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TRANSPORT THEORY FOR STRONG ELECTRIC FIELDS

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A new approach to transport theory for strong electric fields is presented. The model used in the standard transport theory is shown to be incapable of ensuring the establishment of the steady current in strong electric fields. The interaction of the system with the environment is suggested to maintain the steady state. Taking this interaction into account the direct expression for the distribution function of electrons interacting with the lattice vibrations and exposed to the uniform strong electric field is derived.

INTRODUCTION

In transport theory many authors (e. g. [1—3]) consider the problem of strong electric fields identical with the problem of solving the Boltzmann equation to higher approximations in the magnitude of the applied field. Their treatment is based on the assumption of the validity of the Boltzmann equation even for strong fields but the substantiation of this assumption is justified only by the fact that the theory gives reasonable results [2]. The validity of the Boltzmann equation is expected even in those papers (e. g. [4—7]) which assume that in strong electric fields electrons are in equilibrium with the momentum shifted in the direction of the applied field and that the temperature differs from that of the lattice due to the prevalence of the energy exchange between electrons over the exchange between the electron and the lattice vibrations. In these papers the electron temperature is determined from the Boltzmann equation in which electron-electron collisions are included.

Since papers deriving the Boltzmann equation from the quantum-mechanical standpoint (e. g. [8—10]) insure its validity to the first order in the magnitude of the applied field the question arises what entitles many authors to use it in case of a strong electric field. This problem together with the derivation of the direct expression for the distribution function in strong electric fields are the objects of the presented paper, in which under the term „strong field" we mean an electric field of such an intensity which permits the description in the effective mass approximation but leads to the appearance of non-linear terms in the expression for current density.

For the sake of simplicity electron-electron interaction will be omitted in our consideration. Though this makes it impossible to analyze the model used in the papers [4—7], we can analyze the model in which the distribution function is not substantially effected by the electron-electron collisions. And the results mentioned in paper [2] speak in favour of the latter model.

BOLTZMANN EQUATION

We shall assume that the electrons taking part in the charge transfer are exposed to an external electric field and are in an interaction with the lattice vibrations. When computing the current density we must find the average value of the electron velocity which is represented by the trace of the product from the velocity operator and the density matrix of the system considered. The density matrix is determined by the Neumann equation

$$\frac{\partial \varrho}{\partial t} = \frac{i}{\hbar} [\varrho(t), H_0 + H_L + H_i + H_F], \tag{1}$$

in which $H_0 = (2m)^{-1}p^2$ is the Hamiltonian of the quasi-free electron with the effective mass m, H_L is the Hamiltonian of the lattice vibrations which can be written as a sum of simple harmonic oscillator Hamiltonians, each characterized by an angular frequency $\omega_{\vec{\sigma}}$ with the phonon vector $\vec{\sigma}$. $H_F = -eEx$ represents the interaction of the electron with the uniform electric field of the intensity E applied in the x-direction.

$$H_i = \sum_{\vec{\sigma}} V_{\vec{\sigma}}\, a_{\vec{\sigma}} \exp\{i\vec{\sigma}\,.\,\vec{r}\} + V_{\vec{\sigma}}^{*}\, a_{\vec{\sigma}}^{*} + \exp\{-i\vec{\sigma}\,.\,\vec{r}\}$$

represents the electron-lattice interaction energy with $V_{\vec{\sigma}}$ characterizing the coupling and $a_{\vec{\sigma}}^{+}$, $a_{\vec{\sigma}}$ standing for the dimensionless creation and annihilation operators of phonon with frequency $\omega_{\vec{\sigma}}$.

Kubo [11] has drawn attention to the fact that formal solution of the equation (1) satisfying the initial condition $\varrho(-\infty) = \varrho_0(H_0 + H_L + H_i)$ (with ϱ_0 being the equilibrium density matrix) can be written in the form

$$\varrho(t) = \varrho_0 + \frac{i}{\hbar} \int_{-\infty} \exp\left\{\frac{i}{\hbar}(H_0 + H_L + H_i)(t' - t)\right\} \times$$

$$\times [\varrho(t'), H_F] \exp\left\{-\frac{i}{\hbar}(H_0 + H_L + H_i)(t' - t)\right\} dt'.$$

After $\varrho(t')$ is expressed in form of the power series in H_F this integral equation can be iterated and the terms of the expansion of ϱ can be successively computed. Since $H_0 + H_L$ does not commute with H_i mathematical difficulties occur in the treatment mentioned. Though H_i is a small perturbation the higher order terms of the expansion $\exp\left\{\frac{i}{\hbar}(H_0 + H_L + H_i)t\right\}$ in powers of H_i cannot be simply omitted since t is integrated over an infinite domain. When alternating fields are considered the direct expression for the term linear in H_F can be computed by means of the developed graph technique. In case of a uniform electric field the direct computation is not possible and the treatment leads to the kinetic equation representing the generalization of the Boltzmann equation [12].

The extension of this treatment to higher order terms in the magnitude of the applied field would be difficult. As far as the electron-lattice interaction is concerned we shall therefore work with the approximation which is used in the treatment leading to the Boltzmann equation.

If a new operator is introduced

$$H_i'(t) = \exp\left\{\frac{i}{\hbar}(H_0 + H_L)t\right\} H_i \exp\left\{-\frac{i}{\hbar}(H_0 + H_L)t\right\}$$

and similarly H_F' and ϱ' are defined, equation (1) gives

$$\frac{\partial \varrho'(t)}{\partial t} = \frac{i}{\hbar} [\varrho'(t), H_i'(t) + H_F'(t)]. \tag{2}$$

The formal solution of equation (2) can be written in the form

$$\varrho'(t) = \varrho'(t_0) + \frac{i}{\hbar} \int_{t_0}^{t} [\varrho'(t'), H_i'(t') + H_F'(t')]\, dt', \tag{3}$$

We shall use the representation in which $H_0 + H_L$ is diagonal. The eigenvalues of H_0 will be denoted by $\varepsilon(\vec{k})$ and corresponding eigenfunctions, which are normalized plane waves, will be denoted by $|\vec{k}\rangle$. The eigenfunctions and eigenvalues of H_L will be denoted by $|N\rangle$ and E_N respectively. The corresponding quantities for Hamiltonian of the oscillator having the frequency $\omega_{\vec{\sigma}}$ will be signified by $|N_{\vec{\sigma}}\rangle$ and $E(N_{\vec{\sigma}}) = (N_{\vec{\sigma}} + 1/2)\hbar\omega_{\vec{\sigma}}$, $N_{\vec{\sigma}}$ taking on all non-negative integral values.

