadruplet it gives

$$\Delta^- - \Delta^{++} = 3(\Delta^0 - \Delta^+), \tag{1}$$

where as usual the symbol of a particle denotes its mass.

The well-known SU(3) formula for electromagnetic mass differences of baryons has been derived by Coleman and Glashow [2] under an assumption equivalent to that of U-spin invariance:

$$n - p + \Sigma^+ - \Sigma^- + \Xi^- - \Xi^0 = 0. \tag{2}$$

Equation (2) is in excellent agreement with experimental data.

Essentially by the same way the relations among the electromagnetic mass differences in the $3/2^+$ decuplet have been obtained by Rosen [3].

Postulating the transformation properties of the electromagnetic current under the SU(6) group, Sakita [4] and Chan and Sarker [5] have derived the sum rules involving the electromagnetic mass differences of the baryon octet and decuplet.

The effect of magnetic dipole terms with appropriate transformation properties have been included into the calculations by Kuo and Yao [6] and independently by Dolgov and others [7]*.

Within the framework of the *tadpole* model [8] electromagnetic mass splittings of baryons and pseudoscalar mesons have been calculated by Coleman and Glashow [8], Coleman and Schnitzer [9] and Socolow [10]. The predictions of this model, apart from the $K^+ - K^0$ difference, are in agreement with experimental data. Nieh [11] proposed a generalization of the *tadpole* model and obtained a reasonable value of the kaon difference, too.

Katsumori [12] calculated the baryon octet and decuplet e^2-order electromagnetic mass splittings using unified electromagnetic form factors supposed to be a generalization of the Nishijima and Gell—Mann formula. For the baryon octet he obtained the Coleman—Glashow formula and for the baryon decuplet, apart from the Weinberg—Treiman formula, two new ones. These formulae are less restrictive than the above mentioned group-theoretical relations (references [3] — [7]).

Using the quark model Ishida [13] and Rubinstein [14] derived some new relations among masses of baryons and baryon resonances, and on the other hand, have obtained also the analogous relations for pseudoscalar and vector mesons.

The general mass formulae have been derived by Faustov [15], where the Fubini—Furlan dispersion relations for the current commutator have been used. The application of these formulae to the baryon decuplet gives the same electromagnetic mass sum rules as those obtained previously by Katsumori [12].

Also the **S**-matrix perturbation method by Frautschi and Dashen [16] has been used for the study of electromagnetic mass differences [17], [18], [19]. It explains some features of this problem, e. g. the octet dominance and *equal-spacing* rule for electromagnetic splittings in the $3/2^+$ decuplet. Using this method Dashen [20] calculated the nucleon electromagnetic mass shift in excellent agreement with the experimental value. This result has been regarded as a great succes of the bootstrap philosophy, but recently detailed investigations [21], [22] have shown some inaccuracies in Dashen's original calculation. The results obtained by the Frautschi-Dashen method depend to such an extent on so far unknown features of the dynamics of hadrons that even the sign of Dashen's result is questionable.

* All group-theoretical sum rules for electromagnetic mass differences can be found in the interesting paper by Harari [31] together with some critical comments.

On the basis of a simple potential model Barton [21] has shown that the neutron—proton mass difference is negative in an SU(2) potential bound-state model and that it is probably necessary to take into account also higher mass channels if one wants to remove the above mentioned trouble.

Following this result of Barton, Barton and Dare [23] and Pišút and the author [24] calculated the baryon octet and decuplet electromagnetic mass differences by means of the simple SU(3) bound-state model in qualitative agreement with experiment. This model is of course not an original one. It has been used as an illustration in [16] and [20] and more detailed by Barton [21]. In the papers [25], [26] this method was used to calculate the magnetic moments of baryons.

In the present paper, as mentioned above, the SU(6) bound state model analogous to that in reference [24] will be used, in order to obtain sum rules for electromagnetic mass differences of baryons and $3/2^+$ baryon resonances.

In section 2 an explicit statement of the model is given. The sum rules implied by the present calculation are analyzed in section 3. Finally, in section 4 the deviations from SU(3), SU(6) and quark-model sum rules are estimated.

