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ON THE THEORY OF A GAS LASER WITH A CONFOCAL RESONATOR

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Theories of [1, 2] of a passive resonator help us obtain diffraction losses, eigenfrequencies and the form of the field on resonator mirrors. Since, however, the theories neglect the influence of medium in the resonator and losses (different from those of diffraction) on mirrors, they cannot directly give the generation conditions.

The theory of [3] accepts the medium influence by phenomenologically introduced polarizability X and losses on mirrors with reflectivity ϱ. The formulation of the Huygens principle in terms of the Frauenhoffer diffraction problem [5] causes that the theory of [3] is the most suitable for the resonators little different from those with plane and parallel mirrors. This theory gives the generation conditions and resonant frequencies for the mentioned type of laser. Since, however, polarizability X is not determined by simultaneously solved equations of the field and those of material, then [3] cannot describe the nonlinear frequency effects caused by the existence of active medium in the resonator.

Describing the active medium by polarization P, which is determined by the density matrix method, enables us to estimate in the theory in [4] the nonlinear frequency effects. In [4] the field solution is assumed in the form of plane waves. Due to the fact it is not evident whether it is suitable to use Lamb's results for resonator with spherical mirrors, in the case of which it is useful to describe the field in the form of spheroidal waves that are determined with the help of the finite Fourrier's transformation [2]. Since the boundary condition problem is circumvented by describing the losses on mirrors by phenomenologically introduced equivalent specific conductivity σ, then Lamb's theory cannot directly describe the effects in the laser applied in plasma diagnostics.

In paper [10] the effects caused by the existence of plasma in the laser resonator are discussed by changes of the refractive index of the investigated plasma. The influence of active medium and losses on mirrors are neglected.

In the present paper a laser with mirrors of reflectivity ϱ_1, ϱ_2 is described. The medium in the resonator is characterized by the polarization vector $\boldsymbol{P}$

and the current density one $\boldsymbol{j}$. These vectors are determined by the simultaneous solution of the field equations and the material ones by the iterative method. The formulation of the Huygens's principle in the Fresnel diffraction terms [5] causes that the theory is the most suitable for such resonators with spherical mirrors as the confocal ones are.

Further generalisation of the theory (from the semiclassical point of view) for purposes of laser application in plasma diagnostics must take into account also the influence of other particles than the actives ones, electrons and ions.

Since we describe the electromagnetic field by Maxwell's equations, active medium by density matrix method [4], plasma behaviors by magneto-hydrodynamic equations [8], then it is evident that the presented theory does not leave classical boundaries, respectively the semiclassical ones of the above mentioned theories.

The aims of this paper are to outline the theory that can directly describe by a single method phenomena in a confocal resonator which is used in plasma diagnostics (for its small diffraction losses and its easy adjustment of mirrors).

Further, by means of the theory (that corresponds in some respects to a more real laser model) we have the possibility to estimate the validity of some results determined by theories with other models of laser than the presented one (by obtaining known results given by other theories).

Finally, the paper tries, for purposes of applications, to determine such results as polarizability X in analytical form.

FORMULATION OF THE PROBLEM

For laser applications relationships between the radiated electromagnetic field and other laser parameters must be known. Mutual dependencies can be determined by solving the Maxwell equations supplemented by appropriate boundary conditions and equations characterizing a material that is passed by radiation.

Since $\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}$ and $\boldsymbol{B} = \mu_0 \boldsymbol{H}$, then if a space charge effect is negligible, the Maxwell equations are reduced to the wave equation:

$$\Delta \boldsymbol{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \boldsymbol{E}}{\partial t^2} = - \frac{1}{\varepsilon_0} \operatorname{grad} \operatorname{div} \boldsymbol{P} + \mu_0 \frac{\partial \boldsymbol{j}}{\partial t} + \mu_0 \frac{\partial^2 \boldsymbol{P}}{\partial t^2}.$$

If the electric field intensity is expanded in the spectrum

$$\boldsymbol{E}(\boldsymbol{r}, t) = \operatorname{Re} \int E_\omega(r) e^{-i\omega t} d\omega,$$