We shall assume that the lattice vibrations remain in thermal equilibrium in spite of their coupling to the electrons which receive the energy from the external electric field. This enables us to average over the variables of the lattice which simplifies the general treatment leading to the coupled equations for the electron and phonon distributions. The assumption mentioned is well substantiated for temperatures above 35° K [3]. Anyhow, we are not interested in the lower temperatures since in that case the scattering on lattice vibrations need not prevail over the scattering on impurities. Then the distribution function of electrons $f(\vec{k})$ represents the diagonal elements of the matrix f defined by

$$\langle\vec{k}|f|\vec{k}\rangle = \sum_N \langle\vec{k}N|\varrho|\vec{k}N\rangle$$

and satisfying the relation

$$\langle\vec{k}N|\varrho|\vec{k}'N'\rangle = \langle\vec{k}|f|\vec{k}'\rangle\, P(N)\, \delta_{N,N'},$$

in which

$$P(N) = \exp\left\{-\frac{E_N}{k_0T}\right\}\left[\sum_{N'} \exp\left\{-\frac{E_{N'}}{k_0T}\right\}\right]^{-1}$$

[9]. Since H_F does not contain lattice variables, equation (3) gives the following result for f:

$$\langle\vec{k}|f'(t)|\vec{k}\rangle - \langle\vec{k}|f'(t_0)|\vec{k}\rangle = \frac{i}{\hbar}\int_{t_0}^{t} \langle\vec{k}|[f'(t'),\, H_F'(t')]|\vec{k}\rangle\, dt' +$$

$$+ \frac{i}{\hbar}\int_{t_0}^{t} \sum_N \langle\vec{k}N|[\varrho'(t'),\, H_i'(t')]|\vec{k}N\rangle\, dt'. \qquad (4)$$

If $\varrho'(t')$ is replaced by $\varrho'(t_0) + \varrho_1'(t_0, t')$, in which $\varrho_1'(t_0, t')$ is of the first order in H_i' and zeroth order in H_F', the same treatment as in [9] gives for t-t_0 large enough

$$\langle\vec{k}|f'(t)|\vec{k}\rangle - \langle\vec{k}|f'(t_0)|\vec{k}\rangle = (t - t_0)\left[\frac{\partial f(\vec{k}, t_0)}{\partial t}\right]_S + \frac{i}{\hbar}\int_{t_0}^{t} \langle\vec{k}|[f'(t'),\, H_F'(t')]|\vec{k}\rangle\, dt' \qquad (5)$$

with $\left[\frac{\partial f}{\partial t}\right]_S$ representing the collision term of the Boltzmann equation. Since the diagonal elements satisfy the following relations

$$\langle\vec{k}|[f'(t'),\, H_F'(t')]|\vec{k}\rangle = \langle\vec{k}|[f(t'),\, H_F]|\vec{k}\rangle,\quad \langle\vec{k}|f'(t)|\vec{k}\rangle = \langle k|f(t)|\vec{k}\rangle,$$

equation (5) can be written in the form

$$\int_{t_0}^{t}\left\{\frac{df(\vec{k}, t')}{dt'} - \frac{i}{\hbar}\langle\vec{k}|[f(t'),\, H_F]|\vec{k}\rangle - \left[\frac{\partial f(\vec{k}, t_0)}{\partial t}\right]_S\right\} dt' = 0$$

When the expression under the integral equals zero, the equation is always satisfied. This gives for the steady state

$$\left[\frac{\partial f(\vec{k})}{\partial t}\right]_F + \left[\frac{\partial f(\vec{k})}{\partial t}\right]_S = 0.$$

in which

$$\left[\frac{\partial f(\vec{k})}{\partial t}\right]_F = \frac{i}{\hbar}\langle\vec{k}|[f, H_F]|\vec{k}\rangle = -\frac{ie\vec{E}}{\hbar}\cdot\langle\vec{k}|[f, \vec{r}]|\vec{k}\rangle = -\frac{e}{\hbar}\vec{E}\cdot\nabla_{\vec{k}} f(\vec{k}).$$

This proves that the Boltzmann equation can be obtained in the approximation in which the drift term has its origin directly in the expression $\frac{i}{\hbar}\int_{t_0}^{t}[\varrho'(t'),$ $H_F'(t')]\, dt'$, but the collision term is obtained from the term $\frac{i}{\hbar}\int_{t_0}^{t}[\varrho'(t'),\, H_i'(t')]dt'$ when $\varrho'(t')$ has been substituted by the expression $\varrho'(t_0) + \varrho_1(t_0, t')$. However precise the drift term is, a certain inaccuracy remains in the Boltzmann equation due to the approximate character of the collision term.

CORRECTION TERMS

First of all we are interested in the change of situation after $\varrho'(t')$ has been approximated with accuracy to the first order in H_F'. This approximation gives rise to the contribution which is derived from the following expression

$$\Delta\varrho'(t) = \left(\frac{i}{\hbar}\right)^3 \int_0^{t-t_0} dt' \int_0^{t'} dt'' \int_0^{t''} dt''' \times [[[\varrho'(t_0), H_i'(t''' + t_0)], H_F'(t'' + t_0)], H_i'(t' + t_0)], \qquad (6)$$

which has not been included in the standard treatment. We shall assume that the contributions of terms cintaining non diagonal matrix elements of f can be neglected. This assumption can be substantiated by the fact that in standard transport theory these terms are proved to be of higher order in H_i than the diagonal ones [8].

If the following relations are utilized

$$\langle N_{\vec{\sigma}}|a_{\vec{\sigma}}^{+}|N_{\vec{\sigma}} - 1\rangle = \langle N_{\vec{\sigma}} - 1|a_{\vec{\sigma}}|N_{\vec{\sigma}}\rangle = \sqrt{N_{\vec{\sigma}}},$$

$$\langle N_{\vec{\sigma}}|a_{\vec{\sigma}}|N_{\vec{\sigma}} + 1\rangle = \langle N_{\vec{\sigma}} + 1|a_{\vec{\sigma}}^{+}|N_{\vec{\sigma}}\rangle = \sqrt{N_{\vec{\sigma}} + 1},$$

$$\langle\vec{k}|\exp(i\vec{\sigma}\cdot\vec{r})|\vec{k}'\rangle = \delta_{\vec{k},\vec{k}'+\vec{\sigma}},$$

$$\langle\vec{k}|\exp(-i\vec{\sigma}\cdot\vec{r})|\vec{k}'\rangle = \delta_{\vec{k},\vec{k}'+\vec{\sigma}}$$

and the notation

$$A = \frac{1}{\hbar}\,[\varepsilon(\vec{k}+\vec{\sigma}) - \varepsilon(\vec{k}) - \hbar\omega_{\vec{\sigma}}],$$

