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FREQUENCY DEPENDANCE OF THE ULTRASONIC ATTENUATION IN Si

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The measurements of the attenuation of longitudinal ultrasonic waves in silicon single crystals in ⟨111⟩ direction have been made for the frequency range from 30 Mc/sec to 210 Mc/sec at the room temperature. Our results are in contradiction to the Sylwestrowicz's measurements for the range 30—100 Mc/sec, according to whom the frequency dependance is of the form $\alpha = \alpha_0 \exp cf$, where α_0 and c are constants. We have found that such a dependance can approximate the measured values only if the conversion losses are neglected. After the corrections of measured values have been made, the dependance of attenuation on the frequency took the form $\alpha = cf^2$. This result means that in the single crystal Si at the room temperature the scattering of ultrasonic waves on the thermal phonons is predominant.

INTRODUCTION

Attenuation of longitudinal ultrasonic waves in silicon has been recently measured in frequency range from 20 Mc/sec to 100 Mc/sec by Sylwestrowicz [1], who empirically found the dependance of attenuation on the frequency f in the form

$$\alpha = \alpha_0 e^{cf} \tag{1}$$

where α_0 and c are constants. He assumes that for such a dependance the scattering on clusters or on the precipitates of a new phase may be responsible, such a mechanism was discussed by Ying and Truel [2]. The rigorous solution of such multiple scattering mechanism has not been given yet. It is based only on an empirical approach and supported by experimental results.

According to various authors [3, 4, 5] the scattering on the thermal phonons is dominant at room temperature in single crystals of Si and Ge which leads to the dependance of α on ω in the form [3]:

$$\alpha = \frac{E_0(D/3)}{2V^3\varrho}\left[\frac{\omega^2\tau}{1+\omega^2\tau^2}\right] \tag{2}$$

where E_0 is thermal energy per unit volume, τ is the relaxation time for the interchange of the thermal energy between the various phonon modes, D is a constant whose value can be determined from the third-order elastic moduli, ϱ is density and V is Debye's average velocity.

From the relation (2) it is seen that in the range of low frequency, $\omega\tau \ll 1$, the quadratic dependance of α on ω can be expected. Lamb, Redwood and Steinshleifier [4] measured the attenuation for frequencies 100—1000 Mc/sec and found the relation between α and f of the type $\alpha = cf^n$, where n at room temperature is approximately 2. They found the deviation from this dependance at low temperatures. This can be explained by the fact that the term $(\omega\tau)^2$ in denominator of relation (2) cannot be neglected. The measurements by Mason and Bateman [3] of the dependance of α in Si on the temperature are also in good agreement with the relation (2). Miller examined germanium single crystals which are similar to silicon crystals in the structure, and has found quadratic dependance of the attenuation on frequency at room temperature. The increase of attenuation at frequencies below 30 Mc/sec in Ge Miller has explained by spurious losses which arise on the side walls of the specimen.

According to the above mentioned works one could expect that in Si at the room temperature and frequency range from 20 Mc/sec to 100 Mc/sec the dependance of α on f should also be of the type $\alpha = cf^2$. However, Sylwestrowicz on the base of his measurements proposed for this range the dependance of the type $\alpha = \alpha_0 \exp cf$. Because we have had some doubt about his method of the evaluation of the measured values of the attenuation and also in effort to examine the connection between his measurements and those by Lamb, Redwood and Steinshleifier [4] for the range 100—1000 Mc/sec, we decided to measure the dependance of attenuation on the frequency at room temperature in the range 30—210 Mc/sec.

EXPERIMENTAL RESULTS

The attenuation of longitudinal ultrasonic waves has been examined using a pulse technique. The absorption coefficient in dBcm⁻1 is defined as

$$\alpha = 20 \log U_n/U_{n+1}$$

where U_n is an amplitude of the n-th echo. We have measured directly the ratio of subsequent amplitudes by an attenuator with continuously variable attenuation which was situated in the intermediate frequency part of the receiver. Three silicon single crystals, whose data are in Table 1, have been examined. These specimens differ in the type of electrical conductivity and carriers concentrations. The specimen No 2 is electrically non-uniform with different types of the conductivity at its opposite ends.