(similarly: polarization and current density vector $\boldsymbol{P}$ and $\boldsymbol{j}$, respectively), then the wave equation can be written as:

$$\Delta \boldsymbol{E}_\omega + \mu_0 \varepsilon_0 \omega^2 \boldsymbol{E}_\omega = - \frac{1}{\varepsilon_0} \operatorname{grad} \operatorname{div} \boldsymbol{P}_\omega - i\mu_0 \omega \boldsymbol{j}_\omega - \mu_0 \omega^2 \boldsymbol{P}_\omega, \quad (1)$$

where solutions of the equation (1) must satisfy simultaneously the boundary conditions, which determine the intensity distribution functions $\boldsymbol{f}_{\omega_\alpha}(x_\alpha, y_\alpha)$ on lasser resonator mirrors:

$$\boldsymbol{E}_\omega(\boldsymbol{r}_\alpha) \sim \boldsymbol{f}_{\omega_\alpha}(x_\alpha, y_\alpha), \quad \alpha = 1,2, \quad (2)$$

here $\boldsymbol{r}_\alpha \equiv (x_\alpha, y_\alpha, z_\alpha)$ are position vectors of points on mirror surfaces.

Equations (1), (2) must be supplemented by material equations for components of a polarization vector and a current density vector $\boldsymbol{P}_\omega$ and $\boldsymbol{j}_\omega$, respectively.

If the right side of the equation (1) can be considered as a perturbation, then it is conveniet to express the components of vectors $\boldsymbol{P}_\omega$ and $\boldsymbol{j}_\omega$ as pover series in $\boldsymbol{E}_\omega$. Thus we get

$$P_{\omega_i} = \alpha^0_{\omega_i} + \int (\sum_k \alpha^1_{\omega_{ik}} E_{\omega_k})\, d\omega + \ldots, \quad i, k = x, y, z \quad (3)$$

$$j_{\omega_i} = \beta^0_{\omega_i} + \int (\sum_k \beta^1_{\omega_{ik}} E_{\omega_k})\, d\omega + \ldots, \quad (4)$$

$$- \frac{1}{\varepsilon_0} (\operatorname{grad} \operatorname{div} \boldsymbol{P}_\omega)_i = \gamma_{\omega_i} + \int (\sum_k \gamma^1_{\omega_{ik}} E_{\omega_k})\, d\omega + \ldots, \quad (5)$$

where „slow-changing" functions α, β, γ are components of a vector, eventually a tensor, which can be calculated from material equations by comparison to the system of equations (3), (4), (5).

Material equation system and the relationships (1)—(5) can be solved with the help of the iterative method: if we substitute the n^th^ approximation $E^{(n)}_\omega$ from equations (1), (2) into both material equations and the ones in (3), (4), (5), then we shall obtain the $(n + 1)^t$ approximation for $\boldsymbol{P}_\omega$, $\boldsymbol{j}_\omega$. We shall obtain the zeroth approximation by the assumption:

$$j_{\omega_i} \equiv 0, \quad P_{\omega_i} \equiv 0.$$

THE ZEROTH APPROXIMATION

If the influence of $\boldsymbol{j}$ and $\boldsymbol{P}$ on the field $\boldsymbol{E}$ is neglected and $\boldsymbol{E}$ is assumed to be $(E, 0, 0)$*) (it is a case of a linear polarized radiation in the resonator

*) We have dropped index ω.

with Brewster's windows), then the system of equations (1)—(5) is reduced to the equations:

$$\Delta E + k_0^2 E = 0, \quad k_0^2 = \mu_0 \varepsilon_0 \omega^2, \tag{6}$$

$$E(\mathbf{r}_\alpha) = K_\alpha f_\alpha(x_\alpha, y_\alpha), \quad \alpha = 1,2, \tag{7}$$

where K_1, K_2 are constants of proportionality.

There are two independent particular solutions of equation (6), an outgoing spherical wave and an incoming one $R^{-1}\exp(ik_0R)$, $R^{-1}\exp(-ik_0R)$, respectively. The general solution of the equation (6) justifying the boundary conditions will be assumed as a superposotion of the mentioned spherical waves. An asymptotic form of the solution can be assessed analogically to that of the diffraction problem.