$$B = \frac{1}{\hbar}\,[\varepsilon(\vec{k}-\vec{\sigma}) - \varepsilon(\vec{k}) + \hbar\omega_{\vec{\sigma}}]$$

is introduced, the appearance of the following contribution

$$\begin{aligned}\langle\vec{k}|\Delta f|\vec{k}\rangle = \frac{6\pi(t-t_0)}{\hbar^3}\sum_{N,\vec{\sigma}} |V_{\vec{\sigma}}|^2 \Big\{ & -N_{\vec{\sigma}}\langle\vec{k}|f(t_0)|\vec{k}\rangle P(N)\langle\vec{k}+\vec{\sigma}|H_F|\vec{k}+\vec{\sigma}\rangle\frac{\mathscr{D}(A)}{A} - \\ & -(N_{\vec{\sigma}}+1)\langle\vec{k}|f(t_0)|\vec{k}\rangle P(N)\,\langle\vec{k}-\vec{\sigma}|H_F|\vec{k}-\vec{\sigma}\rangle\frac{\mathscr{D}(B)}{B} + \\ & + N_{\vec{\sigma}}\,\langle\vec{k}+\vec{\sigma}|f(t_0)|\vec{k}+\vec{\sigma}\rangle\, P(\ldots N_{\vec{\sigma}}-1\ldots)\langle\vec{k}+\vec{\sigma}|H_F|\vec{k}+\vec{\sigma}\rangle\frac{\mathscr{D}(A)}{A} + \\ & + (N_{\vec{\sigma}}+1)\langle k-\vec{\sigma}|f(t_0)|\vec{k}-\vec{\sigma}\rangle\, P(\ldots N_{\vec{\sigma}}+1\ldots)\,\langle\vec{k}-\vec{\sigma}|H_F|\vec{k}-\vec{\sigma}\rangle\frac{\mathscr{D}(B)}{B} - \\ & - N_{\vec{\sigma}}\langle\vec{k}+\vec{\sigma}|f(t_0)|\vec{k}+\vec{\sigma}\rangle\, P(\ldots N_{\vec{\sigma}}-1\ldots)\langle\vec{k}|H_F|\vec{k}\rangle\frac{\mathscr{D}(A)}{A} - \\ & - (N_{\vec{\sigma}}+1)\langle\vec{k}-\vec{\sigma}|f(t_0)|\vec{k}-\vec{\sigma}\rangle\, P(\ldots N_{\vec{\sigma}}+1\ldots)\,\langle\vec{k}|H_F|\vec{k}\rangle\frac{\mathscr{D}(B)}{B} + \\ & + N_{\vec{\sigma}}\,\langle\vec{k}|f(t_0)|\vec{k}\rangle\, P(N)\langle\vec{k}|H_F|\vec{k}\rangle\frac{\mathscr{D}(A)}{A} + \\ & + (N_{\vec{\sigma}}+1)\,\langle\vec{k}|f(t_0)|\vec{k}\rangle\, P(N)\,\langle\vec{k}|H_F|\vec{k}\rangle\frac{\mathscr{D}(B)}{B}\Big\},\end{aligned}$$

to the expression $\langle\vec{k}|f(t) - f(t_0)|\vec{k}\rangle$ can be easily proved. By $\mathscr{D}(A)$ the expressions of the type $[\pi A^2(t-t_0)]^{-1}\{1 - \cos[A(t-t_0)]\}$ and type $(\pi A)^{-1}\sin[A(t-t_0)]$ are denoted. These expressions for $t - t_0$ large enough lose their dependence on $t - t_0$ and in integrals of smooth functions of A act as $\delta(A)$. Since the diagonal matrix elements from H_F do not depend on $\vec{k}$, for Δf we have $\langle\vec{k}|\Delta f|\vec{k}\rangle = 0$. This is the reason why the expression (6) need not be considered when deriving the Boltzmann equation.

If in $\varrho'(t')$ from equation (4) also the second order term is included a contribution appears with its origin in

$$\Delta'\varrho' = \left(\frac{i}{\hbar}\right)^4 \int_0^{t-t_0} dt' \int_0^{t'} dt'' \int_0^{t''} dt''' \int_0^{t'''} d\bar{t}\, \times$$

$$\times\,[[[[\varrho'(t_0), H_i'(\bar{t}+t_0)], H_F'(t'''+t_0)], H_F'(t''+t_0)], H_i'(t'+t_0)].$$

Due to the occurence of the four-fold time integral from 16 terms represented by the above written commutator the derivation of the contribution would be rather tedious. We shall only mention some properties of this contribution when non-diagonal matrix elements of f are again neglected. Since expressions of the form

$$\gamma(\vec{k}, \vec{\sigma}, \vec{k}')\,[\langle\vec{k}|H_F|\vec{k}'\rangle\langle\vec{k}'|H_F|\vec{k}\rangle - \langle\vec{k}\pm\vec{\sigma}|H_F|\vec{k}'\rangle\langle\vec{k}'|H_F|\vec{k}\pm\vec{\sigma}\rangle]$$

appear instead of

$$\alpha(\vec{k}, \vec{\sigma})\,[\langle\vec{k}|H_F|\vec{k}\rangle - \langle\vec{k}\pm\vec{\sigma}|H_F|\vec{k}\pm\vec{\sigma}\rangle]$$

occuring in the lower approximation, the contribution is no more zero. For $t - t_0$ large enough its dependence on time is again linear, i. e. the realization of the steady state is admitted. The latter result gives rise to doubt as to the validity of the assumptions used since the treatment which follows proves the unrealizability of the steady state in the model considered.

