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THE REPRESENTATION OF INERTIAL PARTICLES IN THE LIE ALGEBRA OF THE LORENTZ GROUP

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Some Lie properties of the general Lorentz group are investigated and an application of them to the space-time structure of the special relativity theory is given.

All the matrices dealt with are supposed to be real. The Lie group of regular $(n \times n)$-matrices $X \equiv [x^{ik}]$ is denoted by $GL(n, R)$. Let $G \equiv [g_{ab}]$ be a fixed regular diagonal $(n \times n)$-matrix. The matrices X with elements x^{ik} satisfying the $\frac{n(n+1)}{2}$ equations

$$F^{kl} \equiv g_{ab}x^{ak}x^{bl} - g^{kl} = 0; \quad k \leq l \tag{1}$$ (1)

form a subgroup $\mathfrak{G}(G)$ of $GL(n, R)$. This can be easily established observing that (1) is equivalent to $X^*GX = G$, where X^* denotes the transpose of X. In other words, $\mathfrak{G}(G)$ is the general Lorentz group of matrices X which leave invariant the quadratic form

$$\xi \to \xi^* G \xi \tag{2}$$

on R^n. Note that $\mathfrak{G}(E)$ (E the unity matrix) is the orthogonal group $\mathfrak{O}(n)$ and $\mathfrak{G}(L)$, with L, the diagonal matrix of the form

$$\xi_1^2 + \xi_2^2 + \xi_3^2 - c^2\xi_4^2, \quad (c > 0), \tag{3}$$

is the usual full Lorentz group.

Next we shall explicitly show that $\mathfrak{G}(G)$ is a Lie subgroup of $GL(n, R)$ and point out a concrete local chart of $\mathfrak{G}(G)$ containing the unity element E.

Lemma. *The Jacobian of* (1), *i. e.*

(1) Summation over repeated indices. No „geometrical" difference is made between upper and lower indices.

$$\det\left(\frac{\partial F^{kl}}{\partial x^{ij}}\right) \quad \textit{with } k \leq l,\ i \leq j \tag{4}$$

taken at the point E, is non-zero.

Proof. Direct differentiation of (1) gives

$$\frac{\partial F^{kl}}{\partial x^{ij}}(E) = g_{il}\delta_{jk} + g_{ki}\delta_{jl} \tag{5}$$

(δ is the usual Kronecker symbol). This is a matrix of order $\frac{n(n+1)}{2}$ for $i \leq j$, $k \leq l$. Let us suppose there exists a non-trivial system of numbers y^{kl} $(k \leq l)$ satisfying the $\frac{n(n+1)}{2}$ linear equations

$$\sum_{k \leq l} (g_{il}\delta_{jk} + g_{ki}\delta_{jl})\, y^{kl} = 0; \quad i \leq j. \tag{6}$$

If we define $y^{kl} = 0$ for $k > l$ and denote $Y \equiv [y^{kl}]$ then (6) asserts that the matrix $G(Y^* + Y)$ has zero elements on its principal diagonal and above it. Furthermore it is also symmetric and hence $G(Y^* + Y) = 0$ i. e. $Y = 0$. Thus the determinant (4) is necessarily non-zero.

Applying the implicit function theorem one can find a neighbourhood $\mathscr{U}_0$ of the origin $\mathcal{O}$ in R^N $\left(N = \frac{n(n-1)}{2}\right)$, a neighbourhood $\mathscr{U}(E)$ of the matrix E in $\mathfrak{G}(G)$ ($\mathfrak{G}(G)$ provided with the topology induced by the natural topology in R^{n^2}) and a homeomorphism φ_0: $\mathscr{U}(E) \to \mathscr{U}_0$. This φ_0 has the properties:

$$i > j \Rightarrow [\varphi_0^{-1}(x^{ij})]^{kl} = \begin{matrix} x^{kl} \text{ for } k > l \\ h^{kl}(x^{ij}) \text{ for } k \leq l \end{matrix} \tag{7a}$$

where h^{kl} $(k \leq l)$ are the (analytic) functions obtained by „solving the equations (1) with respect to x^{kl} $(k \leq l)$", and

$$\varphi_0(E) = \mathcal{O}. \tag{7b}$$

Thus the pair $(\mathscr{U}(E), \varphi_0)$ defines a local chart on $\mathfrak{G}(G)$. It can be easily shown that the family of charts $(A \,.\, \mathscr{U}(E), \varphi_A)$ for all $A \in \mathfrak{G}(G)$, where $\varphi_A(X) = \varphi_0(A^{-1}X)$, provides $\mathfrak{G}(G)$ with the structure of an analytic submanifold of $GL(n, R)$. Moreover $\mathfrak{G}(G)$ is a topological group with the topology induced by the topology in $GL(n, R)$. Hence it is an N-dimensional Lie subgroup of $GL(n, R)$.