2. STATEMENT OF MODEL

In the spirit of the self-constistent **S**-matrix theory we shall regard the 56-plet of baryons (B) as a bound state in the (BM) system, where M denotes the meson 35-plet. We take the coupling to be given by SU(6) symmetry. The reduction coefficients for the decomposition $35 \otimes 56$ are taken from [27] and [28]. The wave functions for B_8 and B_{10} may then be written in the form

$$\Psi(B_8) = \sqrt{\frac{2}{5}}\left[\frac{1}{3}\sqrt{5}\,(P_8B_8)_s + \frac{2}{3}(P_8B_8)_a\right] + \sqrt{\frac{2}{15}}\,(V_8B_8)_a + \\ + \frac{2}{3}(P_8B_{10})_8 + \frac{1}{\sqrt{45}}(V_1B_8), \qquad (3)$$

$$\Psi(B_{10}) = \sqrt{\frac{8}{45}}\,(P_8B_8)_{10} + \frac{2}{3}(P_8B_{10})_{10} + \frac{2}{\sqrt{15}}(V_8B_{10})_{10} + \frac{1}{3}(V_1B_{10}),$$

where the subscripts s and a denote the symmetric and antisymetric octet states, respectively. All states are normalized to unity.

The bound states on the right hand side of equations (3) may be rewritten by means of SU(3) Clebsch—Gordan coefficients tabulated e. g. in reference [30]. We obtain, for example:

$$\Psi(Y^+) = \sqrt{\frac{8}{45}}\left[\frac{1}{2}(\pi^+\Lambda - \eta\Sigma^+) + \frac{1}{\sqrt{6}}(\bar{K}^0 p - K^+\Xi^0) + \right.$$
$$\left. + \frac{1}{2\sqrt{3}}(\pi^+\Sigma^0 - \pi^0\Sigma^+)\right] + \frac{2}{3}\left[\frac{1}{\sqrt{6}}(\pi^+Y^0 - \pi^0 Y^+) + \frac{1}{\sqrt{3}}K^+\Xi^{*0} - \right.$$
$$\left. - \frac{1}{2}K^-\Delta^{++} + \frac{1}{\sqrt{12}}\bar{K}^0\Delta^+\right] + \frac{2}{\sqrt{15}}\left[\frac{1}{\sqrt{6}}(\varrho^+Y^0 - \varrho^0 Y^+) + \right.$$
$$\left. + \frac{1}{\sqrt{3}}K^{*+}\Xi^{*0} - \frac{1}{2}K^{*-}\Delta^{++} + \frac{1}{\sqrt{12}}\bar{K}^{*0}\Delta^+\right] + \frac{1}{3}\omega Y^+, \qquad (4)$$

where a symbol of the type $\pi^+\Lambda$ denotes the relevant bound state.

Computing the mean values of the Hamiltonian* in the states of the type (4) and calculating the mass differences, we obtain (isotopic spin invariance of medium strong interaction is assumed):

$$-140N + 4\Sigma_1 + 8\Delta_1 + 24\Delta_2 + 8Y_1 =$$
$$= 9K + 10T_1 + 3K^* + 6T_2 - 26W_1 + 8W_2 - 6W_3 \qquad (5)$$
$$N - 28\Sigma_1 + 7\Xi + 4T + 2\Delta_1 + 6\Delta_2 + 4Y_1 + 2\Xi^* =$$
$$= 12K + 8T_1 + 12W_2$$
$$28\Sigma_1 - 116\Xi + 16T + 8Y_1 + 8\Xi^* =$$
$$= 39K + 22T_1 - 3K^* - 6T_2 + 26W_1 + 40W_2 + 6W_3$$
$$4\Sigma_1 + 4\Xi - 44T - 4Y_1 - 4\Xi^* =$$
$$3K + 4T_1 - 3K^* - W_1 + 4W_2 - 3W_3$$
$$4N + 4\Sigma_1 - 136\Delta_1 + 24\Delta_2 + 8Y_1 =$$
$$9K + 10T_1 + 3K^* + 6T_2 - 8W_4 - 10W_5 - 6W_6$$
$$4N + 4\Sigma_1 + 8\Delta_1 - 24\Delta_2 + 8Y_1 =$$

* The Hamiltonian is defined in the same way as in [24].