STEADY CURRENT PROBLEM

Strong fields require the iteration process in which the expansion of the density matrix only in powers of electron-phonon interaction occurs. If a new density operator is introduced by means of the relation

$$\varrho''(t) = \exp\left\{\frac{i}{\hbar}H_L t\right\}\exp\left\{\frac{i}{\hbar}(H_0 + H_F)t\right\}\times$$

$$\times\,\varrho(t)\exp\left\{-\frac{i}{\hbar}(H_0 + H_F)t\right\}\exp\left\{-\frac{i}{\hbar}H_L t\right\}$$

and if by a similar relation H_i'' is defined, the density matrix equation takes the form

$$\frac{\partial\varrho''(t)}{\partial t} = \frac{i}{\hbar}\,[\varrho''(t), H_i''(t)]. \qquad (7)$$

If $\varrho''(t)$ is expressed in form of series in powers of H_i''

$$\varrho''(t) = \varrho''(t_0) + \varrho_1''(t_0, t) + \varrho_2''(t_0, t) + \ldots$$

in which ϱ_1'' is of the first order and ϱ_2'' of the second order in H_i'', the following expressions can be obtained from the equation (7)

$$\varrho_1''(t_0, t) = \frac{i}{\hbar} \int_0^{t-t_0} [\varrho''(t_0), H_i''(t_0 + t')]\, dt',$$

$$\varrho_2''(t_0, t) = \left(\frac{i}{\hbar}\right)^2 \int_0^{t-t_0} dt' \int_0^{t'} dt'' \,[[\varrho''(t_0), H_i''(t_0 + t'')], H_i''(t_0 + t')].$$

Then the distribution function with an accuracy to second order terms in H_i'' can be written in the form

$$\langle\vec{k}|f''(t)|\vec{k}\rangle = \langle\vec{k}|f''(t_0)|\vec{k}\rangle + \langle\vec{k}|f_1''(t_0, t)|\vec{k}\rangle + \langle\vec{k}|f_2''(t_0, t)|\vec{k}\rangle,$$

with

$$\langle\vec{k}|f_1''(t_0, t)|\vec{k}\rangle = \sum_N \langle\vec{k}N|\varrho_1''(t_0, t)|\vec{k}N\rangle$$

and $\langle\vec{k}|f_2''(t_0, t)|\vec{k}\rangle$ similarly defined. Since the diagonal in $|N\rangle$ matrix elements of H_i'' equals zero, $\langle\vec{k}|f_1''(t_0, t)|\vec{k}\rangle$ equals zero too and only $\langle\vec{k}|f_2''(t_0, t)|\vec{k}\rangle$ is to be investigated. In this term the matrix elements of H_i'' must be expressed by means of the matrix elements of H_i. This treatment shows a difference in comparison with the treatment of paper [9] leading to the Boltzmann equation. Instead of $\exp\left\{\frac{i}{\hbar} H_0 t\right\}$ the term $\exp\left\{\frac{i}{\hbar}(H_0 + H_F)t\right\}$ appears, in which H_0 does not commute with H_F. It is known (e. g. [13]) that the operator function $\exp[a\mathscr{A} + \mathscr{B})]$ can be expressed in the form $\exp(a\mathscr{B})\mathscr{K}(a) \exp(a\mathscr{A})$ with $\mathscr{K}(a)$ satisfying the differential equation

$$\frac{\partial \mathscr{K}}{\partial a} = \exp(-a\mathscr{B})\mathscr{A}\exp(a\mathscr{B})\mathscr{K} - \mathscr{K}\mathscr{A}.$$

Since

$$\exp(-a\mathscr{B})\mathscr{A}\exp(a\mathscr{B}) \parallel \sum_{n=0}^{\infty} \frac{(-a)^n}{n!} \underbrace{[\mathscr{B}[\mathscr{B}\ldots[\mathscr{B}}_{n},\mathscr{A}]\ldots]$$

the solution of the equation (8) is in general very complicated. As far as $\mathscr{A} = H_0$ and $\mathscr{B} = H_F$ are chosen the problem is simplified, since from commutators of the type $[\ldots[\mathscr{A}, \mathscr{B}], \mathscr{B}]\ldots\mathscr{B}]$ only $[\mathscr{A}, \mathscr{B}]$ and $[[\mathscr{A}, \mathscr{B}], \mathscr{B}]$ differ

from zero and $[[\mathscr{A}, \mathscr{B}], \mathscr{B}] = c_0$ represents the c-number. Equation (8) then takes the form

$$\frac{\partial \mathscr{K}}{\partial a} = [\mathscr{A}, \mathscr{K}] + a[\mathscr{A}, \mathscr{B}]\mathscr{K} + \frac{a^2}{2} c_0 \mathscr{K},$$

which has the following solution

$$\mathscr{K} = \exp(a\mathscr{A})\mathscr{L}\exp(-a\mathscr{A})\exp\left(\frac{a^3 c_0}{6}\right)$$

with

$$\mathscr{L} = \exp\left\{\int_0^a \mu \exp(-\mathscr{A}\mu)[\mathscr{A}, \mathscr{B}]\exp(\mathscr{A}\mu)\,d\mu\right\}.$$

In the effective mass approximation $[\mathscr{A}, \mathscr{B}] \sim p_x$ commutes with $\mathscr{A}$ and the considered operator function can be written in the form

$$\exp\left\{\frac{i}{\hbar}(H_0 + H_F)t\right\} = \exp\left\{-\frac{i c_0 t^3}{6\hbar^3}\right\}\exp\left\{\frac{i}{\hbar} H_F t\right\} \times$$

$$\times \exp\left\{\frac{i}{\hbar} H_0 t\right\}\exp\left\{-\frac{i e E p_x t^2}{2\, m\hbar}\right\}.$$

Then the term $\exp\left\{-\frac{i c_0 t^3}{6\hbar^3}\right\}$ does not appear in the result due to the fact that it is eliminated by the complex conjugated term.

The treatment similar to that of the paper [9] leads to the expression for $\langle\vec{k}|f_2''(t_0, t)|\vec{k}\rangle$ in which the time integrals of the form

$$\int_0^{t-t_0} dt' \int_0^{t'} dt'' \exp\{i[\omega(t'' + t_0) + \Omega\,(t'' + t_0)^2]\} \times \exp\{-i[\omega(t' + t_0) + \Omega\,(t' + t_0)^2]\}$$

occur.

We could hardly succeed in their direct evaluation and therefore we choose the approximation used in the previous caption, in which $\exp\left\{\pm i \frac{eE\sigma_x}{2m\hbar}(t + t_0)^2\right\}$ is to be expressed with an accuracy to second order term in powers of E. Doing so we can find that for $t - t_0$ large enough the zeroth order term in E gives the contribution of the form $(t - t_0)\left[\frac{\partial f}{\partial t}\right]_S$, in which $\left[\frac{\partial f}{\partial t}\right]_S$

stands for the standard collision term. In the contributions from the higher order terms the higher powers of time appear; this makes the realization of the steady state impossible. Thus the model used is not suitable for strong electric fields.

GENERALIZED THEORY

When looking for the suitable model we must take into consideration the interaction of the system with its environment. When higher order terms in the intensity of the applied field are included the effect of heating the crystal appears, which in the experiment is compensated by cooling the sample in which the steady current is realized. This effect was not taken into account in the theory mentioned above.