Lemma. *The functions in* (7a) *satisfy the equations*

$$\frac{\partial h^{ab}}{\partial x^{ij}}(\mathcal{O}) = -\, g_{ib}\tilde{g}_{ja}$$

for $a \leqq b,\ i > j$, *with* $\tilde{g}_{ab} = 0$ *for* $a \neq b$, $\tilde{g}_{aa} = \frac{1}{g_{aa}}$ *for* $a = 1, 2, \ldots n$.

Proof. Differentiation of (1) provides

$$\frac{\partial F^{kl}}{\partial x^{ij}}(E) + \sum_{a \leqq b} \frac{\partial F^{kl}}{\partial x^{ab}}(E) \frac{\partial h^{ab}}{\partial x^{ij}}(\mathcal{O}) = 0;\ k \leqq l,\ i > j,$$

i. e. using (5)

(8) $$g_{il}\delta_{jk} + g_{ki}\delta_{jl} + \sum_{a \leqq b} (g_{al}\delta_{bk} + g_{ka}\delta_{bl}) \frac{\partial h^{ab}}{\partial x^{ij}}(\mathcal{O}) = 0.$$

Note here that $g_{ki}\delta_{jl} = 0$ for all $k \leqq l,\ i > j$. Given a fixed pair $(i > j)$, (8) is a system of $\frac{n(n+1)}{2}$ equations possessing a unique solution (cf. the lemma above). Hence it suffices to show

(9) $$\sum_{a \leqq b} (g_{al}\delta_{bk} g_{ib}\tilde{g}_{ja} + g_{ka}\delta_{bl} g_{ib}\tilde{g}_{ja}) = g_{il}\delta_{jk}.$$

This, however, is evident: The first summand in the bracket is zero for each $k \leqq l$ and $a \leqq b$. The second one is non-zero only if $l = i,\ k = j$ with both $a = l,\ b = k$ and its value is g_{il}. The same, of course, is true about the right hand side. Thus the lemma is proved.

The Lie algebra $gL(n, R)$ of $GL(n, R)$ consists of all the $(n \times n)$-matrices and the product is given by $(A, B) \to AB - BA$ (multiplication of matrices). Each $A \in gL(n, R)$ can be written in the vector form

$$A = \sum_{k,l} a^{kl} \frac{\partial}{\partial x^{kl}}(E).$$

Let $\mathfrak{g}(G)$ be the Lie algebra of $\mathfrak{G}(G)$. It is a subalgebra of $gL(n, R)$ and the homeomorphism φ_0 defines a canonical basis

(10) $$U_{ij} = \sum_{k,l} u^{kl}_{ij} \frac{\partial}{\partial x^{kl}}(E), \quad i > j$$

where U_{ij} are the vectors in $\mathfrak{g}(G)$ associated with the coordinates given by the mapping φ_0, i. e.

$$U_{ij}(f)(E) = \frac{\partial (f \cdot \varphi_0^{-1})}{\partial x^{ij}}(\mathcal{O}), \quad i > j$$

for each function f differentiable in a neighbourhood of E in $GL(n, R)$.

Applying (7a) and the preceding lemma one finds

$$U_{ij}(f)(E) = \frac{\partial f}{\partial x^{ij}}(E) + \sum_{k \leqq l} \frac{\partial f}{\partial x^{kl}}(E) \frac{\partial h^{kl}}{\partial x^{ij}}(\mathcal{O}) =$$

$$= \left(\frac{\partial}{\partial x^{ij}} - \sum_{k \leqq l} g_{il}\tilde{g}_{jk} \frac{\partial}{\partial x^{kl}} \right)(f)(E), \quad i > j.$$

Comparison with (10) gives

$u^{kl}_{ij} = \delta^k_i \delta^l_j$ for $k > l$,

$u^{kl}_{ij} = -g_{il}\tilde{g}_{jk}$ for $k \leqq l$.