If we assume that each of the area elements dS of the mirror is a radiation source with the amplitude $u(x, y)$ of the field intensity, then the general solution of the equation (6) can be approximated by the forms [5]:

$$E(\mathbf{r}) \approx a_1 \int_{S_1} u_1(x_1, y_1) \frac{e^{ik_0R_1}}{R_1} \, dS_1^{(n)} + a_2 \int_{S_2} u_2(x_2, y_2) \frac{e^{-ik_0R_2}}{R_2} \, dS_2^{(n)}, \tag{8}$$

$$R_1 = |\mathbf{r}_1 - \mathbf{r}| \gg 1/k_0,$$

$$E(\mathbf{r}_1) \approx u_1(x_1 y_1) + a_2 \int_{S_2} u_2(x_2, y_2) \frac{e^{-ik_0R_{21}}}{R_{21}} \, dS_2^{(n)}, \quad -R_2 = |\mathbf{r}_2 - \mathbf{r}| \gg 1/k_0,$$

$$E(\mathbf{r}_2) \approx a_1 \int_{S_1} u_1(x_1, y_1) \frac{e^{ik_0R_{12}}}{R_{12}} \, dS_1^{(n)} + u_2(x_2, y_2), \quad R_{12} = |\mathbf{r}_1 - \mathbf{r}_2| = -R_{21},$$

where the constants $a_1 = -a_2$ are estimated in such a way that the plane wave also satisfies the equation (8), d$S_\alpha^{(n)}$ is the projection of an area element dS_α in the direction determined by the vector outgoing from the mirror element dS_α and incoming at point $\mathbf{r}$. The integration is performed over mirror surface S_α. (See Fig. 1.)

The form of the amplitude u_α is determined by resonator geometry, i. e., we shall determine it by solving the boundary conditions. If we assume that the dependence of the source amplitude $u_\alpha(x_\alpha, y_\alpha)$ on the distribution function $f_\alpha(x_\alpha, y_\alpha)$ is given by the expression:

$$u_\alpha x_\alpha, y_\alpha) = A_\alpha(x_\alpha, y_\alpha) f_\alpha(x_\alpha, y_\alpha),$$

then by means of relations (7), (8) we can write:

$$A_1(x_1, y_1) f_1(x_1, y_1) + a_2 \int_{S_2} A_2(x_2, y_2) f_2(x_2, y_2) \frac{e^{-ik_0R_{21}}}{R_{21}} \, dS_2^{(n)} = K_1 f_1(x_1, y_1), \tag{9}$$

$$A_2(x_2, y_2) f_2(x_2, y_2) + a_1 \int_{S_1} A_1(x_1 y_1) f_1(x_1, y_1) \frac{e^{ik_0R_{12}}}{R_{12}} \, dS_1^{(n)} = K_2 f_2(x_2, y_2).$$

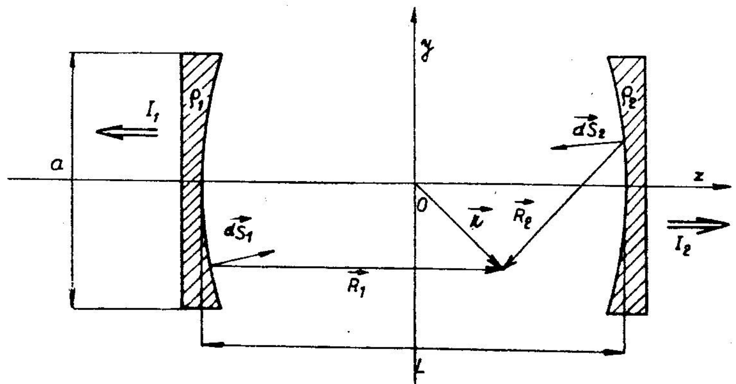

Fig. 1.