Table 1

Sample number	1	2		3
Length [cm]	6.77	6.55		5.5
Diameter [cm]	2.1	1.65		1.9
Orientation	⟨111⟩	⟨111⟩		⟨111⟩
Type of conductivity	n	n	p	p
Concentration of carriers [cm^{-3}]	$2.28 \cdot 10^{14}$	$5.1 \cdot 10^{12}$	$1.04 \cdot 10^{13}$	$4 \cdot 10^{12}$
Hall mobility [$cm^2V^{-1}s^{-1}$]	1800	1800	500	500
Density of dislocations [cm^{-2}]	$1 \cdot 10^4$	$1 \cdot 10^4$		$1 \cdot 10^3$

Remark: Nonparalelism of the faces of the samples max 1 μm nearly periphery of the samples.

X-cut quartz crystals of fundamental frequency 11.6 Mc/sec and 29.8 Mc/sec with diameters from 8 to 20 mm have been used as transducers. The acoustic bond between the transducers and the samples has been realised by silicon oil of viscosities 200 and 500 cP.

The succesion of echoes received on transmitter in pulse method has amplitudes $U_n \sim K_1^{n-1}K_2^n \exp(-2nL\alpha)$, where n is an integer, L length of the sample and K_1 and K_2 are coefficients of reflection on the end-faces of the sample. If the echoes are received on the second transducer, which is in the opposite to the transmitter, the succesion of amplitudes has the form $U_n \sim (K_1K_2)^{n-1} \exp[-(2n-1)L\alpha]$. Absorption coefficient α can be calculated in both cases from the relation

$$\alpha_m = \ln \frac{U_n}{U_{n+1}} = 2\alpha L - \ln K_1 - \ln K_2. \qquad (3)$$

Since there are three unknown quantities α, K_1, K_2 in (3), it is necessary to provide three independent measurements, as it was suggested by Redwood [7]. Then α can be calculated as

$$\alpha = \frac{\alpha_1 + \alpha_3 - \alpha_2}{2L} \qquad (4)$$

where α_1 is the value of α_m in relation (3) for the case when coefficient of reflection $K_2 = 1$ (i. e. a transducer is sealed only on the first face of the specimen), α_2 is the value of α_m in the case when two transducers are sealed on the sample and the conditions on the first face are unaltered, and α_3 is the value of α_m for the case $K_1 = 1$ (the first transducer is removed and the conditions on the second face are the same as in the case when α_2 was measured).

The value of α determined in this way includes also the spurious losses which in general case arise from the difraction of ultrasonic waves and from the conversion of longitudinal waves to the transversal waves along the side walls of the sample. Since the attenuation in Si is relatively very low, the detailed analysis of these losses is necessary especially for the frequencies below 80 Mc/sec. We can distinguish two special cases:

a) The dimensions of a sample in the plane of the transducer are much larger then the diameter of the transducer. In this case the influence of side walls can be neglected. The spurious losses are caused only by the difraction. Analysis of these losses was given by Seki, Granato and Truel [6].

b) The transducer overlaps the end face of the sample, i. e. the ultrasonic wave is guided. In this case energy is drained by mode conversion on the side walls of the sample. The interference of different modes is due for deviations from exponential character of the envelope of echoes amplitudes. Analysis of these losses was given by Redwood [7].