$$9K + 10T_1 + 3K^* + 6T_2 - 8W_4 - 10W_5 - 6W_6$$
$$2N + 4\Sigma_1 + 2\Xi - 4T + 4\Delta_1 + 12\Delta_2 - 52Y_1 + 16\Xi^* =$$
$$= 15K + 4T_1 + 9K^* + 15W_5 + 9W_6$$
$$4\Sigma_1 + 4\Xi - 8T + 32Y_1 - 112\Xi^* =$$
$$= 21K - 2T_1 + 15K^* - 6T_2 + 8W_4 + 40W_5 + 24W_6$$
$$-70\Sigma_2 + 24\Delta_3 + 4Y_2 =$$
$$6\pi + 6\varrho + 29W_2 + 4W_2 + 15W_3$$
$$2\Sigma_2 - 48\Delta_3 + 4Y_2 =$$
$$6\pi + 6\varrho + 8W_4 + 25W_5 + 15W_6$$
$$2\Sigma_2 + 24\Delta_3 - 68Y_2 =$$
$$6\pi + 6\varrho + 8W_4 + 25W_5 + 15W_6,$$

where

$N = n - p$ | $Y_1 = Y^- - Y^+$
$\Sigma_1 = \Sigma^- - \Sigma^+$ | $Y_2 = 2(Y^- - Y^0) - (Y^- - Y^+)$
$\Sigma_2 = 2(\Sigma^- - \Sigma^0) - (\Sigma^- - \Sigma^+)$ | $\Xi^* = \Xi^{*-} - \Xi^{*0}$
$\Xi = \Xi^- - \Xi^0$ | $\pi = \pi^+ - \pi^0$
$T = \sqrt{3}(\Lambda\Sigma)$ | $K = K^+ - K^0$
$\Delta_1 = \Delta_0 - \Delta^+$ | $T_1 = \sqrt{3}(\eta\pi)$
$\Delta_2 = \Delta^- - \Delta^{++}$ | $\varrho = \varrho^+ - \varrho^0$
$\Delta_3 = 2(\Delta^0 - \Delta^+) - (\Delta^0 - \Delta^{++})$ | $K^* = K^{*+} - K^{*0}$

$$T_2 = \sqrt{3}(\Phi_0\varrho)$$

and W_1, W_2, W_3, W_4, W_5, W_6 are the mean values of the Coulomb energy operator between one negatively and one positively charged particle in the $(P_8B_8)_{a,s}$, $(P_8B_{10})_8$, $(V_8B_8)_{10}$, $(P_8B_{10})_{10}$, $(V_8B_{10})_{10}$ states, respectively. The term $(\Lambda\Sigma)$ denotes the second order electromagnetic $\Sigma^0 \to \Lambda$ transition mass. The meaning of symbols $(\eta\pi)$ and $(\Phi_0\varrho)$ is analogous. We denote the $T = Y = 0$ member of the vector meson octet as Φ_0.

As a first approximation we supposed that the quantities W for all pairs of particles belonging to the same SU(3) state are equal. The reason for such an approximation is the following. After a simple calculation it can be shown that the electromagnetic mass differences of baryons as N, Σ_1, ..., Δ_1 etc. are determined mainly by the electromagnetic mass differences of pseudoscalar and vector mesons π, K, ϱ, K^*, and the influence of inaccuracies of the Coulomb interaction energies is relatively small. Futher, on the basis of Barton's result (reference [21]) we neglect the magnetic interaction terms.

The self-consistency conditions (5) represent the set of linear algebraic equations for computing unknown quantities N, $\Sigma_1, \ldots$ etc. The quantities π, K, ϱ, K^* and W occur in these equations as the *driving* terms. The solution of equations (5) is the following:

$$N = \frac{1}{756}(-243K - 198T_1 - 81K^* - 90T_2 + 143W_1 - 92W_2 + 33W_3 + 76W_4 + 35W_5 + 21W_6)$$

$$\Sigma_1 = -\frac{1}{21}(21K + 14T_1 + 3K^* + 2T_2 + W_1 + 14W_2 - 3W_4)$$

$$\Sigma_2 = -\frac{1}{756}(108\pi + 108\varrho + 319W_1 + 44W_2 + 165W_3 + 56W_4 + 175W_5 + 105W_6)$$

$$\Xi = -\frac{1}{756}(513K + 306T_1 + 27K^* - 18T_2 + 179W_1 + 412W_2 + 33W_3 - 32W_4 + 35W_5 + 21W_6)$$

$$T = \frac{1}{504}(-54K - 72T_1 + 54K^* - W_1 - 92W_2 + 33W_3 + 4W_4 + 35W_5 + 21W_6)$$

$$\Delta_1 = \frac{1}{1512}(-486K - 396T_1 - 162K^* - 180T_2 + 13W_1 - 100W_2 + 3W_3 + 236W_4 + 175W_5 + 105W_6)$$

$$\Delta_2 = 3\Delta_1$$

$$\Delta_3 = -\frac{1}{1512}(216\pi + 216\varrho + 92W_1 + 4W_2 + 15W_3 + 280W_4 + 875W_5 + 525W_6)$$

$$Y_1 = -\frac{1}{504}(396K + 192T_1 + 180K^* + 48T_2 + W_1 + 68W_2 + 3W_3 - 64W_4 + 175W_5 + 105W_6)$$

$$Y_2 = \Delta_3$$

$$\Xi^* = -\frac{1}{756}(351K + 90T_1 + 189K^* - 18T_2 + 8W_1 + 52W_2 + 6W_3 + 22W_4 + 350W_5 + 210W_6). \quad (6)$$

