# SOME APPLICATIONS OF THE THEORY OF THE GAS LASER

RUDOLF HAJOSSY, Bratislava

The confocal resonator of the gas laser (Fig. 1) contains both the active lasering medium characterized by polarizability $X$, length $l_L$ and the investigated plasma with properties described by the specific conductivity $G$, length $l_s = L - l_L$, where $L$ is the distance between the square spherical mirrors with the radius of the curvature $L$ and the side of the square $a$. Besides the mirrors are characterized by the complex reflectivity $\varrho_1$, $\varrho_2$ ($\varrho_1$, $\varrho_2$ correspond to the electric field intensity amplitude).

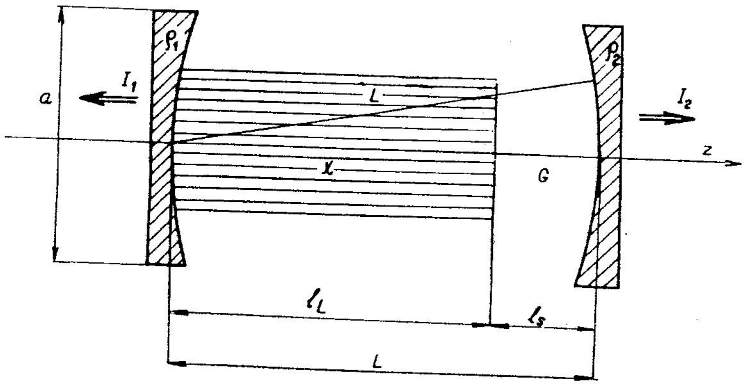


Fig. 1.

In such a type of laser the resonant condition for the mode of the frequency $\omega_{mnq}$, as it follows from theory [1] can be expressed in the form:

$$1 = \varrho_1 \varrho_2 C_{mn}^2, \tag{1}$$

where the diffraction constant $C_{mn}$ is determined by the relations

$$C_{mn}^2 = (1 - \alpha_m 10^{-\beta_m s} - \alpha_n 10^{-\beta_n s}) \exp 2i \left[ \bar{k}L - (m + n + 1)\frac{\pi}{2} \right], \tag{2}$$

$$s = \frac{a^2 \bar{k}}{2\pi L}, \quad m, n, q = 0, 1, \ldots$$



Here $\alpha$, $\beta$ are real constants (e. g., $\alpha_0 \sim 10 . 9$ and $\beta_0 \sim 4 . 94$). Furthermore, $\bar{k}$ denotes the average wave number expressed by:

$$\bar{k} \approx \frac{\omega}{c} \left[ 1 + \frac{1}{\varepsilon_0 L} \left( X_r l_L - G_i \frac{ls}{\omega} \right) + \frac{i}{2\varepsilon_0 L} \left( X_i l_L + G_r \frac{ls}{\omega} \right) \right], \tag{3}$$

both the real component of the polarizability and the imaginary one $X_r$, $X_i$, respectively, can be obtained from the equations:

$$X = -i\eta\, [1 - \Phi(\xi)] \exp \xi^2, \quad \xi = \frac{\gamma_{ab} + ib}{2\sqrt{\beta}}, \quad \beta = \frac{\Omega^2 T}{2c^2 M}, \tag{4}$$

$$b = \Omega - \omega, \quad \eta = \frac{\gamma^2}{\hbar} N \sqrt{\pi/\beta},$$

here $N$ is the population inversion density (that of the steady-state without the simultaneous ligth generation of the laser), the electric dipolemomentum matrix element of the polarized atom is denoted as $\gamma$, the parameter $\gamma_{ab}$ is $\pi$ times the natural linewidth of the lasering transition, $\Omega$ is the peak frequency of the atomic resonance curve, $\omega$ denotes the cavity frequency, $T$ is the temperature of the polarized atoms of mass $M$ and $\Phi$ ($\xi$) is the error integral. (To avoid the complications in the denotations it is convenient to use the energetic units for temperature $T$.)

Relations (4) are, in essential, the first Lamb's approximation of polarizability $X$ given in analytical form. In [1] it is shown that equations (4) are correct only under the conditions

$$1 \gg 4\pi T/ML\lambda\gamma_{ab}^2, \qquad b \gg (2\pi/\lambda\gamma_{ab})^3 (T/M)^2 L^{-1}.$$

Due to the above fact the result for $b \lesssim 10^8$Hz has only order level precision.