The elements $T = [t^{ij}] \in \mathfrak{g}(G)$ can be expressed in the form

(11) $$t^{kl} = \sum_{i>j} \vartheta^{ij} u^{kl}_{ij} = \begin{cases} \vartheta^{kl} & \text{for } k > l \\ -\sum_{i>j} g_{il}\tilde{g}_{jk}\vartheta^{ij} & \text{for } k \leqq l. \end{cases}$$

Note that the last expression is zero if $k = l$.

Proposition. *The matrix* $T \in gL(n, R)$ *is an element of the Lie algebra* $\mathfrak{g}(G)$ *if and only if*

(12) $$T^* + GTG^{-1} = O.$$

Proof. Let $T \in \mathfrak{g}(G)$. Then the (a, b) — element of the matrix on the left hand side of (12) is

(13) $$t^{ba} + g_{ai}t^{ij}\tilde{g}_{jb} = t^{ba} + g_{aa}t^{ab}\tilde{g}_{bb}. \quad (^2)$$

If $b > a$ this is equal to

$$\vartheta^{ba} - g_{aa}g_{bb}\tilde{g}_{aa}\vartheta^{ba}\tilde{g}_{bb} = 0.$$

If $b < a$, (13) gives

$$-\, g_{aa}\tilde{g}_{bb}\vartheta^{ab} + g_{aa}\vartheta^{ab}\tilde{g}_{bb} = 0.$$

The case $a = b$ is evident.

Conversely let t^{ab} satisfy (13). A similar consideration yields (11) q. e. d. Each element T of the Lie algebra $gL(n, R)$ generates a one-parameter subgroup $\Gamma_T = \{\Gamma_T(\theta)\}_{\theta \in R}$ of $GL(n, R)$ with $\left[\frac{d}{d\theta}\Gamma(\theta)\right]_{\theta=0} = T$, i.e. $\Gamma_T(\theta) = e^{T\theta}$. Particularly $T \in \mathfrak{g}(G)$ induces $e^{T\theta} \in \mathfrak{G}(G)$ for all $\theta \in R$. The basis $U_{ij}(i > j)$ of $\mathfrak{g}(G)$ generates $\frac{n(n-1)}{2}$ one-parameter subgroups $\Gamma_{ij} = \{e^{U_{ij}\theta}\}_{\theta \in R}$.

(2) No summation applied in the rest of the proof.

The exponential mapping exp: $\mathfrak{g}(G) \to \mathfrak{G}(G)$ given by $\exp T = \Gamma_T(1) = e^T$ provides a homeomomorphism of a neighbourhood of the origin in $\mathfrak{g}(G)$ onto a neighbourhood of E in $\mathfrak{G}(G)$ (cf. [1]).

For the sake of simplicity and physical interpretation we shall restrict our following considerations to the case $G = L$. $\mathfrak{G}^+$ will denote the proper Lorentz group, i. e. the component in $\mathfrak{G}(L)$ containing E. It consists of space and time orientation preserving Lorentz transformations. The matrices $\Gamma_{ij}(\theta)$ can be given now an explicit form. We have $\Gamma_{ij}(\theta) = e^{U_{ij}\theta}$ or, after having solved the corresponding differential equations,

$$\Gamma_{21}(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \text{ or}$$

$$\Gamma_{41}(\theta) = \begin{pmatrix} \cosh\theta/c & 0 & 0 & c\sinh\theta/c \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1/c\,.\,\sinh\theta/c & 0 & 0 & \cosh\theta/c \end{pmatrix}$$

respectively, with similar expresions for $\Gamma_{31}(\theta)$, $\Gamma_{32}(\theta)$, or for $\Gamma_{42}(\theta)$, $\Gamma_{43}(\theta)$ respectively. Hence the one-parameter group Γ_{ij} $(4 > i > j)$ represents all the space rotations in the (i, j)-coordinate plane while Γ_{4j} $(j = 1, 2, 3)$ correspond to parallel frames moving along the j-th axis. (3)