The experimental data about the electromagnetic mass differences of vector mesons are at present rather poor. It is also difficult to determine the accurate values of quantities W. Therefore it is useful to derive from equations (5) formulae which do not contain the *driving* terms. This question will be dealt with in the next section.

3. SUM RULES

Eliminating the *driving* terms from equations (5) we obtain the following formulae for the electromagnetic mass splittings:

$$n - p + \Sigma^+ - \Sigma^- + \Xi^- - \Xi^0 = 0 \quad (7a)$$

$$3(\Delta^0 - \Delta^+) = \Delta^- - \Delta^{++} \quad (7b)$$

$$\Delta^0 - \Delta^+ + Y^+ - Y^- + \Xi^{*-} - \Xi^{*0} \quad (7c)$$

$$Y^+ + Y^- - 2Y^0 = \Delta^+ + \Delta^- - 2\Delta^0 \quad (7d)$$

$$(\Sigma^- - \Sigma^0) + \sqrt{3}(\Lambda\Sigma) - (\Xi^- - \Xi^0) = (Y^- - Y^0) - (\Xi^{*-} - \Xi^{*0}) = \frac{1}{14}[(\pi^+ - \pi^0) - (K^+ - K^0) + \sqrt{3}(\eta\pi) + (\varrho^+ - \varrho^0) - (K^{*+} - K^{*0}) + \sqrt{3}(\Phi_0\varrho)]. \quad (7e)$$

The relations (7a) and (7b) are the well-known Coleman-Glashow and Weinberg-Treiman formulae. The formulae (7c) and (7d) are the same as those obtained previously by Katsumori [12] and Faustov [15]. These formulae follow from the SU(3) symmetry scheme and quark model relations, but in addition these models give also more restrictive rules which cannot be obtained from the present model as exact sum rules. In the next section equations analogous to those more restrictive formulae will be derived. Contrary to SU(3) and quark model mass formulae these equations contain also *driving* terms. In this manner we can show that these formulae are broken and we shall also indicate the dynamical reasons why they are broken.

In addition to the mass formula (7e) the SU(6) Fermi-Yang model of pseudoscalar and vector mesons [32] gives the following relations:

$$[(\Sigma^- - \Sigma^0) - (\Xi^- - \Xi^0) + \sqrt{3}(\Lambda\Sigma)] + 14[(Y^- - Y^0) - (\Xi^{*-} -$$

$$-\Xi^{*0})] + 54\,[(\varrho^+ - \varrho^0 - (K^{*+} - K^{*0}) + \sqrt{3}\,(\Phi_0\varrho)] = 0 \qquad (7f)$$

$$3[(\varrho^+ - \varrho^0) - (K^{*+} - K^{*0}) + \sqrt{3}\,(\Phi_0\varrho)] = (\pi^+ - \pi^0) - (K^+ - K^0) + \sqrt{3}\,(\eta\pi).$$

The relations (7e) and (7f) are only satisfied simultaneously if

$$\Xi^{*-} - \Xi^{*0} = Y^- - Y^0 \qquad (8a)$$

$$\sqrt{3}\,(\Lambda\Sigma) = (\Xi^- - \Xi^0) - (\Sigma^- - \Sigma^0),$$
$$\sqrt{3}\,(\eta\pi) = (K^+ - K^0) - (\pi^+ - \pi^0), \qquad (8b)$$
$$\sqrt{3}\,(\Phi_0\varrho) = (K^{*+} - K^{*0}) - (\varrho^+ - \varrho^0).$$

The relations (8a) is a unitary symmetry rule ([6], [7]) and the relations (8b) are SU(3) formulae for second order transition masses [33].