Since the concrete form of the distribution function is unknown, then for the single-valued solution we need two additional equations. These can be obtained by the assumption that the outgoing wave*) is created by the reflection of the incoming one in the mirror, which is characterized by the reflectivity function $\varrho_1(x_1, y_1)$. Analogically, the incoming wave is given by reflection of the outgoing one in the mirror with the function $\varrho_2(x_2, y_2)$ [3]:

$$A_1(x_1, y_1) f_1(x_1, y_1) = \varrho_1(x_1, y_1) \, a_2 \int_{S_2} A_2(x_2, y_2) f_2(x_2, y_2) \frac{e^{-ik_0R_{21}}}{R_{21}} \, dS_2^{(n)}, \tag{10}$$

$$A_2(x_2, y_2) f_2(x_2, y_2) = \varrho_2(x_2, y_2) \, a_1 \int_{S_1} A_1(x_1, y_1) f_1(x_1, y_1) \frac{e^{ik_0R_{12}}}{R_{12}} \, dS_1^{(n)}.$$

*) Since in the equations of (8) there are two independent groups of singular points (surfaces S_1, S_2), then the denotations of the spherical waves as an outgoing wave and an incoming one are not single-valued (the same as the denotation of plane waves as rightgoing and leftgoing ones is not single-valued). Therefore in the following the outgoing wave is defined as the wave with the singular points S_1 (it is an analog of a rightgoing wave from the point of view of Fig. 1) and the incoming wave is defined as the one with the singular points S_2 (an analog to the leftgoing wave in Fig. 1).

In general, the field in the resonator cavity is determined by the solving of the integral equations (8)—(10).

To assess the basic behaviours of the resonator it is enough to take the first term of the expansion of the functions ϱ_α, A_α. Besides it is natural to assume because of symmetric geometry of the resonator that the fields on both mirrors are proportional to the same distribution function $f(x, y)$, [1, 2]. If the proportionality constants in equation (7) are chosen in the form

$$K_1 = A_1 + C_1 A_2, \quad K_2 = A_2 + C_2 A_1$$

then the equation system in (9), (10) can be written as

$$a_1 \int_{S_1} f(x_1, y_1) \frac{e^{ik_0 R_{12}}}{R_{12}} \, dS_1^{(n)} = C f(x_2, y_2), \quad C = C_1 = C_2, \tag{11}$$

$$1 = \varrho_1 \varrho_2 C^2, \tag{12}$$

$$\left| \frac{A_1}{A_2} \right|^2 = \frac{\varrho_1 C}{(\varrho_2 C)^*}. \tag{13}$$

If both the geometry of the resonator and the reflectivity ϱ_α of the mirrors are known, then we can get the form of the field distribution function on the resonator mirrors, the ratio of the amplitudes of the incoming wave to the outgoing one and also the appropriate values of the wave number by solving the equations (11), (12), (13).

Approximate values of the field inside the resonator are obtained from the equations (8), if the field amplitudes and the wave number components be replaced by solutions of the equation systems (11), (12), (13).

The equations of (8), (11), (12) for the confocal resonator with the distance L between the square spherical mirrors of the radius of curvature L and the side of the square a were solved by the authors in [2]. The resonant conditions (12) for the confocal resonator of the described type has the form

$$1 \approx \varrho_1 \varrho_2 (1 - \alpha_m 10^{-\beta_m S} - \alpha_n 10^{-\beta_n S}) \exp 2i \left[k_0 L - (m + n + 1) \frac{\pi}{2} \right] = \varrho_1 \varrho_2 C_{mn}^2, \tag{14}$$

where $s = a^2 k_0 / 2\pi L$; $m, n = 0, 1, \ldots$, and α, β are constants (e. g. $\alpha_0 \sim 10.9$ and $\beta_0 \sim 4.94$).

From the condition (14) it is evident that even the ideal reflecting mirrors do not give the steady-state generation, because in the case of the finite dimensions of the mirrors the following condition: $|C_{mn}|^2 < 1$ holds. To satisfy the generation conditions in (14), the wave number must be complex, but as it follows from equation (6), this condition is not satisfied in the zeroth approximation.

THE FIRST APPROXIMATION

As it follows from the foregoing, to obtain a stationary solution of the equations (1)—(5) we must not ignore the influence of $\boldsymbol{j}$ and $\boldsymbol{P}$ on the field $\boldsymbol{E}$.