The subgroup of $\mathfrak{G}^+$ consisting of matrices of the type

$$\begin{pmatrix} P_3 & 0 \\ 0 & 1 \end{pmatrix},$$

where P_3 is a (3×3)-orthogonal matrix with det $P_3 > 0$, is denoted by $\mathfrak{D}$. It is clearly a Lie subgroup of $\mathfrak{G}^+$ its Lie algebra being the vector subspace $\mathfrak{r}$ of $\mathfrak{g}(L)$ generated by the vectors U_{21}, U_{31}, U_{32}. The vectors U_{41}, U_{42}, U_{43} generate a vector subspace $\mathfrak{m} \subset \mathfrak{g}(L)$ so that $\mathfrak{g}(L) = \mathfrak{r} \oplus \mathfrak{m}$. Clearly $\mathfrak{r}$ is a subalgebra of $\mathfrak{g}(L)$ but this is not true about $\mathfrak{m}$. Nevertheless there is a (local) homeomorphism of a neighbourhood of the origin in $\mathfrak{m}$ onto a neighbourhood of the unity class in the space $\mathfrak{G}^+/\mathfrak{D}$ of right cosets $\mathfrak{D}.\mathrm{X}$. This homeomorphism is a restriction of the mapping

(14) $$\pi \exp : \mathfrak{m} \to \mathfrak{G}^+/\mathfrak{D},$$

where π is the projection in $\mathfrak{G}^+/\mathfrak{D}$ and $\mathfrak{G}^+/\mathfrak{D}$ is provided with the induced coset topology (cf. [1] Ch. II. Lemma 4.1).

(3) „Parallel" means here always including orientation.

Our next task is to show that in this special case the mapping (14) is a homeomorphism on the whole of $\mathfrak{m}$ onto $\mathfrak{G}^+/\mathfrak{D}$. We shall first prove that (14) is a one-to-one mapping of $\mathfrak{m}$ onto $\mathfrak{G}^+/\mathfrak{D}$.

One can give a physical interpretation to the space $\mathfrak{G}^+/\mathfrak{D}$. The matrices of $\mathfrak{G}^+$ represent inertial observers of the special relativity theory, one observer being pointed out as corresponding to the unity matrix E. We shall call him the original observer. Each coset of $\mathfrak{G}^+/\mathfrak{D}$ represents a class of observers moving with a common 3-velocity vector but their frames (of orthogonal space coordinates) arbitrarily turned. Thus a coset of $\mathfrak{G}^+/\mathfrak{D}$ can be characterized by inertial observers without frames: we shall identify them with inertial material particles and call them simply particles. A right coset of $\mathfrak{G}^+/\mathfrak{D}$ will be called an IP-coset.

An inertial particle can be equipped with a canonical frame — a frame with its axes parallel to those of the original observer. This canonical frame of the particle defines a Lorentz matrix of special kind. Let us call it an IP-matrix. From the intuitive point of view it is quite natural that the correspondence between IP-cosets and IP-matrices is a one-to-one. Nevertheless we shall give a mathematically strict proof of this statement (cf. the proposition bellow).

It is known that each $\mathrm{X} \in \mathfrak{G}^+$ can be written as $\mathrm{X} = P\,.\,S$, where $P \in \mathfrak{D}$ and S is an IP-matrix. Moreover each IP-matrix $(\neq E)$ has the form (cf [2])

$$\begin{pmatrix} E_3 + \dfrac{q-1}{v^2} W_3 & -q\,\mathbf{v}^* \\ -q/c^2\,\mathbf{v} & q \end{pmatrix}$$

with $v = \sqrt{v_1^2 + v_2^2 + v_3^2} < c$; $q = \left(1 - \dfrac{v^2}{c^2}\right)^{-\frac{1}{2}}$; $W_3 \equiv [v_i v_j]$,

where v_1, v_2, v_3 are the components of the velocity vector $\mathbf{v}$ of the particle with respect to the coordinate system of the original observer.

Proposition. *Each IP-coset contains one and only one IP-matrix.*

Proof. As stated above, each coset of $\mathfrak{G}^+/\mathfrak{D}$ contains an IP-matrix. Suppose a coset contains two IP-matrices, i. e. $S_2 = PS_1$ for some IP-matrices S_1, S_2; $P \in \mathfrak{D}$. Then direct calculation gives

(15) $$PS_1 = \begin{pmatrix} P_3 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} E_3 + \dfrac{q-1}{v^2\,\mathbf{v}} W_3 & -q\mathbf{v}^* \\ -q/c^2 & q \end{pmatrix} =$$

$$= \begin{pmatrix} P_3 + \dfrac{q-1}{v^2} P_3 W_3 & -qP_3\,\mathbf{v}^* \\ -q/c^2\mathbf{v} & q \end{pmatrix}$$

and comparing the lower rows of PS_1 and of S_2 one gets $S_1 = S_2$.