Lax in paper [10], dealing with the direct expression for the density matrix in a weak electric field, has drawn attention to the fact that the interaction with the environment can be taken into account by adding the term $\frac{\varrho - \varrho_0}{\tau}$ into the density matrix equation. In weak electric fields already weak interaction ($\tau \to \infty$) of this type makes the realization of the steady current possible. In the strong field the interaction with τ definite must be expected to maintain the steady current. Since Kubo's formal solution of the density matrix equation (used also in the paper [10]) could be hardly used in case of a uniform electric field a different treatment is to be chosen.

We shall assume that the interaction with the environment is responsible for the fact that the system from its initial equilibrium (corresponding to the absence of the electric field) can later attain the steady state. We shall thus look for the solution of the equation

$$\frac{\partial \varrho}{\partial t} = \frac{i}{\hbar}[\varrho, H_0 + H_F + H_i + H_L] - \frac{\varrho - \varrho_0}{\tau}, \tag{10}$$

which satisfies the initial condition $\varrho(-\infty) = \varrho_0$. For the sake of simplicity we shall assume that ϱ_0 is Boltzmann equilibrium density matrix, i. e. we shall investigate semiconductors in which low concentration of conduction electrons permits the mentioned assumption.

The formal solution of the equation (10) can be expected in the form

$$\varrho(t) = \varrho_0 + \frac{i}{\hbar}\int_{\infty}^{t} \exp\left(\frac{t' - t}{\tau}\right) \exp\left\{\frac{i}{\hbar}(t' - t)(H_0 + H_L + H_F)\right\} \times$$

$$\times [\varrho(t'), H_i] \exp\left\{-\frac{i}{\hbar}(t' - t)(H_0 + H_L + H_F)\right\} dt' + \tilde{\varrho}(t).$$

Then $\varrho(t)$ must satisfy the equation

$$\frac{\partial \tilde{\varrho}(t)}{\partial t} = \frac{i}{\hbar}[\varrho_0, H_0 + H_L + H_F] + \frac{i}{\hbar}[\tilde{\varrho}(t), H_0 + H_L + H_F] - \frac{\tilde{\varrho}(t)}{\tau}$$

with the initial condition $\tilde{\varrho}(-\infty) = 0$. It can be easily proved that $\tilde{\varrho}(t)$ of the following form is available

$$\tilde{\varrho}(t) = \frac{i}{\hbar}\int_{-\infty}^{t} \exp\left(\frac{t' - t}{\tau}\right) \exp\left\{\frac{i}{\hbar}(t' - t)(H_0 + H_L + H_F)\right\} \times$$

$$\times [\varrho_0, H_0 + H_L + H_F] \exp\left\{-\frac{i}{\hbar}(t' - t)(H_0 + H_L + H_F)\right\} dt'.$$

After $\exp\left\{\frac{i}{\hbar}(H_0 + H_F)t\right\}$ is expressed by means of (9) and the relation

$$\left\langle \vec{k}\left| \exp\left\{\pm \frac{i}{\hbar} H_F t\right\}\right| \vec{k}'\right\rangle = \left\langle \vec{k}\left| \exp\left\{\pm \frac{i}{\hbar} eExt\right\}\right| \vec{k}'\right\rangle = \delta_{k_y, k_y'} \delta_{k_z, k_z'} \delta_{k_x, k_x' \mp \frac{eEt}{\hbar}}$$

is used, we can write

$$\langle \vec{k}N|\varrho(t)|\vec{k}N\rangle = \langle \vec{k}N|\varrho_0|\vec{k}N\rangle + \frac{i}{\hbar}\int_{-\infty}^{t} \exp\left(\frac{t' - t}{\tau}\right) \times$$

$$\times \langle \vec{k}'N|[\varrho(t'), H_i]|\vec{k}'N\rangle\, dt' + \frac{i}{\hbar}\int_{-\infty}^{t} \exp\left(\frac{t' - t}{\tau}\right) \times$$

$$\times \langle \vec{k}'N|[\varrho_0, H_0 + H_L + H_F]|\vec{k}'N\rangle\, dt', \tag{11}$$

with $\vec{k}' = \left(k_x + \frac{eE(t' - t)}{\hbar}, k_y, k_z\right)$. Since $e\hbar^{-1}E(t' - t)$ does not contain space coordinates the following relations are valid

$$\langle \vec{k}'N|[\varrho(t'), H_i]|\vec{k}'N\rangle = \langle \vec{k}N|[\varrho(t'), H_i]|\vec{k}N\rangle$$

$$\langle \vec{k}'N|[\varrho_0, H_0 + H_L + H_F]|\vec{k}'N\rangle = \langle \vec{k}N|[\varrho_0, H_0 + H_L + H_F]|\vec{k}N\rangle$$

and the equation (11) gives

$$\langle \vec{k}N|\varrho(t)|\vec{k}N\rangle = \langle \vec{k}N|\varrho_0|\vec{k}N\rangle + \frac{i}{\hbar}\int_{-\infty}^{t} \exp\left(\frac{t'-t}{\tau}\right) \times$$

$$\times \langle \vec{k}N|[\varrho(t'), H_i] + [\varrho_0, H_0 + H_L + H_F]|\vec{k}N\rangle \, dt'. \qquad (12)$$

It is obvious that the diagonal matrix elements of the first order terms in H_i give zero. In order to ensure the accuracy to second order terms in H_i we have to replace $\varrho(t')$ in (12) by the first order term in H_i which will be denoted by $\varrho(1, t')$. From the property of H_i it is obvious that the following non-diagonal matrix elements of $\varrho(1, t')$ are to be expressed

$$\langle \vec{k}N|\varrho(1, t')|\vec{k}_1N_1\rangle \equiv \langle \vec{k}N|\varrho(1, t')|\vec{k} \pm \vec{\sigma}, \ldots N_{\vec{\sigma}} \mp 1 \ldots\rangle.$$