If in the zeroth approximation $j_i \equiv 0$, $P_i \equiv 0$ and if simultaneously $\boldsymbol{E}^0$ $(E, 0, 0)$ was determined by the equations (8), (9), (10), then in the first approximation — as it will be shown later — the equations (3), (4), (5) have, by some preconditions, the form:

$$P_x \approx X E_x, \; j_x = G E_x, \; -\frac{1}{\varepsilon_0} (\text{grad div } \boldsymbol{P})_x \approx \frac{4}{\varepsilon_0} \left(\frac{k_0 a}{L} \right)^2 X E_x. \tag{15}$$

Then the wave equation (1) can be written as:

$$\Delta E + k_1^2 E = 0, \tag{16}$$

where

$$k_1^2 = \mu_0 \varepsilon_0 \omega_1^2 \left\{ 1 + i \frac{G}{\omega_1 \varepsilon_0} + \frac{X}{\varepsilon_0} \left[1 + \frac{1}{\mu_0 \varepsilon_0} \left(\frac{2 k_0 a}{\omega_1 L} \right)^2 \right] \right\}.$$

Evidently, the field in the first approximation is determined by the equations (2), (16) which are formally equivalent to (6), (7). In a general case, the wave number of the equation (16) can depend on coordinates, therefore we must replace the vector $\boldsymbol{k}_1$ by its average value $\boldsymbol{k}_1$ to allow the utilization of the relations of the zeroth approximation.

Then the first approximation is determined either by the relations of (8), (9), (10), or (8), (11), (12), (13) so that the terms of the zeroth approximation are replaced by those of the first one. Besides we must determine the „constants" G, X, which determine the differences between the wave numbers k_1 and k_0.

POLARIZABILITY X IN THE FIRST APPROXIMATION

The influence of excited atoms on field properties is described by the polarization vector $\boldsymbol{P}$, which can be calculated by the density matrix method [4].

In the first approximation this method gives for the two-level model:

$$\boldsymbol{P}(\boldsymbol{r}, t) = -2 \,\text{Re} \frac{i\nu^2}{\hbar} N \int_0^\infty d\tau \int d^3 v \, W(\boldsymbol{v}) \, \boldsymbol{E}(\boldsymbol{r} - \boldsymbol{v}\tau, t - \tau) \times$$

$$\times \exp -(\gamma_{ab} + i\Omega)\tau], \quad (17)$$

where ν is the dipole moment matrix element.

$$N = \frac{\lambda_a}{\gamma_a} - \frac{\lambda_b}{\gamma_b},$$

where λ is the number density of the atoms excited to the higher level, or to the lower one denoted by a or b, respectively, γ are phenomenologically introduced decay rates (γ_{ab} is equal to one-half of the sum γ_a and γ_b), Ω is the circular frequency of transition, $W(\mathbf{v})$ is the velocity distribution (normalized to the unit), $\tau = t - t'$ is the time difference between the moment of the polarization of an atom and the moment of its observation at the point $\mathbf{r}$.

We assume that there is no dependence of the population inversion density N on the coordinates. The contribution of the spontaneous decay of the higher level a to the number density λ_b of the lower level b is neglected.

If E° from equation (8) is substituted into (17), then the part of the polarization vector $\mathbf{P}$ corresponding to the outgoing wave has the form

$$P'(\mathbf{r}) = -\frac{i\nu^2}{\hbar} N \int_0^\infty \mathrm{d}\tau \int \mathrm{d}^3 v \int_{S_1} \mathrm{d}S_1^{(n)} W(\mathbf{v}) a_1 u_1(x_1, y_1) \times$$
$$\times \frac{\exp\{ik_0 \,|\mathbf{R}_1 - \mathbf{v}\tau| - [\gamma_{ab} + i(\Omega - \omega)]\tau\}}{|\mathbf{R}_1 - \mathbf{v}\tau|}. \quad (18)$$

Since practically for all R_1 the following holds: $R_1 \gg v\tau \sim 10^{-3}$ cm we can write:

$$|\mathbf{R}_1 - \mathbf{v}\tau| \approx R_1 \left[1 - \frac{v_\parallel \tau}{R_1} + \frac{1}{4}\left(\frac{v_\parallel \tau}{R_1}\right)^2 + \frac{1}{2}\left(\frac{v_\perp \tau}{R_1}\right)^2 + \ldots\right], \quad (19)$$

where the velocity vector $\mathbf{v}$ is decomposed into components $v_\parallel$ and $v_\perp$ parallel and perpendicular to $\mathbf{R}_1$ respectively.