Hence there is a one-to-one correspondence between the points of the open ball $v_1^2 + v_2^2 + v_3^2 < c^2$ and the IP-cosets. (15) gives an explicit expression of this correspondence: If $X = [x^{ij}] \in \mathfrak{G}^+$, the triple (v_1, v_2, v_3) corresponding to the class of X is given by

$$v_j = -\frac{c^2 x^{4j}}{x^{44}}. \tag{16}$$

Now one can find the explicit form of the mapping (14) simply by computing the elements of the fourth row of the matrix $\exp T$, $T \in \mathfrak{m}$. For this purpose let $T = t_1 U_{41} + t_2 U_{42} + t_3 U_{43}$. The matrix

$$\exp T\theta = \Gamma_T(\theta) \quad (\theta \in R)$$

is the solution of the system of differential equations

$$\frac{\mathrm{d}}{\mathrm{d}\theta} \Gamma_T(\theta) = \Gamma_T(\theta) \,.\, T$$

with $\Gamma_T(0) = E$. Denoting $\Gamma_T(\theta) = [\gamma_{ik}(\theta)]$ and $t = \sqrt{t_1^2 + t_2^2 + t_3^2}$ one gets

$$\gamma_{4k}(1) = \frac{t_k}{ct} \sinh c\,t; \quad k < 4, \quad t \neq 0$$

$$\gamma_{44}(1) = \cosh c\,t;$$

or, with respect to (16) and $e^O = E$,

$$(17) \qquad \begin{aligned} v_k &= -\frac{c\,t_k}{t} \operatorname{tgh} c\,t && \text{for } t \neq 0 \\ v_k &= 0 && \text{for } t = 0. \end{aligned}$$

This formulae can be inverted in a unique way

$$(18) \qquad \begin{aligned} t_k &= -\frac{v_k}{cv} \operatorname{arccosh} q && \text{for } v \neq 0 \\ t_k &= 0 && \text{for } v = 0 \end{aligned}$$

Thus the mapping (14) is a one-to-one. It is also a homeomorphism as one can easily see from (17) and (18) realizing that the topology in $\mathfrak{G}^+/\mathfrak{D}$ is such that π is continuous and open. We may sum this up in the

СИНОПСИСЫ

Гавел В., *Сопряженные сети аксиального и аксиально-радиального типа относительно конгруэнции канонических прямых постоянного индекса*, Mat.-fyz. časop. *16* (1966), 3—9. (Нем.)

Выведены условия существования конечного числа сопряженных сетей на поверхности со связностью (в смысле А. Швеца), содержащихся в системе аксиальных кривых относительно конгруэнции канонических прямых постоянного индекса. Аналогично разобран аксиально-радиальный случай.

Карпе Р., *Обобщение формулы Хермеса*, Mat.-fyz. časop. *16* (1966), 10—12. (Словацк.; рез. русск.)

Статья содержит обобщение одного известного комбинаторного соотношения.

Драшковичова Г., *Об отношении „между'' в решетках*, Mat.-fyz. časop. *16* (1966), 13—20. (Нем.)

Некоторые свойства отношения „между'' в метрических решетках обобщаются на случай общих решеток. Дана характеристика дедекиндовых и дистрибутивных решеток с нулем с помощью тернарных операций.

Нойбрунн Т., *Затечание об абсолютной непрерывности мер*, Mat.-fyz. časop. *16* (1966), 21—30. (Русск.)

В работе изучаются некоторые вопросы, имеющие связь с абсолютной непрерывностью и с асимптотической абсолютной непрепывностью мер. Доказывается теорема о τ-доминированной системе σ-идеалов.

Бартош П., Кауцки Й., *Об одном способе получения комбинаторных тождеств*, Mat.-fyz. časop. *16* (1966), 31—40. (Англ.)

Статья содержит два доказательства теоремы, с помощью которой дается вывод некоторых комбинаторных тождеств.

Медек В., *Об одной интерпретации конечной аффинной плоскости над полем классов вычетов по модулю p*, Mat.-fyz. časop. *16* (1966), 41—44. (Словацк.; рез. англ.)

Моделью конечной аффинной плоскости служит множество некоторых точек на прямой круговой цилиндрической поверхности; прямыми являются подмножества на ее геодезических линиях.