Since

$$\langle \vec{k}N|\varrho(1, t)|\vec{k}_1N_1\rangle = \langle \vec{k}N|\varrho_0(1)|\vec{k}_1N_1\rangle + \frac{i}{\hbar}\int_{-\infty}^{t} \exp\left(\frac{t'-t}{\tau}\right) \times$$

$$\times \left\langle \vec{k}N \middle| \exp\left\{\frac{i}{\hbar} H_F(t'-t)\right\} \middle| \vec{k}'N \right\rangle \left\langle \vec{k}'N \middle| \exp\left\{\frac{i}{\hbar}(H_0 + H_L)(t'-t)\right\} \middle| \vec{k}'N \right\rangle \times$$

$$\times \left\langle \vec{k}'N \middle| \exp\left\{-\frac{ieEp_x}{2m\hbar}(t'-t)^2\right\} \middle| \vec{k}'N \right\rangle \langle \vec{k}'N|([\varrho_0(1), H_0 + H_L + H_F] +$$

$$+ [\varrho_0(0), H_i])|\vec{k}_1'N_1\rangle \left\langle \vec{k}_1'N_1 \middle| \exp\left\{\frac{ieEp_x}{2m\hbar}(t'-t)^2\right\} \middle| \vec{k}_1'N_1 \right\rangle \times$$

$$\times \left\langle \vec{k}_1'N_1 \middle| \exp\left\{-\frac{i}{\hbar}(H_0 + H_L)(t'-t)\right\} \middle| \vec{k}_1'N_1 \right\rangle \times$$

$$\times \left\langle \vec{k}_1'N_1 \middle| \exp\left\{-\frac{i}{\hbar} H_F(t'-t)\right\} \middle| \vec{k}_1N_1 \right\rangle dt'$$

with

$$\vec{k}' = \left(k_x + \frac{eE(t'-t)}{\hbar}; k_y; k_z\right),$$

$$\vec{k}_1' = \left(k_{1x} + \frac{eE(t'-t)}{\hbar}; k_{1y}; k_{1z}\right),$$

the following result can be obtained

$$\langle \vec{k}N|\varrho(1, t)|\vec{k}_1N_1\rangle = \langle \vec{k}N|\varrho_0(1)|\vec{k}_1N_1\rangle + \frac{i}{\hbar}\int_{-\infty}^{t} \exp\left(\frac{t'-t}{\tau}\right) \times$$

$$\times \exp\left\{\frac{i}{\hbar}[\varepsilon(\vec{k}, N) - \varepsilon(\vec{k}_1, N_1)](t'-t)\right\} \exp\left\{\frac{ieE}{2m\hbar}(k_{1x} - k_x)(t'-t)^2\right\} \times$$

$$\times \langle \vec{k}N|[\varrho_0(1), H_0 + H_L + H_F] + [\varrho_0(0), H_i]|\vec{k}_1N_1\rangle \, dt',$$

in which the zeroth and first order terms of ϱ_0 in powers of H_i are denoted by $\varrho_0(0)$ and $\varrho_0(1)$ respectively and the eigenvalues af $H_O + H_L$ are denoted by $\mathcal{E}(\vec{k}, N)$. When the following fact is utilized

$$\langle \vec{k}N|[\varrho_0(0), H_i] + [\varrho_0(1), H_0 + H_L]|\vec{k} \pm \vec{\sigma}, \ldots N_{\vec{\sigma}} \mp 1 \ldots\rangle = 0,$$

equation (12) gives

$$\langle \vec{k}N|\varrho|\vec{k}N\rangle = \langle \vec{k}N|\varrho_0|\vec{k}N\rangle + \frac{i}{\hbar}\int_{-\infty}^{t} \exp\left(\frac{t'-t}{\tau}\right) \times$$

$$\times \langle \vec{k}N|[\varrho_0(0), H_F] + [\varrho_0(1), H_i] + [\varrho_0(2), H_0 + H_L + H_F]|\vec{k}N\rangle \, dt' -$$

$$- \frac{1}{\hbar^2}\sum_{\vec{k}_1N_1}\int_{-\infty}^{t} dt' \exp\left(\frac{t'-t}{\tau}\right)\int_{-\infty}^{t} dt'' \exp\left(\frac{t''-t'}{\tau}\right) \times$$

$$\times \Bigg[\exp\left\{\frac{i}{\hbar}[\varepsilon(\vec{k}, N) - \varepsilon(\vec{k}_1, N_1)](t''-t')\right\} \exp\left\{\frac{ieE}{2m\hbar}(k_{1x} - k_x)(t''-t')^2\right\} \times$$

$$\times \langle \vec{k}N|[\varrho_0(1), H_F]|\vec{k}_1N_1\rangle\langle \vec{k}_1N_1|H_i|\vec{k}N\rangle - \langle \vec{k}N|H_i|\vec{k}_1N_1\rangle\langle \vec{k}_1N_1|[\varrho_0(1), H_F]|\vec{k}N\rangle \times$$

$$\times \exp\left\{\frac{i}{\hbar}[\varepsilon(\vec{k}_1, N_1) - \varepsilon(\vec{k}, N)](t''-t')\right\} \exp\left\{\frac{ieE}{2m\hbar}(k_x - k_{1x})(t''-t')^2\right\}\Bigg].$$

The second term from the right side of this equation obviously gives

$$\frac{i}{\hbar}\int_{-\infty}^{t} \exp\left(\frac{t'-t}{\tau}\right) \langle \vec{k}N|[\varrho_0(0) + \varrho_0(2), H_F]|\vec{k}N\rangle \, dt'.$$

To express the third term we can utilize the following relations

$$\langle \vec{k}N|[\varrho_0, H_F]|\vec{k}_2N_2\rangle = ieE\left(\frac{\partial}{\partial k_x} + \frac{\partial}{\partial k_{2x}}\right)\langle \vec{k}N|\varrho_0|\vec{k}_2N_2\rangle,$$

$$\left(\frac{\partial}{\partial k_x} + \frac{\partial}{\partial (k_x \pm \sigma_x)}\right) \langle \vec{k}N|H_i|\vec{k} \pm \vec{\sigma}, \ldots N_{\vec{\sigma}} \mp 1 \ldots\rangle = 0$$

$$\langle \vec{k}N|H_i|\vec{k} \pm \vec{\sigma}, \ldots N_{\vec{\sigma}} \mp 1 \ldots\rangle\langle \vec{k} \pm \vec{\sigma}, \ldots N_{\vec{\sigma}} \mp 1 \ldots|H_i|\vec{k}N\rangle = |V_{\vec{\sigma}}|^2 \mathcal{N}(\pm \vec{\sigma}),$$

$$\mathcal{N}(+\vec{\sigma}) = N_{\vec{\sigma}}, \; \mathcal{N}(-\vec{\sigma}) = N_{\vec{\sigma}} + 1,$$

$$\langle \vec{k}N|\varrho_0(1)|\vec{k}_1 N_1\rangle = [\varrho_0(\varepsilon(\vec{k}, N)) - \varrho_0(\varepsilon(\vec{k}_1, N_1))][\varepsilon(\vec{k}, N) - \varepsilon(\vec{k}_1, N_1)]^{-1} \langle \vec{k}N|H_i|\vec{k}_1 N_1\rangle,$$

$$\langle \vec{k}N|\varrho_0(2)|\vec{k}N\rangle = \sum_{\vec{\sigma}} |V_{\vec{\sigma}}|^2 \mathcal{N}(\pm \vec{\sigma}) \left\{\frac{1}{k_0 T} \varrho_0(\varepsilon(\vec{k}, N))[\varepsilon(\vec{k} \pm \vec{\sigma}, \ldots N_{\vec{\sigma}} \mp 1 \ldots) - \varepsilon(\vec{k}, N)]^{-1} + [\varrho_0(\varepsilon(\vec{k} \pm \vec{\sigma}, \ldots N_{\vec{\sigma}} \mp 1 \ldots)) - \varrho_0(\varepsilon(\vec{k}, N))][\varepsilon(\vec{k} \pm \vec{\sigma}, \ldots N_{\vec{\sigma}} \mp 1 \ldots) - \varepsilon(\vec{k}, N)]^{-2}\right\}.$$