The velocity distribution of the polarized atoms is assumed to be Maxwellian. Provided we have an axial symmetric resonator, it is most natural to choose the distribution function in the cylindrical coordinates of the velocity space [6]:

$$\mathrm{d}^3 v W(\mathbf{v}) = \left(\frac{M}{2T}\right)^{3/2} v_\perp \exp\left[-\frac{M}{2T}(v_\parallel^2 + v_\perp^2)\right] \mathrm{d}v_\perp \,\mathrm{d}v_\parallel \,\mathrm{d}\varphi. \quad (20)$$

If the integration over the variables $v_\parallel$ and $v_\perp$ is performed for infinite boundaries, then by the assumption*)

$$1 \gg \frac{2Tk_0\gamma_{ab}^{-2}}{MR_1} \sim 10^{-1} - 10^{-3},$$

$$b = \Omega - \omega \gg \frac{k_0^3 \gamma_{ab}^{-3}}{R_1}\left(\frac{T}{M}\right)^2 \sim (10^8 - 10^{11})\,\mathrm{Hz}, \quad (21)$$

from equations (18), (19), (20) it follows [7] that:

$$P'(\mathbf{r}) \approx -\frac{i\nu^2}{\hbar} N \int_{S_1} a_1 u_1 \frac{e^{ik_0 R_1}}{R_1} \mathrm{d}S_1^{(n)} \int_0^\infty \exp\left[-\frac{k_0^2}{4}\left(\frac{2T}{M}\right)\tau^2 - \gamma_{ab}\tau - i(\Omega - \omega)\tau\right] \mathrm{d}\tau, \quad (22)$$

but this is just what is predicted by the first approximation in [4], wherein a similar term is calculated by the assumption that the field is a superposition of plane waves, hence the iterative process gives correct results, moreover, it permits to show the degree of suitability of the plane wave approximation.

The coefficient at the outgoing wave in (22) is independent of the coordinates, therefore we obtain the same at the incoming wave, too.

By performing the integration in the relation (22) we obtain the poralizability X of equation (15):

$$X \approx -i\eta[1 - \Phi(\xi)] \exp \xi^2, \quad (23)$$

where $\eta = (\gamma^2 N/\hbar)\sqrt{\pi/\beta}$, $\beta = k_0^2 T/2M$, $\xi = (\gamma_{ab} + ib)/2\sqrt{\beta}$, $\varphi = \operatorname{arctg} b/\gamma_{ab}$, $\Phi(\xi)$ is the error integral.

For the real component and the imaginary one of the polarizability we obtain [7] the relations:

$$X_r \approx \frac{1}{2}\eta \left\{\exp|\xi|^2 . \sin\frac{\gamma_{ab} b}{2\beta} - \frac{2}{\sqrt{\pi}} \sum_{l=0}^{\infty} \frac{2^l}{(2l+1)!!} |\xi|^{2l+1} \sin(2l+1)\varphi\right\} \quad (24)$$

*) Integrand in the relation (22) ought to have correctly the form:

$$\exp\left[-k^2\tau^2 \Big/ 4\left(\frac{M}{2T} - i\frac{k\tau^2}{R}\right) - \gamma_{ab}\tau - i(\Omega - \omega)\tau\right].$$

By comparison of this term with the form of the integrand in (22) we can get the conditions of (21). By a numerical estimation of the conditions we used the following values: $M \sim 20 \,.\, 10^{-27}$ kg, $T \sim 400$ °K, $\lambda \sim .\, 6\mu$, $\gamma_{ab}^{-1} \sim 10^{-7} - 10^{-8}$ sec, $R \sim 10^{-1}$ m.