From (7) and (8a) we get:

$$\Delta^- - \Delta^0 = Y^- - Y^0$$
$$\Delta^0 - \Delta^+ = Y^0 - Y^+$$
$$\Delta^- - \Delta^0 = \Xi^{*-} - \Xi^{*0}.$$

These sum rules have been obtained in the unitary symmetry scheme ([6], [7]) where the transformation properties of charge and magnetic moment operators were postulated in a particular way.

Now, let us return to the discussion of equations (7). They have been obtained after the elimination of *driving* terms, without making any assumptions as regards the magnitude of *driving* terms occuring in various states. If we make some additional assumptions about the *driving* terms, we can obtain further sum rules. It has to be stressed that assumptions of this kind are dynamical and they can be verified by more detailed calculations.

If we, for instance, suppose that

$$W_0 = W_1 = W_2 = W_3 \quad \text{and} \quad W_D = W_4 = W_5 = W_6,$$

we obtain

$$4(n - p) + 2(\Sigma^- - \Sigma^0) - (\Sigma^- - \Sigma^+) = 6\,(\Delta^0 - \Delta^+) - (\Delta^0 - \Delta^{++}). \qquad (9)$$

Using the following experimental values [29]:

$$n - p = 1\,.\,29 \text{ MeV}$$
$$2\,(\Sigma^- - \Sigma^0) - (\Sigma^- - \Sigma^+) = 1\,.\,79 \pm .\,16) \text{ MeV}$$
$$\Delta^0 - \Delta^{++} = (.\,45 \pm .\,85) \text{ MeV},$$

we obtain from this new sum rule

$$\Delta^0 - \Delta^+ = (1\,.\,23 \pm .\,14) \text{ MeV}.$$

Any comparison with experiment is impossible at present, because the $\Delta^0 - \Delta^+$ difference was not measured yet.

If we further assume that also $W_0 = W_D$, we obtain

$$\Delta^0 - \Delta^+ = n - p \qquad (10a)$$

$$2\,(\Sigma^- - \Sigma^0) - (\Sigma^- - \Sigma^+) = 2\,(Y^- - Y^0) - (Y^- - Y^+) \qquad (10b)$$

$$(\Xi^- - \Xi^0) - (\Sigma^- - \Sigma^0) = \frac{1}{7}\,[(\pi^+ - \pi^0) - (K^+ - K^0)] + \frac{3}{28}\,[K^{*+} - K^{*0}) -$$
$$- (K^+ - K^0)]. \qquad (10c)$$

The sum rules (10a) and (10b) can be obtained also by pure group theoretical methods under specific assumptions about the transformation properties of the charge operator in the SU(6) group (see references [7] and [34]).

By the same method it is possible to obtain formulae relating the electromagnetic mass differences of pseudoscalar and vector mesons. We suppose that the pseudoscalar mesons (P_8) are composed of (P_8V_8) states and that the vector mesons (V_8) are composed of (P_8P_8) and (V_8V_8) states. The other possible two mesons channels are forbidden by G-parity. The baryon-antibaryon states which could be also important are completely neglected. We hope to return to this rather delicate question later [32]. We shall write therefore

$$\Psi\,(P_8) = (P_8V_8),$$
$$\Psi\,(V_8) = \sin\vartheta\,(P_8P_8) + \cos\vartheta\,(V_8V_8). \qquad (11)$$

If we suppose that all mean values of the Coulomb energy are equal, we obtain the following simple relations (independent from the mixing angle ϑ):

$$\pi^+ - \pi^0 = \varrho^+ - \varrho^0 \qquad (12a)$$
$$K^+ - K^0 = K^{*+} - K^{*0}. \qquad (12b)$$

These relations are the same as those implied by the quark model [14]. It is probable that the rules (12a) and (12b) are badly broken, because in this case there are no other *driving* terms than Coulomb energies and their inequality may have a great effect.

However, the relation (12a) does not contradict the present experimental data ($\pi^+ - \pi^0 = 4\,.\,61$; $\varrho^+ - \varrho^0 = 2 \pm 3$ MeV; reference [29]).

On the basis of equation (12b) we can rewrite the formula (10c) in the following form:

$$7\,[(\Xi^- - \Xi^0) - (\Sigma^- - \Sigma^0)] = (\pi^+ - \pi^0) - (K^+ - K^0). \qquad (13)$$

If we substitute the experimental values

$$\pi^+ - \pi^0 = 4.61 \text{ MeV}$$

$$K^+ - K^\circ = -(4.06 \pm .12) \text{ MeV}$$

into (13), we immediatly obtain the following estimate:

$$(\Xi^- - \Xi^0) - (\Sigma^- - \Sigma^0) = 1.25 \text{ MeV},$$

whereas the experiment gives $(\Xi^- - \Xi^0) - (\Sigma^- - \Sigma^0) = (1.6 \pm .2)$ MeV.