Then we obtain

$$\langle \vec{k}N|\varrho|\vec{k}N\rangle = \langle \vec{k}N|\varrho_0|\vec{k}N\rangle - \frac{e\tau E}{\hbar}\left\{\frac{\partial \varrho_0(\varepsilon(\vec{k}, N))}{\partial k_x} + \sum_{\vec{\sigma}} |V_{\vec{\sigma}}|^2 \times \right.$$

$$\times \mathcal{N}(\pm \vec{\sigma}) \frac{\partial}{\partial k_x}\left[\frac{1}{k_0 T} \varrho_0(\varepsilon(\vec{k}, N))\{\varepsilon(\vec{k} \pm \vec{\sigma}, \ldots N_{\vec{\sigma}} \mp 1 \ldots) - \varepsilon(\vec{k}, N)\}^{-1} + \right.$$

$$+ \{\varrho_0(\varepsilon(\vec{k} \pm \vec{\sigma}, \ldots N_{\vec{\sigma}} \mp 1 \ldots)) - \varrho_0(\varepsilon(\vec{k}, N))\}\{\varepsilon(\vec{k} \pm \vec{\sigma}, \ldots N_{\vec{\sigma}} \mp 1 \ldots) -$$

$$\left.\left. - \varepsilon(\vec{k}, N)\}^{-2}\right]\right\} - \frac{ieE}{\hbar^2} \sum_{\vec{\sigma}} |V_{\vec{\sigma}}|^2 \mathcal{N}(\pm \vec{\sigma}) \int_{-\infty}^{t} dt' \exp\left(\frac{t' - t}{\tau}\right) \int_{-\infty}^{t'} dt'' \exp\left(\frac{t'' - t'}{\tau}\right) \times$$

$$\times \left[\exp\left\{\frac{i}{\hbar}(\varepsilon(\vec{k}, N) - \varepsilon(\vec{k} \pm \vec{\sigma}, \ldots N_{\vec{\sigma}} \mp 1 \ldots))(t'' - t')\right\} \exp\left\{\pm \frac{ieE\sigma_x}{2m\hbar}(t'' - t')^2\right\} - \right.$$

$$\left. - \exp\left\{\frac{i}{\hbar}(\varepsilon(\vec{k} \pm \vec{\sigma}, \ldots N_{\vec{\sigma}} \mp 1 \ldots) - \varepsilon(\vec{k}, N))(t'' - t')\right\} \exp\left\{\mp \frac{ieE\sigma_x}{2m\hbar}(t'' - t')^2\right\}\right] \times$$

$$\times \left(\frac{\partial}{\partial k_x} + \frac{\partial}{\partial (k_x \pm \sigma_x)}\right)([\varrho_0(\varepsilon(\vec{k} \pm \vec{\sigma}, \ldots N_{\vec{\sigma}} \mp 1 \ldots)) - \varrho_0(\varepsilon(\vec{k}, N))]\,[\varepsilon(\vec{k} \pm \vec{\sigma}, \ldots N_{\vec{\sigma}} \mp 1 \ldots) - \varepsilon(\vec{k}, N)]^{-1}).$$

After the new variable $\bar{t} = t'' - t'$ is introduced, the following relation can be used when computing the time integrals

$$\int_{-\infty}^{0} \exp\left(\frac{\bar{t}}{\tau}\right) \exp(i\omega\bar{t}) \exp(i\Omega\bar{t}^2)\, d\bar{t} = \left[\sqrt{\frac{\pi}{2\Omega}}\{C(\sqrt{\Omega}\,\bar{t}) + iS(\sqrt{\Omega\bar{t}})\} \times \right.$$

$$\left. \times \exp\left(\frac{\bar{t}}{\tau}\right) \exp(i\omega\bar{t})\right]_{-\infty}^{0} - \sqrt{\frac{\pi}{2\Omega}}\left(i\omega + \frac{1}{\tau}\right) \int_{-\infty}^{0} \exp\left(\frac{\bar{t}}{\tau}\right) \exp(i\omega\bar{t}) \times$$

$$\times \{C(\sqrt{\Omega}\,\bar{t}) + iS(\sqrt{\Omega}\,\bar{t})\}\, d\bar{t} = -\sqrt{\frac{\pi}{2\Omega}}\left(i\omega + \frac{1}{\tau}\right) \int_{-\infty}^{0} \exp\left(\frac{\bar{t}}{\tau}\right) \exp(i\omega\bar{t}) \times$$

$$\times \{C(\sqrt{\Omega}\,\bar{t}) + iS(\sqrt{\Omega\bar{t}})\}\, d\bar{t}.$$

Due to the occurence of the damping factor $\exp(\bar{t}|\tau)$ we shall replace the function $\sqrt{\frac{\pi}{2\Omega}}\{C(\sqrt{\Omega\bar{t}}) + iS(\sqrt{\Omega\bar{t}})\}$ by the approximate expression $\bar{t} + \frac{i}{3}\Omega\bar{t}^3$. When the average over the lattice variables is taken and the time integration is performed we obtain for the part of the distribution function which contributes to the current the following expression

$$\langle \vec{k}|\Delta f|\vec{k}\rangle = -\frac{eE\tau Z_0}{\hbar}\left\{\frac{\partial}{\partial k_x} \exp\left\{-\frac{1}{k_0 T}\varepsilon(\vec{k})\right\} + \sum_{\vec{\sigma}} |V_{\vec{\sigma}}|^2 \bar{N}_{\vec{\sigma}} \times \right.$$

$$\times \frac{\partial}{\partial k_x}\left(\left[\exp\left\{-\frac{1}{k_0 T}(\varepsilon(\vec{k} + \vec{\sigma}) - \hbar\omega_{\vec{\sigma}})\right\} - \exp\left\{-\frac{1}{k_0 T}\varepsilon(\vec{k})\right\}\right][\varepsilon(\vec{k} + \vec{\sigma}) - \right.$$