$$X_i \approx -\frac{1}{2}\eta\left\{\exp|\xi|^2 . \cos\frac{\gamma_{ab}b}{2\beta} - \frac{2}{\sqrt{\pi}}\sum_{l=0}^{\infty}\frac{2^l}{(2l+1)!!}|\xi|^{2l+1}\cos(2l+1)\varphi\right\} \quad (25)$$

or

$$X_r \approx -\frac{\eta}{\pi}\sum_{l=0}^{\infty}(-1)^l\Gamma\left(l+\frac{1}{2}\right)\cdot\frac{\sin(2l+1)\varphi}{|\xi|^{2l+1}}, \quad (26)$$

$$X_i \approx -\frac{\eta}{\pi}\sum_{l=0}^{\infty}(-1)^l\Gamma\left(l+\frac{1}{2}\right)\cdot\frac{\cos(2l+1)\varphi}{|\xi|^{2l+1}}. \quad (27)$$

where Γ is the gamma-function.

GENERAL CONDUCTIVITY G

Plasma diagnostics by means of a gas laser is based in refractive index variation measurements. If a considerable difference between the frequencies at which the index of refractivity is measured and the natural frequencies of electronic displacement can be assumed, then a fairly simple dispersion formula is sufficient for describing the changes of the refractive index with frequencies in the optical range, too [11]. Due to the fact it is possible to describe the influence of plasma on the electromagnetic field in terms of the classical theory. (Measurements [10] confirm the correctness of this assumption.)

If we place into the resonator cavity besides the active lasering tube also an investigated discharge tube then the discharge medium can be characterized by the current density vector $\boldsymbol{j}$.

We neglect the influence of neutral particles on the variation of the refractive index, which is accurate only if the conditon $|dn_0| \ll |dn|$. 10 is fulfilled, where dn_0, dn is the change in number density of neutral particles and electrons, respectively. The condition is estimated for investigated plasma of gas, the refractive index of which at $n_0 \sim 2.7.10^{19}$ cm^{-3} is 1.0003, and the measurement is done on the wavelength $\lambda \sim .6\mu$.

For the rough magneto-hydrodynamic model we may write the system of linearized equations of [8], [9]:

$$\frac{m}{ne^2}\frac{\partial \boldsymbol{j}}{\partial t} = \boldsymbol{E} + \mu_0[\boldsymbol{v}\boldsymbol{H}_0] - \frac{\mu_0}{en}[\boldsymbol{j}\boldsymbol{H}_0] - \frac{\boldsymbol{j}}{\sigma},$$

$$nM\frac{\partial \boldsymbol{v}}{\partial t} = \mu_0[\boldsymbol{j}\boldsymbol{H}_0],$$

where $\boldsymbol{H}_0$ is the constant magnetic field intensity within the investigated medium, σ is the specific conductivity, n denotes the electron density number. Further it is assumed that the electroneutral plasma is composed of both the single-ions of the mass M and the electrons of the mass m.

If we introduce the unit vector $\boldsymbol{h} = \boldsymbol{H}_0/H_0$ and constants with the frequency dimension

$$\omega_p^2 = \frac{ne^2}{m\varepsilon_0}, \quad \omega_e = \frac{e\mu_0 H_0}{m}, \quad \omega_i = \frac{e\mu_0 H_0}{M}, \quad \gamma = \frac{ne^2}{m\sigma}, \quad (28)$$

then for the spectral component $\boldsymbol{j}_\omega$ (we drop index ω in the following) the system of material equations is reduced to the relation:

$$\boldsymbol{j} = i\frac{\omega_p^2}{\omega}\varepsilon_0\boldsymbol{E} + \frac{\omega_i\omega_e}{\omega^2}[\boldsymbol{h}[\boldsymbol{j}\boldsymbol{h}]] + i\frac{\omega_e}{\omega_i}[\boldsymbol{h}\boldsymbol{j}] - i\frac{\nu}{\omega}\boldsymbol{j}. \quad (29)$$

The general specific conductivity components G can be approximated by equation (29) into which $\boldsymbol{E}^0$ from (8) is substituted. $\boldsymbol{E}^0$ was derived by the assumption $j_i \equiv 0$, i. e., $H_i \equiv 0$, too. If the magnetic field is oriented in the direction of the radiation spread, then the existence of this field is the reason for a rotation of the polarization plane. To be able to assess the magnitude of Faraday's effect we assume $\boldsymbol{h}(0, 0, 1)$ and $\boldsymbol{E}^0$ $(E, 0, 0)$. The latter vector is valid in the form:

$$\left(\frac{E^+ + E^-}{2}, \frac{E^+ - E^-}{2i}, 0\right),$$

whereas $E^+ \equiv E^-$, i. e., each linearly polarized wave is decomposed into a right-circulating component and a left-circulating one, E^+ and E^-, respectively.