Components of conductivity $G$ can be obtained from the relations:

$$G^{\mp} \approx i \frac{\omega_p \varepsilon_0}{2\omega} \left( 1 \pm \frac{\omega_e}{\omega} - i \frac{\nu}{\omega} \right), \quad \omega_p^2 = \frac{n\, e^2}{m \varepsilon_0}, \quad \omega_e = \frac{e\, \mu_0 H}{m}. \tag{5}$$

Here $H$ is the intensity of the constant magnetic field, which is oriented along the optical axis of the resonator. (We assume that the magnetic field is only within the investigated plasma range.) $\nu$ is the electron collision frequency, $m$, e, $n$ are the mass, the charge and the number density of electrons, respectively, whereas the signs $-$ or $+$ (in brackets) correspond to the rigth-circular polarized outgoing wave or to the left-circular one, respectively.

Subsequently we get from equations (1), (2), (3) for the resonance conditions:

$$\frac{\omega}{c\varepsilon_0}\left(X_i l_L + \frac{G_r}{\omega} l_s\right) \approx |\varrho_1| \cdot |\varrho_2| - 1, \tag{6}$$

$$\frac{\omega L}{c}\left[1 + \frac{1}{\varepsilon_0 L}\left(X_r l_L - \frac{G_i}{\omega} l_s\right)\right] \approx \pi q + (m + n + 1)\frac{\pi}{2}. \tag{7}$$

Here we neglected the phase-shift in (7), which is a result of the existence of medium boundaries and of the reflections on the mirrors.

From (4), (5), (6) the condition of pumping $\eta$ can be estimated:

If $|\xi| < 1$ then $X_i$ can be assessed as [1]:

$$X_i \approx \frac{\eta_0}{2}\left(\frac{2}{\sqrt{\pi}}\frac{\gamma_{ab}}{4\beta} - \exp\frac{\gamma_{ab}^2}{4\beta}\right), \quad \text{if} \quad b = 0, \tag{4a}$$

$$X_i \approx -\frac{\eta_b}{2}\exp\left(-\frac{b^2}{4\beta}\right), \quad \text{if} \quad b \gg \gamma_{ab}. \tag{4b}$$

Accordingly, we have for the pumping condition

$$\eta_0/\eta_b = \exp\left[-(\gamma_{ab}^2 + b^2)/4\beta\right]. \tag{8}$$

From the relationship in (8) it is obvious that a simultaneous existence of several modes of various frequencies is possible. If e. g. there is between the adjacent modes a difference of about 100 MHz and $\gamma_{ab} \sim 10^7$ Hz, $\sqrt{\beta} \sim 5 \cdot 10^9$ Hz, then the one percent exceeding the minimal pumping $\eta_0$ makes it possible to generate about seven modes.

According to the equation (6) we obtain the following condition for the minimal pumping $\eta_0$:

$$\eta_0 \approx \frac{2\varepsilon_0 c}{\omega l_L}(1 - |\varrho_1| \cdot |\varrho_2|). \tag{9}$$

From (4), (5), (7) we can calculate the first approximation for the beat frequency $\Delta = \omega_{mnq+1} - \omega_{mnq}$.

If $|\xi| < 1$ and $b > 0$ then $X_r$ can be written approximately as in [1]:

$$X_r \approx \frac{\eta_b}{2}\left[-\frac{2}{\sqrt{\pi}}\frac{b^2}{4\beta} + \frac{\gamma_{ab} b}{2\beta}\exp\left(-\frac{b^2}{4\beta}\right)\right]. \tag{10}$$

(If $b < 0$ then an analogical relation takes place which differs only in the sign of the square term.)

If the conductivity contribution and the exponential term in (10) are neglected, then by the assumption: $b_{q+1} = \Omega - \omega_q - \Delta < 0$, $b_q = \Omega - \omega_q > 0$ we have for the beat frequency:



$$\Delta \approx \frac{\pi c}{L} - \frac{\eta_b}{\eta_0}\frac{c}{L}(1 - |\varrho_1| \cdot |\varrho_2|)\left(\frac{b_q^2 - b_q\Delta + \Delta^2/2}{\sqrt{\pi\beta}} - \frac{\gamma_{ac}\Delta}{2\beta}\right) = \Delta^0 - \delta. \tag{11}$$

Evidently, if $b_q \gg \gamma_{ab}$ then $\delta > 0$, i. e., the cavity frequency $\omega$ is pulled to the line center $\Omega$ (frequency-pulling effect [2]). In the He—Ne laser with the following properties: $\lambda \sim 1 \cdot 1\mu$; $L \sim .5$ m; $b_q = \Delta$; $(1 - \varrho_1\varrho_2) \sim 10^{-2}$ we have $\delta \sim 10^5$ Hz, which is in order-agreement with the value $\delta \sim 4 \cdot 10^5$ Hz measured by McFarlane [3].