4. VIOLATIONS OF UNITARY SYMMETRY AND QUARK MODEL SUM RULES

Kuo and Yao [6] and independently Dolgov et al. [7] have considered second order effects of charge and magnetic moment operators with appropriate transformation properties. They obtained apart form the Coleman-Glashow and Weinberg-Treiman relations the following sum rules:

$$\Delta^0 - \Delta^+ = n - p \qquad (14a)$$

$$n - p + \Sigma^+ + \Sigma^- - 2\Sigma^{-0} = \Delta^- - \Delta^0 \qquad (14b)$$

$$\Delta^- - \Delta^0 = Y^- - Y^0 = \Xi^{*-} - \Xi^{*0} \qquad (14c)$$

$$\Delta^0 - \Delta^+ = Y^0 - Y^+. \qquad (14d)$$

Within the framework of our model, the mass formulae (14a) and (14b) are satisfied only approximately. Solving the set of equations (5) and manipulating the results in an obvious way, we obtain the sum rules (12a) and (12b) together with corrections of them.

$$(\Delta^0 - \Delta^+) - (n - p) = \frac{1}{6}(W_D - W_0) \qquad (15a)$$

$$(n - p) + (\Sigma^+ + \Sigma^- - 2\Sigma^0) - (\Delta^- - \Delta^0) = \frac{1}{2}(W_D - W_0). \qquad (15b)$$

Combining the rules (15a) and (9) and using the experimental values listed above we obtain the estimate

$$W_D - W_0 = -(.36 \pm .85) \text{ MeV}.$$

If, within the framework of the SU(6) group, the charge operator is only taken into account and its transformation properties are postulated in a particular way, the following sum rule results [4], [5]:

$$\Xi^- - \Xi^0 = \Sigma^- - \Sigma^0. \qquad (16a)$$

Our model gives:

$$(\Xi^- - \Xi^0) - (\Sigma^- - \Sigma^0) = \frac{1}{7}[(\pi^+ - \pi^0) - K^+ - K^0)] +$$

$$+ \frac{3}{28}[(K^{*+} - K^{*0}) - (K^+ - K^0)] + \frac{5}{42}(W_D - W_0). \qquad (16b)$$

It can be seen that the relation (16a) is broken above all because of the fact that $\pi^+ - \pi^0$ and $K^+ - K^\circ$ have approximately the same magnitude but opposite signs.

In an analogous way it is possible to look at the quark model [14], which gives some new sum rules

$$\Sigma^- - \Sigma^+ = Y^- - Y^+, \qquad (17a)$$

$$\Xi^- - \Xi^0 = \Xi^{*-} - \Xi^{*0},$$

$$\Sigma^0 - \Sigma^+ = Y^0 - Y^+,$$

$$n - p + \Sigma^+ + \Sigma^- - 2\Sigma^0 = \Xi^{*-} - \Xi^{*0},$$

$$\Delta^0 - \Delta^{++} + \Sigma^+ + \Sigma^- - 2\Sigma^0 = 2\,(n - p).$$

Inserting into the left hand sides of (17) solution (6) of equations (5) we obtain:

$$(\Sigma^- - \Sigma^+) - (Y^- - Y^+) = \frac{2}{7}[(\pi^+ - \pi^0) - (K^+ - K^0)] +$$

$$+ \frac{3}{14}[(K^{*+} - K^{*0}) - (K^+ - K^0)] + \frac{4}{7}(W_D - W_0), \qquad (17b)$$

$$(\Xi^- - \Xi^0) - (\Xi^{*-} - \Xi^{*0}) = \frac{2}{7}[(\pi^+ - \pi^0) - (K^+ - K^0)] +$$

$$+ \frac{3}{14}[(K^{*+} - K^{*0}) - (K^+ - K^0)] + \frac{31}{42}(W_D - W_0),$$

$$(\Sigma^0 - \Sigma^+) - (Y^0 - Y^+) = \frac{1}{7}[(\pi^+ - \pi^0) - (K^+ - K^0)] +$$

$$+\frac{3}{28}[(K^{*+}-K^{*0})-(K^{+}-K^{0})]-\frac{1}{21}(W_D-W_0),$$

$$(n-p)+(\Sigma^{+}+\Sigma^{-}-2\Sigma^{0})-(\Xi^{*-}-\Xi^{*0})=\frac{1}{2}(W_D-W_0),$$

$$(\Delta^{0}-\Delta^{++})+(\Sigma^{+}+\Sigma^{-}-2\Sigma^{0})-2(n-p)=W_D-W_0.$$

The comparison of our results with those following from the quark model can be easily seen from Table 1.