$$\left. - \varepsilon(\vec{k}) - \hbar\omega_{\vec{\sigma}}]^{-2} + \frac{1}{k_0 T}\exp\left\{-\frac{1}{k_0 T}\varepsilon(\vec{k})\right\}[\varepsilon(\vec{k}) - \varepsilon(\vec{k} + \vec{\sigma}) + \hbar\omega_{\vec{\sigma}}]^{-1}\right) +$$

$$+ |V_{\vec{\sigma}}|^2(\bar{N}_{\vec{\sigma}} + 1) \frac{\partial}{\partial k_x}\left(\left[\exp\left\{-\frac{1}{k_0 T}(\varepsilon(\vec{k} - \vec{\sigma}) + \hbar\omega_{\vec{\sigma}})\right\} - \right.\right.$$

$$\left. - \exp\left\{-\frac{1}{k_0 T}\varepsilon(\vec{k})\right\}\right][\varepsilon(\vec{k} - \vec{\sigma}) - \varepsilon(\vec{k}) + \hbar\omega_{\vec{\sigma}}]^{-2} +$$

$$\left. \frac{1}{k_0 T}\exp\left\{-\frac{1}{k_0 T}\varepsilon(\vec{k})\right\}[\varepsilon(\vec{k}) - \varepsilon(\vec{k} - \vec{\sigma}) - \hbar\omega_{\vec{\sigma}}]^{-1}\right) + \frac{4}{\hbar}|V_{\vec{\sigma}}|^2 \bar{N}_{\vec{\sigma}} \times$$

$$\times \left[\omega_{\vec{k}+\vec{\sigma}}\left(\frac{1}{\tau^2} + \omega_{\vec{k}+\vec{\sigma}}^2\right)^{-1} + \frac{eE\sigma_x}{2m\hbar}\left(\frac{3\omega_{\vec{k}+\vec{\sigma}}^2}{\tau} - \frac{1}{\tau^3}\right)\left(\left[\frac{3\omega_{\vec{k}+\vec{\sigma}}^2}{\tau} - \omega_{\vec{k}+\vec{\sigma}}^3\right]^2 + \right.\right.$$

$$+\left[\frac{1}{\tau^3}-\frac{3\omega^2_{\vec{k}+\vec{\sigma}}}{\tau}\right]^2\Bigg)^{-1}\Bigg]\left(\frac{\partial}{\partial k_x}+\frac{\partial}{\partial(k_x+\sigma_x)}\right)\Bigg(\Bigg[\exp\left\{-\frac{1}{k_0T}(\varepsilon(\vec{k}+\vec{\sigma})-\hbar\omega_{\vec{\sigma}})\right\}-$$

$$-\exp\left\{-\frac{1}{k_0T}\varepsilon(\vec{k})\right\}\Bigg]\Big[\varepsilon(\vec{k}+\vec{\sigma})-\hbar\omega_{\vec{\sigma}}-\varepsilon(\vec{k})\Big]^{-1}\Bigg)+$$

$$+\frac{4}{\hbar}|V_{\vec{\sigma}}|^2(\bar{N}_{\vec{\sigma}}+1)\left[\omega_{\vec{k}-\vec{\sigma}}\left(\frac{1}{\tau^2}+\omega^3_{\vec{k}-\vec{\sigma}}\right)^{-1}-\frac{eE\sigma_x}{2m\hbar}\left(\frac{3\omega^2_{\vec{k}-\vec{\sigma}}}{\tau}-\frac{1}{\tau^3}\right)\left(\left[\frac{3\omega_{\vec{k}-\vec{\sigma}}}{\tau^2}-\omega^3_{\vec{k}-\vec{\sigma}}\right]^2+\right.\right.$$

$$+\left[\frac{1}{\tau^3}-\frac{3\omega^2_{\vec{k}-\vec{\sigma}}}{\tau}\right]^2\Bigg)^{-1}\Bigg]\left(\frac{\partial}{\partial k_x}+\frac{\partial}{\partial(k_x-\sigma_x)}\right)\Bigg(\Bigg[\exp\left\{-\frac{1}{k_0T}(\varepsilon(\vec{k}-\vec{\sigma})+\hbar\omega_{\vec{\sigma}})\right\}-$$

$$-\exp\left\{-\frac{1}{k_0T}\varepsilon(\vec{k})\right\}\Bigg]\Big[\varepsilon(\vec{k}-\vec{\sigma})+\hbar\omega_{\vec{\sigma}}-\varepsilon(\vec{k})\Big]^{-1}\Bigg)\Bigg\}, \qquad (13)$$

in which the terms of higher than second order in E are neglected and the following notation has been introduced

$$\omega_{\vec{k}+\vec{\sigma}}=\frac{1}{\hbar}[\varepsilon(\vec{k})-\varepsilon(\vec{k}+\vec{\sigma})+\hbar\omega_{\vec{\sigma}}],$$

$$\omega_{\vec{k}-\vec{\sigma}}=\frac{1}{\hbar}[\varepsilon(\vec{k})-\varepsilon(\vec{k}-\vec{\sigma})-\hbar\omega_{\vec{\sigma}}],$$

$$\bar{N}_{\vec{\sigma}}=\left[\sum_N\exp\left(-\frac{E_N}{k_0T}\right)\right]^{-1}\sum_N N_{\vec{\sigma}}\exp\left(-\frac{E_N}{k_0T}\right)=\left[\exp\left(\frac{\hbar\omega_{\vec{\sigma}}}{k_0T}\right)-1\right]^{-1},$$

$$Z_0^{-1}=Tr\left[\exp\left(-\frac{H_0}{k_0T}\right)\right].$$

When damping is not sufficient to make $\tilde{t}+\frac{i}{3}\Omega\tilde{t}^3$ a good approximation of $\sqrt{\frac{\pi}{2\Omega}}\{c(\sqrt{\Omega}\tilde{t})+iS(\sqrt{\Omega}\,\tilde{t})\}$, the higher order term in $\tilde{t}$ — namely $-\frac{1}{10}\Omega^2\tilde{t}^5$ — must be included. This leads to the appearance of the third order terms in E in the expression for $\langle\vec{k}|\Delta f|\vec{k}\rangle$. After $\sum_{\vec{\sigma}}\ldots$ is replaced by $V(2\pi)^{-3}\int\ldots d^3\sigma$ the direct expression for the current density can be obtained by substituting (13) into the expression

$$j_x=\frac{\hbar neV}{(2\pi)^3m}\int k_x\,\langle\vec{k}|\Delta f|\vec{k}\rangle\,d^3k$$

in which n stands for the electron concentration and V for the volume of the sample. The choice of τ must be performed so as to ensure the equivalence of the considered model with the experiment.

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