Мишик Л., *Свойство Дарбу для функций*, Mat.-fyz. časop. *16* (1966), 45—52. (Нем.)

В работе дается доказательство эквивалентности двух определений свойства Дарбу, теорема о свойстве Дарбу для функций многих переменных со свойством Дарбу для каждой переменной отдельно и теорема о свойстве Дарбу частной производной $\frac{\partial f}{\partial x}$ функции $f(x, y)$.

SYNOPSES

Havel V., *Conjugate nets of axial and axially-radial type associated to any congruence of canonical lines with fixed index*, Mat.-fyz. časop. *16* (1966), 3—9. (German.)
The author deduces the conditions for the existence of the finite number of conjugate nets on any surface with connection (in the sense of A. Švec) which are contained in the system of axial curves associated to any congruence of canonical lines with a fixed index. A similar investigation is made for the axially-radial case.

Karpe R., *A generalisation of Hermes' formula*, Mat.-fyz. časop. *16* (1966), 10—12. (Slovak. Russian summary.)
The paper contains a generalisation of a known combinatorial relation.

Draškovičová H., *Some properties of a betweenness relation*, Mat.-fyz. časop. *16* (1966), 13—20. (German.)
Some properties of a betweennes relation in metric lattices are extended to the case of general lattices. Characterization of modular and distributive lattices with the least element by ternary operations is given.

Neubrunn T., *A note on absolute continuity of measures*, Mat.-fyz. časop. *16* (1966), 21—30. (Russian.)
In the paper some questions related to the absolute continuity and to the asymptotic absolute continuity of measures are studied. A theorem on τ-dominated system of σ-ideals is proved.

Bartoš P., Kaucký J., *A genesis for combinatorial identities*, Mat.-fyz. časop. *16* (1966), 31—40. (English.)
The article contains two proofs of a theorem by means of which some combinatorial identities are derived.

Medek V., *On an interpretation of the finite affine plane over the field of the residue classes modulo p*, Mat.-fyz. časop. *16* (1966), 41—44. (Slovak, English summary.)
The model of the finite affine plane is a set of points on a cylindrical surface of revolution; the lines are subsets on the geodesics of this surface.

Mišík L., *On the Darboux property for functions*, Mat.-fyz. časop. *16* (1966), 45—52. (German.)
The present paper brings a proof of the equivalence of two definitions of the Darboux property, a theorem on the Darboux property for functions of several variables having the Darboux property for each variable separately and a theorem on the Darboux property of $\frac{\partial f}{\partial x}$ for the function $f(x, y)$.

Drs L., *Conjugate parallel projections*, Mat.-fyz. časop. *16* (1966), 53—57. (Czech, Russian summary.)
In the article the construction of the parallel projection U_{02}, U_2 of the figure U is shown, using the given parallel projections U_{01}, U_1. Both corresponding processes of projection are determined as well.

Gruska J., *Two operations with formal languages and their influence upon structural unambiguity*, Mat.-fyz. časop. *16* (1966), 58—65. (English.)
In this paper it is proved that the extension and, under certain easily verified assumptions, even reduction have no influence upon structural unambiguity.

Zelinka B., *Introducing an orientation into a given non-directed graph*, Mat.-fyz. časop. *16* (1966), 66—71. (English.)
In this paper the subsets X (or Y) of the vertex set of a non-directed graph G are characterized, which arise from an orientation of G as the set of those vertices at which there is no incoming (resp. outgoing) edge. This is a solution of a problem of A. Kotzig.

Kotzig A., *On even regular graphs of the third degree*, Mat.-fyz. časop. *16* (1966), 72—75. (English.)
It is proved in the paper that any even regular graph of the third degree can be constructed from a graph each component of which is isomorphic to Kuratowski's graph of the third degree by repetitions of a special kind of transformation („extension").

Kluvánek I., *Contribution to the theory of vector measures II*, Mat.-fyz. časop. *16* (1966), 76—81. (Russian.)
Necessary and sufficient conditions are given for the existence on a δ-ring $\mathbf{T}$ or a σ-ring $\mathbf{S}$ of a measure with values in a linear topological locally convex space X which coincides with a given measure with values in X on a ring generating $\mathbf{T}$ or $\mathbf{S}$, respectively.

