ON THE CALCULATION OF GEOELECTRIC RESISTIVITY ANOMALIES OF INFINITE CIRCULAR HALF-CYLINDERS

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In the papers [1], [2] formulae have been deduced for the calculation of the geoelectric resistivity anomalies for the case of the circular infinite cylinder embedded in infinite space and for the case of the circular coaxial half-cylinders embedded in infinite half-space, using a point source of steady electric current. The deduced formulae have a form of infinite sums of improper integrals which cannot be evaluated by known formulae for improper integrals of compound expressions of Bessel functions, but it is necessary to determine their numerical calculation, which is one of the purposes of this paper.

Let us have an infinite long circular half-cylinder the resistivity of which is ϱ_2 and the radius of which is $r_0 = 1$ and which is embedded in infinite homogeneous and isotropic half-space, the resistivity of which is denoted by ϱ_1 (fig. 1). We are to calculate numerically the sum of improper integrals for such a case where the souce electrode **A** is more removed than the potential electrode **M** and both electrodes lie on the straight line running through the origin of the cylindrical coordinate system and being perpendicular to the longitudinal axis of the half-cylinder. Then we have for the calculation of the potential from [2] the equation:

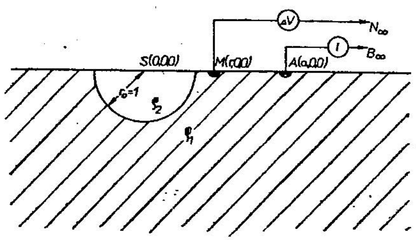

Fig. 1.

$$\mathscr{U}(r, \varphi, z) = \frac{J\varrho_1}{2\pi^2}\left\{\int_0^\infty \sum_{n=-\infty}^{\infty}\left[I_n(rt)K_n(at) + \right.\right.$$
$$\left.\left. + \frac{K_n(rt)K_n(at)I_n(t)I_n'(t)(\varrho_2 - \varrho_1)}{\varrho_1 K_n(t)I_n'(t) - \varrho_2 I_n(t)K_n'(t)}\right]\right\} \times \cos n\varphi \cos tz \, dt, \tag{1}$$

where $I_n(x) = i^{-n}J_n(ix)$; $K_n(x) = \frac{\pi}{2} i^{n+1}H_n^{(1)}(ix)$; $I_n'(x)$; $K_n'(x)$ are Bessel functions and their derivatives with respect to argument x, while r, φ, z; a, φ, z

are cylindrical coordinates of the potential or the source point respectively, J is the intensity of the source current. The first term in eq. (1) expresses the potential of the point source embedded in infinite half-space and it will be calculated by an elementary formula. The anomalous potential is expressed by the second term of equation (1) and for the chosen arrangement of the electrodes will be simplified into the form:

$$(2) \qquad \mathcal{U}^*(r, 0, 0) = \frac{J\varrho_1(\varrho_2 - \varrho_1)}{2\pi^2} \int_0^\infty \sum_{n=-\infty}^{\infty} \frac{K_n(rt)K_n(at)I_n(t)I'_n(t)}{\varrho_1 K_n(t)I'_n(t) - \varrho_2 I_n(t)K'_n(t)}\, dt,$$

which will be the subject of our study.

We may simplify the equation (2) by taking into consideration the formulae of [3]:

$$(3) \qquad I_{-n}(x) = I_n(x); \quad K_{-n}(x) = K_n(x); \quad n = 0, 1, 2, \ldots,$$

by means of which we have:

$$(4) \qquad \mathcal{U}^*(r, 0, 0) = \frac{J\varrho_1(\varrho_2 - \varrho_1)}{2\pi^2} \left[\int_0^\infty \frac{K_0(rt)K_0(at)I_0(t)I'_0(t)}{\varrho_1 K_0(t)I'_0(t) - \varrho_2 I_0(t)K'_0(t)}\, dt + \right.$$

$$\left. + 2 \int_0^\infty \sum_{n=1}^{\infty} \frac{K_n(rt)K_n(at)I_n(t)I'_n(t)}{\varrho_1 K_n(t)I'_n(t) - \varrho_2 I_n(t)K'_n(t)}\, dt \right].$$

The following parameters were chosen for the calculation of the anomalous potential: $r_0 = 1$; $r = 1{,}2$; $a = 2{,}4$; $3{,}6$; $4{,}8$; $6{,}0$; $\varrho_1 = 1\ \Omega$m; $\varrho_2 = 20\ \Omega$m; $50\ \Omega$m; $100\ \Omega$m; $200\ \Omega$m; $0{,}05\ \Omega$m; $0{,}02\ \Omega$m; $0{,}01\ \Omega$m; $0{,}005\ \Omega$m. Further we have put — for the sake of simplicity — $J = 2\pi^2$ amperes. We have chosen the following values of the parameter t for the numerical computation of integrals: 0,1; 0,2; 0,3; 0,4; 0,5; 0,6; 0,7; 0,8; 0,9; 1,0; 2,0; 3,0; 4,0; 5,0; 6,0. Instead of an infinite sum of integrals we have considered only the sum of the first eight terms. We were able to simplify in this way because the subintegral functions have already for $t = 6$ very small values and the series of thus defined terms converges rapidly. Hence we have introduced into the numerical calculation of integrals arranged in tables for parameters $\varrho_1 = 20\ \Omega$m; $\varrho_2 = 1\ \Omega$m; $a = 2{,}4$ the following approximating equations:

$$\mathcal{U}^*(1{,}2;\ 0;\ 0) \approx -380 \int_0^6 \frac{K_0(1{,}2t)K_0(2{,}4t)I_0(t)I'_0(t)}{20K_0(t)I'_0(t) - I_0(t)K'_0(t)}\, dt -$$

$$- 760 \int_0^6 \sum_{n=1}^{7} \frac{K_n(1{,}2t)K_n(2{,}4t)I_n(t)I'_n(t)}{20K_n(t)I'_n(t) - I_n(t)K'_n(t)}\, dt.$$

For numerical calculation of the given integrals we used the functions: $K_0(x)$; $K_1(x)$; …; $K_7(x)$; $I_0(x)$; $I_1(x)$; …; $I_7(x)$ and their first derivatives $K'_n(x)$; $I'_n(x)$.