Table 1

Mass difference	Quark model	Present model	Experiment
$\Delta^0 - \Delta^+$	1.3 MeV	1.3 MeV	
$Y^0 - Y^+$	3.1 MeV	3.1 MeV	
$\Xi^{*-} - \Xi^{*0}$	6.5 MeV	3.7 MeV	(4.2 ± 1.7) MeV
$Y^- - Y^0$	4.9 MeV	3.7 MeV	
$Y^- - Y^+$	7.9 MeV	5.0 MeV	(2.6 ± 1.1) MeV

5. SUMMARY OF RESULTS AND CONCLUSIONS

We shall first list the results following from our simple model. All sum rules obtained here can be classified into four groups:

(i) Sum rules which have been obtained in almost all models produced so far:

$$n-p+\Sigma^{+}-\Sigma^{-}+\Xi^{-}-\Xi^{0}=0,$$

$$\Delta^{-}-\Delta^{++}=3(\Delta^{0}-\Delta^{+}),$$

$$\Xi^{*-}-\Xi^{*0}=Y^{-}-Y^{0},$$

$$\sqrt{3}(\Lambda\Sigma)=(\Xi^{-}-\Xi^{0})-(\Sigma^{-}-\Sigma^{0}).$$

The first is the well-known Coleman-Glashow formula, which is in excellent agreement with experimental data.

(ii) Sum rules which are less restrictive than those obtained by group methods and which are identical with the sum rules derived by Katsumori [12] and Faustov [13]:

$$\Delta^{+}-\Delta^{-}-2\Delta^{0}=Y^{+}+Y^{-}-2Y^{0},$$

$$\Delta^{0}-\Delta^{+}+Y^{+}-Y^{-}+\Xi^{*-}-\Xi^{*0}=0.$$

(iii) Some new sum rules:

$$4(n-p)+2(\Sigma^{-}-\Sigma^{0})-(\Sigma^{-}-\Sigma^{+})=6(\Delta^{0}-\Delta^{+})-(\Delta^{0}-\Delta^{++}),$$

$$7[(\Xi^{-}-\Xi^{0})-(\Sigma^{-}-\Sigma^{0})]=(\pi^{+}-\pi^{0})-(K^{+}-K^{0}).$$

(iv) Other sum rules, derived by group or quark model methods appear to be broken in our model. These sum rules can be divided into two parts. The first part includes sum rules which seem to be only slighty broken:

$$(\Delta^{0}-\Delta^{+})-(n-p)=\frac{1}{6}(W_D-W_0),$$

$$(\Sigma^{+}+\Sigma^{-}-2\Sigma^{0})-(Y^{+}+Y^{-}-2Y^{0})=\frac{2}{3}(W_D-W_0),$$

$$(n-p)+(\Sigma^{+}+\Sigma^{-}-2\Sigma^{0})-(\Delta^{-}-\Delta^{0})=\frac{1}{2}(W_D-W_0),$$

$$(\Delta^{0}-\Delta^{++})+(\Sigma^{+}+\Sigma^{-}-2\Sigma^{0})-2(n-p)=W_D-W_0.$$

The second part includes sum rules which are badly broken. We list them here also with corrections estimated above:

$$(\Xi^{-}-\Xi^{0})-(\Sigma^{-}-\Sigma^{0})=1.25\text{ MeV}$$

(experimental value: $(1.6 \pm .2)$ MeV),

$$(\Sigma^{-}-\Sigma^{+})-(Y^{-}-Y^{+})=2.5\text{ MeV},$$

$$(\Sigma^{0}-\Sigma^{+})-(Y^{0}-Y^{+})=1.2\text{ MeV},$$

$$(\Xi^{-}-\Xi^{0})-(\Xi^{*-}-\Xi^{*0})=2.5\text{ MeV}.$$

Electromagnetic mass differences can give us valuable information on the internal structure of fundamental particles. In this way they can also help us to decide between various models. Unfortunately, the present-day experimental situation does not permit any definite conclusions regarding this type. Only the difference $(\Xi^{-}-\Xi^{0})-(\Sigma^{-}-\Sigma^{0})$ has been measured with sufficient accuracy. According to some results from group theory it should be equal to zero. Our estimate of this difference is in reasonable agreement with the experimental value.