Virsik J., *The representation of inertial particles in the Lie algebra of the Lorentz group*, Mat.-fyz. časop. *16* (1966), 82—90. (English.)
The explicite form of the Lie algebra $\mathfrak{g}(L)$ of the proper Lorentz group $\mathfrak{G}^+$ is derived and it is shown that the map π . exp (cf. (14)) is a homeomorphism (on the whole) of a certain linear subspace $\mathfrak{m}$ of $\mathfrak{g}(L)$ onto the space of right cosets $\mathfrak{G}^+/\mathfrak{D}$. Since under given conditions one can associate with each element of $\mathfrak{G}^+/\mathfrak{D}$ an „inertial particle", this homeomorphism generates a representation of inertial particles in $\mathfrak{m} \subset \mathfrak{g}(L)$. A physical interpretation of the analytic vector field on $\mathfrak{G}^+$ associated with an element of $\mathfrak{m}$ is given.

Šáró Š., *The selfacting coincidence of the spark counters*, Mat.-fyz. časop. *13* (1966), 91—96. (Slovak, English summary.)
Provisional communication. In this paper the principle and the description of the arrangement and a case of a treefold and fourfold selfacting coincidence of the Rosenblum type spark counters are described. The resolving time moves from 10^{-4} s to 10^{-8} s according to the parameters of the device.

Дрс Л., *Сопряженные параллельные проекции*, Mat.-fyz. časop. *16* (1966), 53—57. (Чешск.; рез. русск.)

В статье дается конструкция параллельной проекции U_{02}, U_2 фигуры U из данной проекции U_{01}, U_1 и определение обоих параллельных проектирований.

Груска Й., *Две операции с формальными языками и их влияние на структурную однозначность*, Mat.-fyz. časop. *16* (1966), 58—65. (Англ.)

В статье доказывается, что операция расширения языка и, при некоторых условиях, операция сокращения не имеют влияние на структурную однозначность.

Зелинка Б., *Введение направления в заданном ненаправленном графе*, Mat.-fyz. časop. *16* (1966), 66—71. (Англ.)

В статье характеризованы подмножества X и Y множества вершин ненаправленного графа G, являющиеся — после введения некоторой ориентации G — множеством соответственно вершин, в которые не входит и вершин из которых не выходит ни одно ребро. Статья является решением одной задачи А. Коцига.

Коциг А., *О четных регулярных графах третьей степени*, Mat.-fyz. časop. *16* (1966), 72—75. (Англ.)

В работе доказывается, что всякий четный регулярный граф третьей степени можно построить из графа, каждая компонента которого изоморфна графу Куратовского третьей степени, путем повторения одного единственного вида преобразования (= расширения).

Клуванек И., *К теории векторных мер II*, Mat.-fyz. časop. *16* (1966), 76—81. (Русск.)

Приводятся некоторые необходимые и достаточные условия для того, чтобы меру со значением из линейного топологического локально выпуклого пространства X, заданную на некотором кольце множеств, можно было расширить на порожденное им δ-кольцо или σ-кольцо и чтобы значения этого расширения тоже принадлежали X.

Вирсик Ю., *Представление инерциальных частиц в алгебре Ли группы Лоренца*, Mat.-fyz. časop. *16* (1966), 82—90. (Англ.)

Приводится явный вид алгебры Ли $\mathfrak{g}(L)$ собственной группы Лоренца $\mathfrak{G}^+$. Показывается, что отображение $\pi.\exp$ (см. (14)) является гомеоморфизмом (в целом) определенного линейного подпространства $\mathfrak{m}$ в $\mathfrak{g}(L)$ на пространство правых смежных классов $\mathfrak{G}^+/\mathfrak{D}$. Так как при определенных условиях возможно сопоставить каждому элементу из $\mathfrak{G}^+/\mathfrak{D}$ „инерциальную частицу", этот гомеоморфизм порождает представление инерциальных частиц в $\mathfrak{m} \subset \mathfrak{g}(L)$. Дается физическая интерпретация (аналитического) векторного поля на $\mathfrak{G}^+$, связанного с вектором из $\mathfrak{m}$.

Шаро Ш., *Автосовпадение у искровых счетчиков*, Mat.-fyz. časop. *16* (1966), 91—96. (Словацк.; рез. англ.)

Предварительное сообщение. Приведен принцип и описание установки и случай трехкратного и четырехкратного автосовпадения искровых счетчиков с неоднородным полем. Разрешающее время меняется с параметрами установки от 10^{-4} s до 10^{-8} s.