If it is assumed that ω_e/ω, ν/ω, $\omega_p/\omega \ll 1$, then from the equation (29) we obtain for the x-component of the current density j_x' caused by the outgoing wave

$$j_x' \approx i\frac{\omega_p^2\varepsilon_0}{\omega}\left\{\left(1 - \frac{\omega_e}{\omega} - i\frac{\nu}{\omega}\right)\frac{E^+}{2} + \left(1 + \frac{\omega_e}{\omega} - i\frac{\nu}{\omega}\right)\frac{E^-}{2}\right\}. \quad (30)$$

In the first approximation under the condition that $E_y^0 = 0$ and $E^+ = E^-$ we get for the general conductivity G

$$G \approx i\frac{\omega_p^2\varepsilon_0}{\omega}\left(1 - i\frac{\nu}{\omega}\right). \quad (31)$$

In the further approximation, where it is assumed that $E^+ \neq E^-$, the degeneration of the right-circular component and the left-circular one vanishes, so that each of them can be characterized by the special conductivity constant:

$$G^{\mp} \approx \frac{G}{2}\left(1 \pm \frac{\omega_e}{\omega}\right). \tag{32}$$

Note: The solution $E_y = (E_x^+ - E_x^-)/2i$ is that of the homogeneous wave equation for the y-component of the field. The contribution to conductivity from the particular solution of the nonhomogeneous equation was neglected, since it is of the order $(\omega_e/\omega)^2$.

DETERMINATION OF THE CONTRIBUTION γ

If $\mathbf{E}^\circ$ from (8) and X from (23) are substituted into equation (5), we get:

$$\gamma E \approx -\frac{X}{\varepsilon_0}\frac{\partial^2}{\partial x^2}E \approx 4\,\frac{X}{\varepsilon_0}\left(\frac{k_0 a}{L}\right)^2 E,$$

where the lower order terms of k_0 are neglected and we have used the approximation $(x_\alpha - x)^2/R_x^2 \approx 4a^2/L^2$. Hence

$$\gamma \approx \frac{X}{\varepsilon_0}\left(\frac{2k_0 a}{L}\right)^2. \tag{33}$$

From equation (16) it follows that the contribution of the expression (33) to the wave number is equivalent to the change of the polarizability X in $X(1 + 4a^2/L^2)$, or it is equivalent to the increase of the population inversion density N by $(4a^2/L^2)\,.\,100\,\%$.

Note: All relations are in the MKSA unit system.*

REFERENCES

[1] Fox A. G., Li T., Bell Syst. Techn. Journ. *40* (1961), 453.
[2] Boyd G. D., Gordon J. P., Bell Syst. Techn. Journ. *40* (1961), 489.
[3] Kotik J., Newstein M. C., Journ. Appl. Phys. *32* (1961), 178.
[4] Lamb W. E., Phys. Rev. *134* (1964), 6 A, 1429.
[5] Ландау Л. Д., Лифшиц Е. М., *Теория поля.* Москва 1960.
[6] Ландау Л. Д., Лифшиц Е. М., *Статистическая физика.* Москва 1964, 106.
[7] Градштейн И. С., Рыжик И. М., *Таблицы интегралов, сумм, рядов и произведений.* Москва 1962.

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[8] Русанов В. Д., *Современные методы исследования плазмы.* Москва 1962.
[9] Спитцер Л., *Физика полностью ионизованного газа.* Москва 1965, 43.
[10] Gerardo J. B., Verdeyen J. T., Gusinow M. A., Journ. Appl. Phys. *36* (1965), 3526.
[11] Handbuch der Physik. Bd. 17, Dielektrika. Springer-Verlag, Berlin 1956, 114.

Note: the references [1, 2, 3] belong to: *Лазеры, оптические когерентные квантовые генераторы и усилители.* Издательство иностранной литературы, Москва 1963.

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