The values of the functions $\exp(-x)I_0(x)$; $\exp(-x)I_1(x)$; $\exp x\, K_0(x)$; $\exp x\, K_1(x)$ by means of which we define easily $I_0(x)$; $I_1(x)$; $K_0(x)$; $K_1(x)$ are tabulated in [4] with accuracy to 7 decimal places and with an interpolation error 0,02 in the whole interval $0{,}00 \leqq x \leqq 16{,}00$. For the calculation of the values of the functions of higher orders we have used recurrence formulae:

$$I_{n-1}(x) - \frac{2n}{x} I_n(x) = I_{n+1}(x); \quad K_{n-1}(x) + \frac{2n}{x} K_n(x) = K_{n+1}(x),$$

and we have calculated with all decimal places given in the tables and then we have rounded off the results to 5 decimal places. We don't give the respective tabulation for the sake of brevity. The derivatives of Bessel functions $I'_0(x)$; $I'_1(x)$; …; $I'_7(x)$; $K'_0(x)$; $K'_1(x)$; …; $K'_7(x)$ were to be calculated yet. This was accomplished in an analogical way by means of recurrence formulae:

$$I'_0(x) = I_1(x); \quad I'_n(x) = \tfrac{1}{2}[I_{n-1}(x) - I_{n+1}(x)]$$

$$K'_0(x) = -K_1(x); \quad K'_n(x) = -\tfrac{1}{2}[K_{n-1}(x) + K_{n+1}(x)].$$

Next we have calculated the values of Bessel functions and those of their derivatives for small ($x \leqq 0{,}02$) or for great ($x \geqq 10{,}0$) values respectively by the formulae:

$$\underset{x\to 0}{K_0(x)} \approx \lg \frac{2}{x}; \quad \underset{x\to 0}{K_n(x)} \approx \frac{1}{2}(n-1)!\left(\frac{x}{2}\right)^{-n}$$

$$\underset{x\to 0}{I_0(x)} \approx 1; \quad \underset{x\to 0}{I_n(x)} \approx \frac{1}{n!}\left(\frac{x}{2}\right)^n$$

$$\underset{x\to\infty}{I_n(x)} \approx \frac{\exp x}{\sqrt{2\pi x}}\left[1 + O\left(\frac{1}{x}\right)\right]; \quad \underset{x\to\infty}{K_n(x)} \approx \exp(-x)\sqrt{\frac{\pi}{2x}}\left[1 + O\left(\frac{1}{x}\right)\right].$$

Thus we have obtained all necessary data and then we tabulated for the above chosen parameters. Finally we have evaluated the ratio of the anomalous potential to the potential in the homogeneous half — space and then arranged the results into tab. 1, where we denote by ϱ_1 the resistivity of the half-space and by ϱ_2 the resistivity of the half-cylindrical embedded body.

CONCLUSION

If we take into account the obtained results we may state that the decrease in the values of geoelectrical anomalies with increasing distance of the source electrode and potential electrode for $\varrho_1 > \varrho_2$ is very slow. For $\varrho_1 = 200\ \Omega$m; $\varrho_2 = 1\ \Omega$m the maximal value of the anomaly is about 24 % and its minimal value about 20 %. Though in the case when $\varrho_1 < \varrho_2$ the decrease of the values

Table 1

$\varrho_1 = 1\ \Omega m$	$a = 2,4$	$a = 3,6$	$a = 4,8$	$a = 6,0$
$\varrho_2 = 200{,}000\ \Omega m$	23,9	23,2	22,9	20,3
$\varrho_2 = 100{,}000\ \Omega m$	22,5	21,9	20,0	17,8
$\varrho_2 = 50{,}000\ \Omega m$	20,5	19,2	17,0	14,9
$\varrho_2 = 20{,}000\ \Omega m$	16,4	15,2	13,8	11,3
$\varrho_2 = 0{,}005\ \Omega m$	9,3	7,0	4,8	3,9
$\varrho_2 = 0{,}010\ \Omega m$	9,8	7,7	6,3	5,0
$\varrho_2 = 0{,}020\ \Omega m$	9,6	8,2	6,2	4,7
$\varrho_2 = 0{,}050\ \Omega m$	9,3	8,1	5,9	4,7

of anomalies with increasing distance of the source and potential electrode is greater in this case the maximum anomaly is 10 %, the minimum anomaly is 5 %, but the anomalies are practically not meausurable. Besides we find that for $\varrho_1 < \varrho_2$ the magnitude of anomaly for different resistivities of the half-cylindrical body varies only insignificantly. The results obtained by the above analysis may be summed up as follows:

1. if the resistivity of the half-cylindrical embedded body is — in comparison with resistivity of the surroundings — greater but does not reach tenfold value of the resistivity of the surroundings, we may — with an external source — neglect the influence of the half-cylindrical embedded body.

2. if the resistivity of the half-cylindrical embedded body is smaller — even ten times — we may neglect the influence of the embedded body.

3. in the other cases we must take into account the influence of the half-cylindrical embedded body whereby we must realize that this influence decreases very slowly with increasing distance of source and potential electrodes.

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