This agreement (although it may of course be accidental) allows us to believe that the basic assumptions of our model are essentially correct. This question can of course only be solved when also other masses have been measured experimentally with greater accuracy, thus enabling more complete comparison with the data and this way enabling us to decide between various models.

Although we have obtained the correct sign and the order of magnitude of $(\Xi^- - \Xi^0) - (\Sigma^- - \Sigma^0)$ mass difference, our result is not within the limits put by present-day experimental data. This discrepancy is probably due to the fact that the present model does not take into account the electromagnetic splittings of masses and coupling constants of exchanged particles (or putting it in another way, the electromagnetic corrections to the forces responsible for the binding of particles in a bound state). As has been recently shown in an interesting paper by J. Harte on the Bethe-Salpeter equation model of the nucleon as a bound state, these effects may well be important.

The author is indebted to Professor H. J. Lipkin from the Weizmann Institute of Science for his critical comments on the previous version of this paper and to J. Pišút for valuable discussions.

REFERENCES

[1] Weinberg S., Treiman S. B., Phys. Rev. *116* (1959), 465.
[2] Coleman S., Glashow S. L., Phys. Rev. Lett. *6* (1961), 423.
[3] Rosen S. P., Phys. Rev. Lett. *11* (1963), 100.
[4] Sakita B., Phys. Rev. Lett. *13* (1964), 643.
[5] Chan C. H., Sarker A. Q., Phys. Rev. Lett. *13* (1964), 731.
[6] Kuo T. K., Yao T., Phys. Rev. Lett. *14* (1965), 79.
[7] Dolgov A. D., Okun L. B., Pomeranchuk I. Ya., Solovjev V. V., Phys. Lett. *15* (1965), 84.
[8] Coleman S., Glashow S. L., Phys. Rev. *132* (1964), B 671.
[9] Coleman S., Schnitzer H. J., Phys. Rev. *136* (1964), B 223.
[10] Socolow R. H., Phys. Rev. *137* (1965), B 1221.
[11] Nieh H. T., Phys. Rev. *146* (1966), 1020.
[12] Katsumori H., Prog. Theor. Phys. *33* (1965), 757.
[13] Ishida S., Prog. Theor. Phys. *34* (1965), 64.
[14] Rubinstein H. R., Phys. Rev. Lett. *17* (1966), 41.
[15] Faustov R., Nuovo Cim. *45 A* (1966), 145.
[16] Dashen R., Frautschi S. C., Phys. Rev. *135* (1964), B 1190.
[17] Dashen R., Frautschi S. C., Phys. Rev. Lett. *13* (1964), 497.
[18] Dashen R., Frautschi S. C., Phys. Rev. *137* (1965), B 1331.
[19] Biswas S. N., Bose S. K., Paude S. K., Phys. Rev. *138* (1965), B 163.
[20] Dashen R., Phys. Rev. *135* (1964), B 1196.
[21] Barton G., Phys. Rev. *146* (1966), 1149.
[22] Shaw G. L., Wong D., Phys. Rev. *147* (1966), 1028.
[23] Barton G., Dare D., Phys. Rev. *150* (1966), 1220.
[24] Lichard P., Pišút J., Nuclear Physics *B 1* (1967), 511.
[25] Petráš M., Nuclear Physics *61* (1965), 526.
[26] Lichard P., Pišút J., Czech. J. Phys. *B 16* (1966), 267.
[27] Carter J. C., Coyne J. J., Meshkov S., Phys. Rev. Lett. *14* (1965), 523.
[28] Cook C. L., Murtaza G., Nuovo Cim. *39* (1965), 531.
[29] Rosenfeld A. H. et al., Berkeley preprint 1967/UCRL — 8030.

[30] Swart J. de, Revs. Mod. Phys. *35* (1963), 916.
[31] Harari H., Phys. Rev. *139* (1965), B 132 3.
[32] Lichard P., Pišút J., Nuclear Physics *B 2* (1967), 8.
[33] Okubo S., Sakita B., Phys. Rev. Lett. *11* (1963), 50.
[34] Glashow S. L., Socolow R., Lecture presented at the Seminar on high-energy physics, Trieste 1965.
[35] Harte J., Berkeley preprint 1967/UCRL — 17329.

Received April 21st, 1967

Katedra fyziky
Strojníckej fakulty SVŠT,
Bratislava