With regard to the dimensions of our samples we could fulfil the conditions of the case b). For all measurements we have made a plot of the amplitudes of echoes in logarithmic scale and read the attenuation from the slope of the straight line drawing through the maxima of their envelope. Then we have made corrections on conversion losses according to the formula given by Redwood:

$$\alpha_{COR} = \frac{0.545C_d(1 - L_n^2)}{2\pi f a^2} \cdot \frac{j_{0n}}{j_{01}} \quad [\mathrm{Np\ cm^{-1}}] \qquad (5)$$

where C_d is the phase-velocity of the longitudinal waves (which for ⟨111⟩ direction in Si is $9.35 \cdot 10^5$ cm/sec), L_n is the absolute value of the reflection coefficient for longitudinal ultrasonic wave falling on the wall under an angle α_d which according to Aremberg [9] is given by the equation $\cos \alpha_d = j_{0n}C_d/2\pi f a$, a is the radius of the examined cylindrical sample, j_{0n} is the n-th root of the Bessel-function of the zero-th order.

In many cases the experiment can be arranged in such a way that the diameter of the transducer is smaller than that of the sample, but the influence of the side walls cannot be neglected. We have made measurements also for this case and tried to analyse them in the following way. We have divided the succession of echoes into two groups. In the first one there are the echoes on the amplitudes of which the influence of the side walls is not observable. This group of echoes corresponds to the case a). In the second group all the other echoes are included. That means that we have to do with a *quasi-guided* wave; α determined from the first group of echoes contains spurious losses of difraction kind, while α determined from the second group of echoes must be corrected on the conversion losses. The corrections on the difraction can be made accroding to the analysis of Seki, Granato and Truell [6]. If we subtract the corrected value of α gained in this way from the value of α determined from the second group of echoes by a graphical method described above, we get the corrections on conversion losses. The conversion losses determiend in this way are in good agreement with calculated losses from the relation (5), if we take the value of L_n for the angle of incidence given by the relation $\cos \vartheta = 1.22\,\lambda/D$, where λ is the wavelength of ultrasonic wave and D is the diameter of the transducer. The angle ϑ is the smallest angle of incidence under which the ultrasonic wave can fall on the wall of the sample. It is a supplementary angle to the angle of the beam spreading, which is determined by difraction theory on the assumption that $2a \gg D$. If this assumption is not fulfiled then for the following relation holds: $\cos \alpha_d \leq \cos \vartheta \leq 1.22\,\lambda/D$, where α_d is the angle of incidence of ultrasonic wave on the side wall when $2a = D$. Formula (5) can be applied on the assumption that for the case when the diameter of the transducer is not too much smaller than the diameter of the sample, the ultrasonic field has similar structure as in the case when the transducer overlaps the face of the sample.

We have verified the accuracy of corrections on the conversion losses by measurements of attenuation at given frequency with transducers of different diameters. When we have made the appropriate corrections, we got the same results. In the Table 2 there are values of α_m; α_{COR}; α; α_{COM} for frequencies at which we have measured. Plot of the function α is on the Fig. 1 (full line), the other line belongs to α_m measured on the specimen No 2 without corrections on conversion losses. Reproducibility of our measurements was within 8 %.

Table 2

f[Mc/sec]	α_m[dBcm^{-1}]	α_{COR}[dBcm^{-1}]	α[dBcm^{-1}]	α_{COM}[dBcm^{-1}]
29.8	⌀ .0156	.0075	.0081	.0084
58	.035	.0019	.0331	.0324
81.2	.063	.0008	.062	.063
89.4	.072	negligible	.072	.077
104.4	.105	,,	.1050	.1045
149	.22	,,	.22	.212
174	.27	,,	.27	.29
208.6	.41	,,	.41	.417

α_m — measured attenuation after correcting for reflection loss
α_{COR} — conversion losses calculated according to the formula (5)
α — intrinsic attenuation
α_{COM} — attenuation calculated according to the formula (6)

Error due to nonparalelism is of order less then 10^{-5} dBcm^{-1}. The values in Table 2 have been measured and calculated for the sample No 2.