A mirror characterized by the reflectivity $\varrho_1$ is the source of the ligth intensity $I_1$ for the light beam going out of the resonator. $I_1$ is determined as the square of the incoming wave amplitude $A_2$ reduced $(1 - |\varrho_1|^2)$-times. Similarly, the intensity $I_2 = |A_1|^2 (1 - |\varrho_2|^2)$, where $A_1$ is the outgoing wave amplitude.

By help of the relationship [1]:

$$|A_1/A_2|^2 = \varrho_1 C_{mn}/(\varrho_2 C_{mn})^*,$$

for the ratio of the ligth intensities $I_1/I_2$ we get:

$$\frac{I_1}{I_2} = \left|\frac{\varrho_2}{\varrho_1}\right| \cdot \frac{1 - |\varrho_1|^2}{1 - |\varrho_2|^2}\exp\frac{\Omega}{c\varepsilon_0}\left(X_i l_L + \frac{G_r}{\Omega} l_s\right). \tag{12}$$

As it follows from equations (3) and (5) the phase shift $\Delta\varphi$ occurs between the right-circular polarized outgoing wave and the left-circular one after the beam has passed the length $l_s$ along the constant magnetic field $H$. The phase shift is equivalent to the rotation of the polarization plane of the linearly polarized outgoing wave.

The corresponding rotation is:

$$\varphi \approx \frac{\Delta\varphi}{2} \approx \frac{\omega_p^2\omega_e}{2c\omega^2} l_s, \tag{13}$$

which is in the order-agreement with the expression for the Faraday effect calculated for a plane wave [4].

Some sensitivity conditions for the application of the gas laser to plasma diagnostics can be determined as follows: The above type of plasma diagnostics is based on the physical essentiality of equation (3). Changing the electron density $dn$ (i. e., variation of the conductivity $G$) effects the change of wave number $d\bar{k}$, which can be measured. Since the wave number is complex, experimental technic depends on the measured component. By measuring the absorptive index the imaginary component is determined, while the phase-shift-technic estimates the real part of the wave number.

The dependence of the wave number on the polarizability $X$ of the lasering medium complicates the problem. Evidently, the measurement sensitivity decreases when the contribution of polarizability and conductivity terms in (3) to the wave number are about the same order magnitude.

Provided that change of polarizability is associated with the fluctuations of the pumping $\eta$, then by means of the equations (5), (9), (10) the sensitive condition of the phase-shift-methods can be written in terms of the electron density change $dn$ and that of the pumping ratio $d\,(\eta/\eta_0)$ as:

$$dn \approx \frac{mc\varepsilon_0\omega}{2\sqrt{\pi\beta}l_s}\left(\frac{b}{e}\right)^2(1-|\varrho_1|\cdot|\varrho_2|)d(\eta/\eta_0) \approx \frac{10^{17}}{l_s}d(\eta/\eta_0), \tag{14}$$

where we assume $\nu \sim 10^{10}$ Hz, $\lambda \sim .\,6\mu$, $l_s \sim (.\,1-1)$ m. Using this expression we see that the minimal measured change of the electron density is about $10^8-10^9$ cm$^{-3}$.

The sensitivity condition for absorptive methods is obtained analogically by means of the terms of the imaginary part of the wave number. Then with respect to the equations (3), (5), (4b) the sensitive condition has the form:

$$l_L dX_i \approx -dG_r\frac{l_s}{\omega},$$

or the one corresponding to (14):

$$dn \approx -\frac{mc\varepsilon_0}{\gamma l_s}\left(\frac{\omega}{e}\right)^2(1-|\varrho_1|\cdot|\varrho_2|)\exp\left(-\frac{b^2}{4\beta}\right)d(\eta/\eta_0) \approx -\frac{10^{24}}{l_s}d(\eta/\eta_0), \tag{15}$$

i. e., we see that by use of the absorptive method the minimal measured density change is about $10^{15}$ cm$^{-3}$*.

Note: All relations are in the MKSA unit system.

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*Katedra experimentálnej fyziky*
*Prírodovedeckej fakulty UK,*
*Bratislava*



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