Theorem. *The mapping* $\pi\,.\exp$ *is a homeomorphism of the linear subspace* $\mathfrak{m} \subset \mathfrak{g}(L)$ *onto the space* $\mathfrak{G}^+/\mathfrak{D}$ *of IP-cosets. This homeomorphism is given by* (17) *resp.* (18) *and maps IP-cosets corresponding to particles moving along the k-th axis onto vectors in* $\mathfrak{m}$ *colinear with* U_{4k}. *Moreover it represents the family of particles moving in a given direction as a subspace of colinear vectors in* $\mathfrak{m}$.

Note that in our considerations the inertial particle is completely characterized by its 3-velocity vector and no attention is payed to its position say in the zero moment of the original observer. So we can always suppose the particle passing the origin of the original observer (and also of the others) at this moment.

Up to this time we have used the one-to-one correspondence between particles and IP-cosets provided all the measurings have been made with respect to the original observer. If $\mathbf{p}$ denotes the particle in view and $h(\mathbf{p}, E)$ the corresponding coset of $\mathfrak{G}^+/\mathfrak{D}$ then $h(\mathbf{p}, E)$ is given by the triple (v_1, v_2, v_3) describing the 3-velocity vector components of the particle from the point of view of the original observer. Calculating the velocity vector with respect to another observer, say given by the matrix $X_0 \in \mathfrak{G}^+$, one obtains in general an other triple (v_1', v_2', v_3') defining an another IP-coset. In order to get explicitly this new triple it suffices to calculate the lower row in the matrix YX_0^{-1}, where Y is an arbitrary matrix of the IP-coset given by the triple (v_1, v_2, v_3). Formally it can be shown that the homeomorphism (14) defines a unique analytic structure on $\mathfrak{G}^+/\mathfrak{D}$ with the property that $\mathfrak{G}^+$ is a Lie transformation group of $\mathfrak{G}^+/\mathfrak{D}$ (cf. [1] Th. 4.2).

We may connect with each particle $\mathbf{p}$ and each observer given by $X_0 \in \mathfrak{G}$ an IP-coset $h(\mathbf{p}, X_0)$ defined by

$$h(\mathbf{p}, X_0) = h(\mathbf{p}, E)\,.\,X_0^{-1}.$$

In accordance with the considerations above the triple (v_1, v_2, v_3) corresponding to the IP-coset $h(\mathbf{p}, X_0)$ is nothing else but the 3-velocity components of the particle with respect to the observer represented by the matrix X_0.

On the other hand the linear subspace $\mathfrak{m} \subset \mathfrak{g}(L)$ may be considered as a linear space of right invariant vector fields on $\mathfrak{G}^+$. Hence there is a canonical one-to-one correspondence $T \to \hat{X}_0(T)$ between the vectors of $\mathfrak{m}$ and the vectors of a linear subspace $\mathfrak{m}(X_0)$ of the tangent space to $\mathfrak{G}^+$ at X_0. Let $\log\colon \mathfrak{G}^+/\mathfrak{D} \to \mathfrak{m}$ denote the inverse of the homeomorphism (14). Given a fixed particle $\mathbf{p}$ one can define a continuous vector field on $\mathfrak{G}^+$ by

$$X_0 \to F_{\mathbf{p}}(X_0) = (\hat{X}_0 \log)\, h(\mathbf{p}, X_0).$$

It is not difficult to see that this is even an analytic vector field on $\mathfrak{G}^+$. The field $X_0 \to F_{\mathbf{p}}(X_0)$ is uniquely defined by $F_{\mathbf{p}}(E) = \log h(\mathbf{p}, E)$ and for a fixed $X_0 \in \mathfrak{G}^+$ the correspondence $\mathbf{p} \to F_{\mathbf{p}}(X_0)$ is a one-to-one. The physical meaning of this field can be found in the following: Given $F_{\mathbf{p}}(X_0)$ one calculates its components t_k with respect to the basis $\hat{X}_0$ (U_{4k}) ($k = 1, 2, 3$), uses (17) and gets the components of the 3-velocity of the particle measured by the observer connected with the matrix X_0. In particular $F_{\mathbf{p}}(X_0) = 0$ means that the particle $\mathbf{p}$ is in rest with respect to X_0.

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