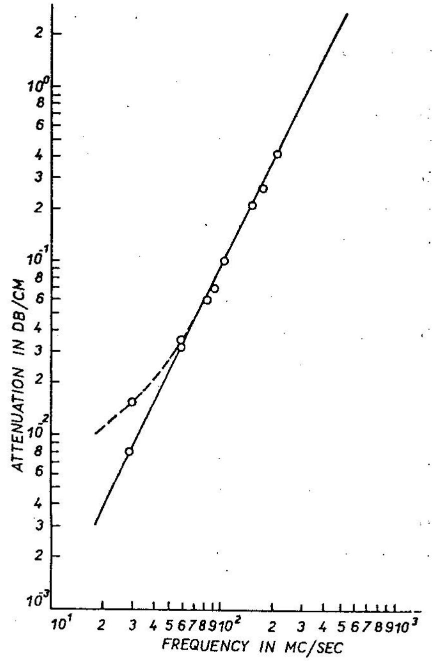

Fig. 1. Plot of the function α (solid line); the dashed line belongs to α_m measured on specimen No 2 wtithou corrections on conversion losses.

DISCUSSION

The values of attenuation which we have measured, corrected on the conversion losses, fulfil the relation

$$\alpha = cf^2 \quad [\text{dBcm}^{-1}] \tag{6}$$

where $c = 9.6 \,.\, 10^{-18}$ dB sec^2/cm. Since the velocity of longitudinal waves in Si in $\langle 111 \rangle$ direction is approximately twice as large as in Ge for the same direction, it is not possible to neglect the conversion losses in silicon single crystals of usual dimensions for frequencies higher than 30 Mc/sec as Miller could for Ge. In our case at frequency 30 Mc/sec these losses represent about 50 % of the measured attenuation.

Our results do not agree with those gained by Sylwestrowicz, as we have already mentioned in the introduction. We think that the difference is in the method of evaluation of experimental results, especially in the estimation of the spurious losses. Sylwestrowicz's method is based on the recognition that the measured value of the attenuation depends on the thickness — and therefore on the fundamental frequency — of the transducer. By extrapolation he found the value of attenuation corresponding to the vanishing thickness of the transducer and so he eliminated the losses caused by the transducer. In order to eliminate the spurious losses in the sample, he made the measurements for two its lengths. For more details of his method see the original paper [8]. However, according to our opinion, by variation of transducer's thickness and length of the sample only the losses which arise by the reflection on the faces of the sample can be eliminated, but it is necessary to take into account the conversion losses which arise on the side walls of the sample for frequencies below 80 Mc/sec, what Sylwestrowicz did not perform. We have verified that neglecting the conversion losses on the side walls leads to the exponential dependance of α_m on f in the frequency range 30—100 Mc/sec, but when the corrections on these losses have been performed the resultant dependance is quadratic and attaches to the dependance gained for higher frequencies. Therefore we can conclude that in the whole frequency range in which we have measured the dominant scattering mechanism in silicon single crystals at room temperature is that on the thermal phonons. We have not observed the difference in attenuation for our three samples due to different content of impurities and dislocations.

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REFERENCES

[1] Sylwestrowicz W. D., J. Appl. Phys. *37* (1966), 535.
[2] Ying and Truell, J. Appl. Phys. *27* (1956), 1086.
[3] Mason W. P., Bateman T. B., J. Acoust. Soc. Amer. *36* (1964), 644.
[4] Lamb, Redwood and Steinshleifier, Phys. Rev. Letters *3* (1959), 28.
[5] Miller B., Phys. Rev. *132* (1963), 2477.
[6] Seki, Granato and Truell, J. Acoust. Soc. Amer. *28* (1956), 230.
[7] Redwood M., Proc. Phys. Soc. *70* B (1957), 721.
[8] Sylwestrowicz W. D., IEEE Trans. Son. Ultrason. SU *11* (1964), 50.
[9] Aremberg D. L., J. Acoust. Soc. Amer. *20* (1948), 1.

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